Vedantu
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Revision Notes for Class 12 Physics

Chapter 1 – Electric Charges and Fields

1. Electric Charge

1.1 Definition- Charge is that property that is associated with the matter due to which it produces and experiences electrical and magnetic effects.

1.2 Type

There exist two types of charges in nature. They are:

i. Positive charge

ii. Negative charge

Charges with the same electrical sign repel each other while charges with opposite electrical signs attract each other.

1.3 Unit and Dimensional Formula

S.I. unit of charge is coulomb (C), $\left(1\text{mC} = 10^{-3}\text{C}, 1\mu\text{C} = 10^{-6}\text{C}, \ln\text{C} = 10^{-9}\text{C}\right)$

C.G.S. the unit of charge is e.s.u. $1\text{C} = 3\times10^{9}$ esu

The Dimensional formula is given by $[\text{Q}] = [\text{AT}]$.

1.4 Point Charge

Whose spatial size is negligible as compared to other distances.

1.5 Properties of Charge

(i) Charge is a Scalar Quantity: Charges can be added or subtracted algebraically.

(ii) Charge is transferable: When a charged body is put in contact with an uncharged body, the uncharged body becomes charged due to transfer of electrons from the charged body to the uncharged body.

(iii) Charge is always associated with mass: Charge cannot exist without mass though mass can exist without charge.

(iv) Charge is conserved: Charge can neither be created nor be destroyed.

(v) Invariance of charge: The numerical value of an elementary charge is independent of velocity.

(vi) Charge produces an electric field and magnetic field: When a charged particle is at rest it only produces an electric field in the space surrounding it. However, if the charged particle is in unaccelerated motion it produces both electric and magnetic fields. And if the motion of the charged particle is accelerated it not only produces electric and magnetic fields but also radiates energy in the space surrounding the charge in the form of electromagnetic waves.

(vii) Charge resides on the surface of conductor: Charge resides on the outer surface of a conductor because like charges repel and try to get as far away as possible from one another and stay at the farthest distance from each other which is the outer surface of the conductor. Therefore, a solid and hollow conducting sphere of the same outer radius will hold a maximum equal charge, and a soap bubble expands on charging.

(viii) Quantization of charge: When a physical quantity can have only discrete values rather than any value, the quantity is said to be quantised. The smallest charge that can exist in nature is the charge of an electron. If the charge of an electron $\left(-1.6\times10^{-19}\mathrm{C}\right)$ is taken as elementary unit i.e. quanta of charge the charge on anybody will be some integral multiple of e i.e., $\mathrm{Q}= \pm ne$ with $\mathrm{n}=0,\ 1,2,3\ldots\ldots$

Charge on a body can never be $0.5e, \pm17.2e$ or $\pm10^{-5}e$ etc.

1.6 Comparison of Charge and Mass

We are familiar with the role of mass in gravitation, and we have just studied some features of electric charge. The comparison between the two are as follows:

S. No	**Charge**	**Mass**
1.	Electric charge can be positive, negative or zero.	Mass of a body is a positive quantity.
2.	Charge carried by a body does not depend upon the velocity of the body.	Mass of a body increases with its velocity as $m = \frac{m_o}{\sqrt{1-\frac{v^2}{c^2}}}$ where c id velocity of light in vacuum, m is the mass of the velocity v and m_0 is rest mass of the body.
3.	The charge is quantized.	The quantization of mass is yet to be established.

4.	Electric charge is always conserved.	Mass is not conserved as it can be changed into energy and vice-versa.
5.	Force between charges can be attractive(unlike charges) or repulsive(like charges) in nature.	The gravitational force between two masses is always attractive.

1.7 Methods of Charging

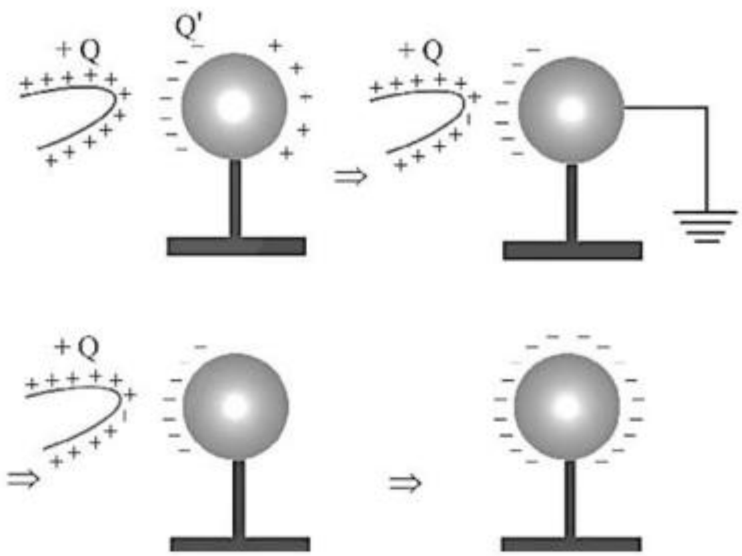
A body can be charged by following methods:

i. By friction:

In friction when two bodies are rubbed together, electrons are transferred from one body to the other. As a result of this one body becomes positively charged while the other is negatively charged, e.g., when a glass rod is rubbed with silk, the rod becomes positively charged while the silk becomes negatively charged. However, ebonite on rubbing with wool becomes negatively charged making the wool positively charged. Clouds also become charged by friction. In charging by friction in accordance with conservation of charge, both positive and negative charges in equal amounts appear simultaneously due to the transfer of electrons from one body to the other.

ii. By electrostatic induction:

If a charged body is brought near an uncharged body, the charged body will attract the opposite charge and repel a similar charge present in the uncharged body. As a result of this one side of the neutral body (closer to charged body) becomes oppositely charged while the other is similarly charged. This process is called electrostatic induction.



Note: Inducting body neither gains nor loses charge.

(iii) Charging by conduction:

Take two conductors, one charged and the other uncharged. Bring the conductors in contact with each other. The charge (whether negative or positive) under its own repulsion will spread over both the conductors. Thus, the conductors will be charged with the same sign. This is called as charging by conduction (through contact).

Note: A truck carrying explosives has a metal chain touching the ground, to conduct away the charge produced by friction.

2. Coulomb's Law

If two stationary and point charges Q_1 and Q_2 are kept at a distance r, then it is found that force of attraction or repulsion between them is Mathematically, Coulomb's law can be written as

$$F = k\frac{q_1 q_2}{r^2}$$

where k is a proportionality constant.

In SI units k has the value,

$$k = 8.988 \times 10^9 \, Nm^2 / C^2$$

$$k = 9.0 \times 10^9 \, Nm^2 / C^2$$

(a) The direction of force is always along the line joining the two charges.

(b) The force is repulsive if the charges have the same sign and attractive if their signs are opposite.

(c) This force is conservative in nature.

(d) This is also called inverse square law.

2.1 Variation of k

Constant k depends upon a system of units and medium between the two charges.

a. In C.GS. for air

$$k = 1, \, F = \frac{Q_1 Q_2}{r^2} \text{ Dyne}$$

b. In S.I. for air $k = \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \frac{N - m^2}{C^2}$,

$$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q_1 Q_2}{r^2} \text{ Newton } (1 \text{ Newton } = 10^5 \text{ Dyne})$$

Note:

- $\varepsilon_0 =$ Absolute permittivity of air or free space

$$=8.85\times10^{-12}\frac{C^2}{N-m^2}\left(=\frac{\text{Farad}}{m}\right)$$

Dimension is $\left[M^{-1}L^{-3}T^{4}A^{2}\right]$

- ε_0 Relates with absolute magnetic permeability (μ_0) and velocity of light (c) according to the following relation $c=\frac{1}{\sqrt{\mu_0\varepsilon_0}}$

2.1.2 Effect of Medium

(a) When a dielectric medium is completely filled in between charges rearrangement of the charges inside the dielectric medium takes place and the force between the same two charges decreases by a factor of K known as dielectric constant, K is also called relative permittivity ε_r of the medium (relative means with respect to free space).

Hence in the presence of medium

$$F_m=\frac{F_{\text{air}}}{K}=\frac{1}{4\pi\varepsilon_0 K}\cdot\frac{Q_1Q_2}{r^2}$$

Here $\varepsilon_0\,\mathrm{K}=\varepsilon_0\varepsilon_\mathrm{r}=\varepsilon$ (permittivity of medium)

Medium	K
Vacuum/air	1

Water	80
Mica	6
Glass	5-10
Metal	∞

2.2 Vector Form of Coulomb's Law

It is helpful to adopt a convention for subscript notation.

$F_{12} =$ force on 1 due to 2 $\quad F_{21} =$ force on 2 due to 1D

Suppose the position vectors of two charges q_1 and q_2 are $\vec{r}_1$ and $\vec{r}_2$, then, electric force on charge q_1 due to charge q_2 is,

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{\left|\vec{r}_1 - \vec{r}_2\right|^3}\left(\vec{r}_1 - \vec{r}_2\right)$$

Similarly, electric force on q_2 due to charge q_1 is

$$\vec{F}_{21} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{\left|\vec{r}_2 - \vec{r}_1\right|^3}\left(\vec{r}_2 - \vec{r}_1\right)$$

Force is a vector, so in vector form the Coulomb's law is written as

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}\hat{r}_{12}$$

Where $\hat{r}_{12}$ is a unit vector, directed toward q_1 from q_2.

Note:

$$\hat{r}_{12} = -\hat{r}_{21}$$

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}\hat{r}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_2q_1}{r^2}\left(-\hat{r}_{21}\right)$$

$$\vec{F}_{12} = -\frac{1}{4\pi\varepsilon_0}\frac{q_2q_1}{r^2}\hat{r}_{21} = -\vec{F}_{21}$$

Remember convention for $\hat{r}$.

Here q_1 and q_2 are to be substituted with sign. Position vector of charges q_1 and q_2 are $\vec{r}_1 = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$ and $\vec{r}_2 = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$ respectively. Where (x_1, y_1, z_1) and (x_2, y_2, z_2) are the co-ordinates of charges q_1 and q_2.

2.3 Principle of Superposition

According to the principle of superposition, the total force acting on a given charge due to a number of charges is the vector sum of the individual forces acting on that charge due to all the charges.

Consider number of charges Q_1, Q_2, Q_3.....are applying force on a charge Q

Net force on Q will be

$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \dots\dots\dots + \vec{F}_{n-1} + \vec{F}_n$$

The magnitude of the resultant of two electric force is given by $F = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$ and the force direction is given by,

$$\tan\alpha = \frac{F_2 \sin\theta}{F_1 + F_2 \cos\theta}$$

3. Electric Field

A positive charge or a negative charge is said to create its field around itself. Thus space around a charge in which another charged particle experiences a force is said to have an electrical field in it.

3.1 Electric Field Intensity $\left(\vec{\mathrm{E}}\right)$

The electric field intensity at any point is defined as the force experienced by a unit positive charge placed at that point.

$$\vec{E} = \frac{\vec{F}}{q_0}$$

Where $q_0 \to 0$ so that presence of this charge may not affect the source charge Q and its electric field is not changed, therefore expression for electric field intensity can be better written as:

$$\vec{E} = \lim_{q_0 \to 0} \frac{\vec{F}}{q_0}$$

(a) Unit and Dimensional formula:

It's S.I. unit is,

$\frac{\text{Newton}}{\text{coulomb}} = \frac{\text{volt}}{\text{meter}} = \frac{\text{Joule}}{\text{coulomb} \times \text{meter}}$ and

C.G.S. unit $=$ Dyne/stat coulomb.

Dimension: $[E]=\left[MLT^{-3}A^{-1}\right]$

(b) Direction of electric field: Electric field (intensity) $\vec{E}$ is a vector quantity. Electric field due to a positive charge is always away from the charge and that due to a negative charge is always towards the charge.

3.2 Relation Between Electric Force and Electric Field

In an electric field $\vec{E}$ a charge (Q) experiences a force $F=QE$. If the charge is positive then force is directed in the direction of the field while if the charge is negative force acts on it in the opposite direction of field.

3.3 Superposition of Electric Field

The resultant electric field at any point is equal to the vector sum of electric fields at that point due to various charges.

$$\vec{E}=\vec{E}_1+\vec{E}_2+\vec{E}_3+\ldots$$

The magnitude of the resultant of two electric fields are given by

$E=\sqrt{E_1^2+E_2^2+2E_1E_2\cos\theta}$ and the direction is given by,

$$\tan\alpha=\frac{E_2\sin\theta}{E_1+E_2\cos\theta}$$

3.4 Point Charge

Point charge produces its electric field at a point P which is distance r from it given by,

$$E_P=\frac{Q}{4\pi\varepsilon_0 r^2}\ \text{(Magnitude)}$$

For positive point charge, E is directed away from it.

For negative point charge, E is directed towards it.

3.5 Continuous Charge Distributions

There is an infinite number of ways in which we can spread a continuous charge distribution over a region of space. Mainly three types of charge distributions will be used. We define three different charge densities.

Symbol	**Definition**	**SI units**
(lambda) λ	Charge per unit length	C/m
(sigma) σ	Charge per unit area	C/m^2
(rho) ρ	Charge per unit volume	C/m^3

If a total charge q is distributed along a line of length ℓ, over a surface area A or throughout a volume V, we can calculate charge densities from.

$$\lambda = \frac{q}{\ell}, \sigma = \frac{q}{A}, \rho = \frac{q}{V}$$

3.6 Properties of Electric Field Lines

1. Electric field lines originate from a positive charge & terminate on a negative charge.

2. The number of field lines originating/terminating on a charge is proportional to the magnitude of the charge.

3. The number of Field Lines passing through the perpendicular unit area will be proportional to the magnitude of the Electric Field there.

4. Tangent to a Field line at any point gives the direction of the Electric Field at that point. This will be the instantaneous path charge will take if kept there.

5. Two or more field lines can never intersect each other.

(they cannot have multiple directions)

6. Uniform field lines are straight, parallel & uniformly placed.

7. Field lines cannot form a loop.

8. Electric field lines originate and terminate perpendicular to the surface of the conductor. Electric field lines do not exist inside a conductor.

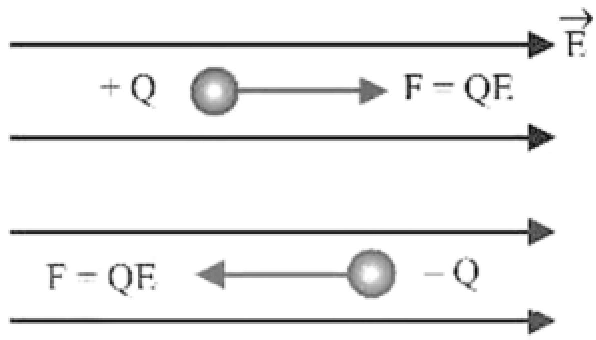
9. Field lines always flow from higher potential to lower potential.

10. If in a region electric field is absent, there will be no field lines.

3.7 Motion of Charged Particle in an Electric Field

(a) When charged particle initially at rest is placed in the uniform field:

Let a charge particle of mass m and charge Q be initially at rest in an electric field of strength E.



(i) Force and acceleration:

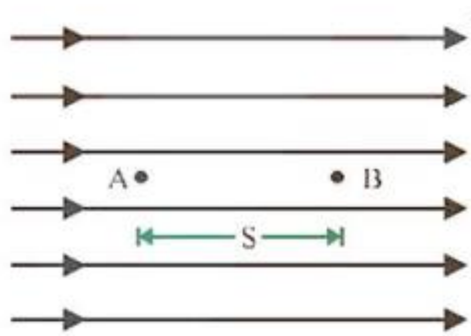
The force experienced by the charged particle is $F = QE$. Positive charge experiences force in the direction of electric field while negative charge experiences force in the direction opposite to the field. [Fig. (A)]

Acceleration produced by this force is $a = \frac{F}{m} = \frac{QE}{m}$

Since the field E in constant the acceleration is constant, thus motion of the particle is uniformly accelerated.

(ii) Velocity:

Suppose at point A particle is at rest and in time t, it reaches the point B



$V =$ Potential difference between A and B

$S =$ Separation between A and B

(a) By using

$v = u + at, \quad v = 0 + Q\frac{E}{m}$

$t \Rightarrow v = \frac{QEt}{m}$

(b) By using,

$v^2 = u^2 + 2as, v^2 = 0 + 2 \times \frac{QE}{m} \times s = \sqrt{\frac{2QEs}{m}}$

(iii) Momentum:

Momentum $p = mv$,

$p = m \times \frac{QEt}{m} = QEt$

(iv) Kinetic energy:

Kinetic energy gained by the particle in time t is

$K = \frac{1}{2}mv^2 = \frac{1}{2}m\frac{(QEt)^2}{m} = \frac{Q^2E^2t^2}{2\,m}$

(b) When a charged particle enters with an initial velocity at a right angle to the uniform field.

When a charged particle enters perpendicularly in an electric field, it describes a parabolic path as shown.

(i) Equation of trajectory:

Throughout the motion, a particle has uniform velocity along x-axis and horizontal displacement (x) is given by the equation $x = ut$

Since the motion of the particle is accelerated along y-axis, we will use the equation of motion for uniform acceleration to determine displacement y.

From $S = ut + \frac{1}{2}at^2$

We have $u = 0$ (along $y-$axis) so $y = \frac{1}{2}at^2$

i.e., displacement along y-axis will increase rapidly with time (since $y \propto t^2$)

From displacement along x-axis, $t = x/u$

So $y = \frac{1}{2}\left(\frac{QE}{m}\right)\left(\frac{x}{u}\right)^2$; this is the equation of parabola which shows $y \propto x^2$.

(ii) Velocity at any instant:

At any instant $t, v_x = u$ and $v_y = \frac{QEt}{m}$.

So,

$$v = |\vec{v}| = \sqrt{v_x^2 + v_y^2} = \sqrt{u^2 + \frac{Q^2E^2t^2}{m^2}}$$

If β is the angle made by v with x-axis than

$$\tan\beta = \frac{v_y}{v_x} = \frac{QEt}{mu}.$$

4. Electric Dipole

4.1 General Information

A system of two equal and opposite charges separated by a small, fixed distance is called a dipole.

(i) Dipole axis: Line joining negative charge to positive charge of a dipole is called its axis. It may also be termed as its longitudinal axis.

(ii) Equatorial axis: Perpendicular bisector of the dipole is called its equatorial or transverse axis as it is perpendicular to the length.

(iii). Dipole length: The distance between two charges is known as dipole length (d).

(iv). Dipole moment: It is a quantity that gives information about the strength of dipole. It is a vector quantity and is directed from negative charge to positive charge along the axis. It is denoted as p and is defined as the product of the magnitude of either of the charge and the dipole length.

i.e., $\vec{p} = q(\vec{d})$

Its S.I. unit is **coulomb-metre** or **Debye** (1 Debye = $3.3\times10^{-30} C\times m$) and its dimensions are $M^0L^1T^1A^1$.

Note:

- A region surrounding a stationary electric dipole has electric field only.
- When a dielectric is placed in an electric field, its atoms or molecules are considered as tiny dipoles.

(a) Electric Potential due to a dipole

$$V_P = \frac{k(-q)}{AP} + \frac{k(+q)}{BP}$$

$r >> d$ (distance ' r ' is large as compared to d)

$AP \approx O''P; \quad BP \approx O'P$

$O''P = r + d/2\cos\theta, \quad OP = r - d/2\cos\theta$

$$V_p = \frac{k(-q)}{(r + d/2\cos\theta)} + \frac{k(+q)}{(r - d/2\cos\theta)}$$

$$= k(+q)\left[\frac{1}{r - d/2\cos\theta} - \frac{1}{r + d/2\cos\theta}\right]$$

$$= kq\left[\frac{r + d/2\cos\theta - r + d/2\cos\theta}{r^2 - \frac{d^2}{4}\cos^2\theta}\right] = \frac{kqd\cos\theta}{r^2 - \frac{d^2}{4}\cos^2\theta}$$

$$V_p = \frac{k(qd)\cos\theta}{r^2 - \frac{d^2}{4}\cos^2\theta} = \frac{kp\cos\theta}{r^2 - \frac{d^2}{4}\cos^2\theta} [\because p = qd]$$

since $r >> d$

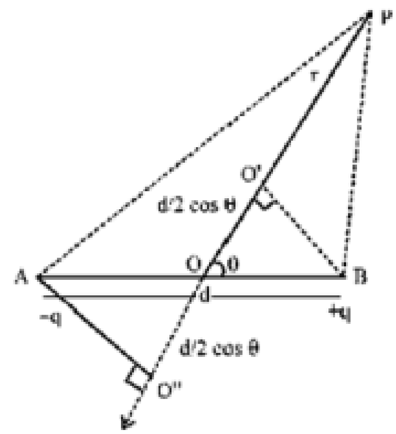
$$V_p = \frac{kp\cos\theta}{r^2} = \frac{1}{4\pi \in_0}\frac{p\cos\theta}{r^2}$$

θ is angle with the axis of dipole; r is distance from centre of dipole.

(b) Electric Field due to dipole

(i) For points on the axis

Let the point P be at distance r from the centre of the dipole on the side of the charge q ,as shown in the below figure.



Then

$$E_{-q} = -\frac{q}{4\pi z_0 (r+a)^2}\hat{p}$$

where p is the unit vector along the dipole axis (from $-q$ to q). Also, $\mathrm{E}_{+q} = \frac{\mathrm{q}}{4\pi\varepsilon_0 (\mathrm{r}-\mathrm{a})^2}\mathrm{p}$

The total field at P is,

$$E = E_{+q} + E_{-q} = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2}\right]\hat{p} = \frac{q}{4\pi\varepsilon_0}\frac{4ar}{\left(r^2-a^2\right)^2}\hat{p}$$

For $r >> a$

$$E = \frac{4qa}{4\pi\varepsilon_0 r^3}\hat{p} \quad (r > > a) \ldots \text{(i)}$$

(ii) For points on the equatorial plane

The magnitudes of the electric fields due to the two charges $+\mathrm{q}$ and $-\mathrm{q}$ are given by,

$$E_{+q} = \frac{q}{4\pi\varepsilon_0} \frac{1}{r^2 + a^2}$$

$$E_{-q} = \frac{q}{4\pi\varepsilon_0} \frac{1}{r^2 + a^2}$$

and are equal.

The directions of E_{+q} and E_{-q} are as shown in fig. (b). Clearly, the components normal to the dipole axis cancel away. The components along the dipole axis add up. The total electric field is opposite to p. We have,

$$\mathrm{E} = -\left(\mathrm{E}_{+q} + \mathrm{E}_{-\alpha}\right)\cos\theta \mathrm{p}$$

$$= -\frac{2\mathrm{qa}}{4\pi\varepsilon_0\left(\mathrm{r}^2 + \mathrm{a}^2\right)^{3/2}} \mathrm{p}$$

At large distances $(r > > a)$, this reduces to

$$E = -\frac{2qa}{4\pi\varepsilon_0 r^3} \hat{p}_{(r>>a)} \ldots \text{(ii)}$$

From Eqs. (i) and (ii), it is clear that the dipole field at large distances does not involve q and a separately; it depends on the product **\$qa\$**. This suggests the definition of dipole is defined by

$$\mathrm{P} = \mathrm{q} \times 2\mathrm{a}\mathrm{p}$$

that is, it is a vector whose magnitude is charge q times the separation **2a (between the pair of charges q**, **-** q) and the direction is along the line from $-q$ to q.

In terms of p, the electric field of a dipole at large distances takes simple forms:

At a point on the dipole axis

$$E=\frac{2p}{4\pi\varepsilon_0 r^3}=\frac{2kp}{r^3}(r>>a)$$

At a point on the equatorial plane.

$$\mathrm{E}=\frac{\mathrm{p}}{4\pi\varepsilon_0 \mathrm{r}^3}=\frac{-\mathrm{kp}}{\mathrm{r}^3}(\mathrm{r}>>\mathrm{a})$$

4.3 Electric Dipole in a Uniform Electric Field

(i) Force and Torque: If a dipole is placed in a uniform field such that dipole (i.e. $\vec{p}$) makes an angle θ with the direction of field then two equal and opposite forces acting on dipole constitute a couple whose tendency is to rotate the dipole hence torque is developed in it and dipole tries to align itself in the direction of the field. Consider an electric dipole in placed in a uniform electric field such that dipole (i.e., $\vec{p}$) makes an angle θ with the direction of the electric field as shown.

(a) Net force on electric dipole $\mathrm{F}_{\mathrm{net}}=0$

(b) …… $\tau=\mathrm{pE}\sin\theta(\vec{\tau}=\vec{\mathrm{p}}\times\vec{\mathrm{E}})$

(ii) Work: From the above discussion it is clear that in an uniform electric field dipole tries to align itself in the direction of electric field (i.c. equilibrium position). To change it's angular position some work has to be done.

Suppose an electric dipole is kept in an uniform electric field by making an angle θ_1 with the field, if it is again turn so that it makes an angle θ_2 with the field, work done in this process is given by the formula

$$\mathrm{W}=\mathrm{qE}\left(\cos\theta_1-\cos\theta_2\right)$$

(iii) Potential energy: In case of a dipole (in a uniform field), potential energy of dipole is defined as work done in rotating a dipole from a direction perpendicular to the field to the given direction i.e. if $\theta_1 = 90^o$ and $\theta_2 = \theta$ then

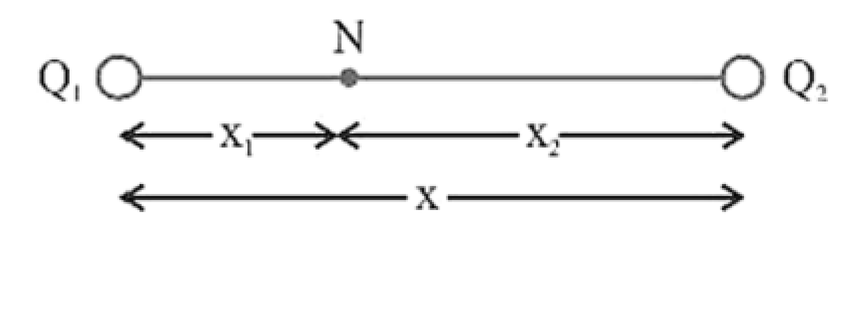
$$W = \delta U = U_\theta - U_{90^o} = -PE\cos\theta$$

$$\Rightarrow U_\theta = -PE\cos\theta\left(\because U_{90} = 0\, or\, U = -PE\right)$$

5. Neutral Point

A neutral point is a point where resultant electrical field is zero. Thus neutral points can be obtained only at those points where the resultant field is subtractive.

(a) At an internal point along the line joining two like charges (Due to a system of two like point charge): Suppose two like charges. Q_1 and Q_2 are separated by a distance x from each other along a line as shown in following figure.



If N is the neutral point at a distance x_1 from Q_1 and at a distance $x_2\left(= x - x_1\right)$ from Q_2 then for natural pt. at N,

| E.F. due to Q_1 |=| $E.F$ due to Q_2| i.e.,

$$\frac{1}{4\pi\varepsilon_0}\cdot\frac{|Q_1|}{x_1^2} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{|Q_2|}{x_2^2} \Rightarrow \frac{|Q_1|}{|Q_2|} = \left(\frac{x_1}{x_2}\right)^2$$

Short trick:

$x_1 = \frac{x}{1+\sqrt{|Q_2|/|Q_1|}}$ and $x_2 = \frac{x}{1+\sqrt{Q_1|/|Q_2|}}$

Note:

In the above formula if $Q_1 = Q_2$, neutral point lies at the centre so remember that resultant field at the midpoint of two equal and like charges is zero.

(b) At an external point along the line joining two unlike charges (Due to a system of two unlike point charge):

Suppose two unlike charges Q_1 and Q_2 separated by a distance x from each other.

Here neutral point lies outside the line joining two unlike charges and also it lies nearer to charge which is smaller in magnitude.

If $|Q_1| \triangleleft Q_2$ then neutral point will be obtained on the side of Q_1, suppose it is at a distance l from Q_1 Hence at neutral point;

$$\frac{k|Q_1|}{\ell^2} = \frac{k|Q_2|}{(x+\ell)^2} \Rightarrow \frac{|Q_1|}{|Q_2|} = \left(\frac{\varepsilon}{x+2}\right)^2$$

Short trick: $\ell = \frac{x}{\left(\sqrt{Q_2|/Q_1|} - 1\right)}$

Note:

In the above discussion if $|Q_1| = |Q_2|$ neutral point will be at infinity.

6. Equilibrium of Charge

(a) Definition: A charge is said to be in equilibrium, if net force acting on it is zero. A system of charges is said to be in equilibrium if each charge is in equilibrium.

(b) Type of equilibrium: Equilibrium can be divided in following type:

(i) Stable equilibrium: After displacing a charged particle from it's equilibrium position, if it returns back then it is said to be in stable equilibrium. If U is the potential energy then in case of stable equilibrium U is minimum.

(ii) Unstable equilibrium: After displacing a charged particle from it's equilibrium position, if it never returns back then it is said to be in unstable equilibrium and in unstable equilibrium, U is maximum.

(iii) Neutral equilibrium: After displacing a charged particle from it's equilibrium position if it neither comes back, nor moves away but remains in the position in which it was kept it is said to be in neutral equilibrium and in neutral equilibrium, U is constant.

(c) Different cases of equilibrium of charge

Suppose three similar charges Q_1, q and Q_2 are placed along a straight line as shown below.

Case -1:

Charge q will be in equilibrium if $|F_1| = |F_2|$ ie.,, $\frac{Q_1}{Q_2} = \left(\frac{x_1}{x_2}\right)^2$;

This is the condition of equilibrium of charge q. After following the guidelines, we can say that charge q is in stable equilibrium and this system is not in equilibrium.

$x_1 = \frac{x}{1+\sqrt{Q_2/Q_1}}$ and $x_2 = \frac{x}{1+\sqrt{Q_1/Q_2}}$

e.g. if two charges $+4\mu C$ and $+16\mu C$ are separated by a distance of $30\,\text{cm}$ from each other than for equilibrium a third charge should be placed between them at a distance

$x_1 = \frac{30}{1+\sqrt{16/4}} = 10\text{ cm or } x_2 = 20\text{ cm}$

Case-2:

Two similar charge Q_1 and Q_2 are placed along a straight line at a distance x from each other and a third dissimilar charge q is placed in between them as shown below

Charge g will be in equilibrium if $|F_1| = |F_2|$

i.e., $\frac{Q_1}{Q_2} = \left(\frac{x_1}{x_2}\right)^2$.

Note:

Same short trick can be used here to find the position of charge q as we discussed in Case-1 i.e.,

$x_1 = \frac{x}{1+\sqrt{Q_2/Q_1}}$ and $x_2 = \frac{x}{1+\sqrt{Q_1/Q_2}}$

It is very important to know that magnitude of charge q can be determined if one of the extreme charge (either Q_1. or Q_2) is in equilibrium ie. if Q_2 is in equilibrium then $|q| = Q_1(x_2/x)^2$ and if Q_1 is in equilibrium then $|q| = Q_2(x_1/x)^2$ (It should be remember that sign of q is opposite to that of Q_1 (or Q_2)).

Case -3:

Two dissimilar charge Q_1 and Q_2 are placed along a straight line at a distance x from each other, a third charge q should be placed outside the line joining Q_1 and Q_2 for it to experience zero net force.

(Let $|Q_2| \lhd Q_1$)

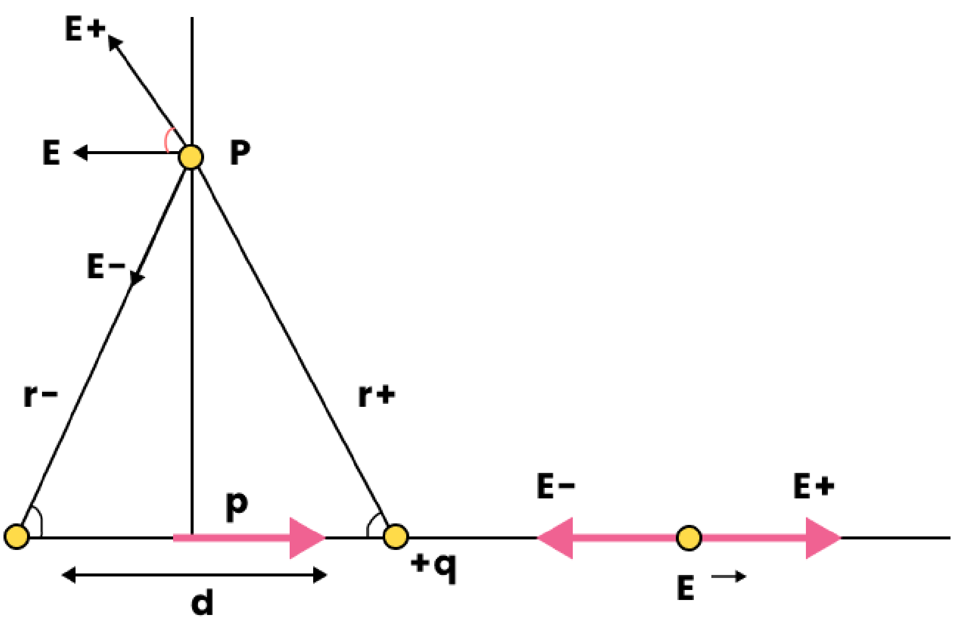
Short Trick :

For its equilibrium. Charge q lies on the side of charge which is smallest in magnitude and

$$d = \frac{x}{\sqrt{Q_1 / Q_2} - 1}$$

7. Electric Dipole

7.1 Electric Field Due to a Dipole



Using the concept that if we know potential electric field can be calculated we have already calculated

$$V_p = \frac{kp\cos\theta}{r^2}$$

To Calculate net electric field at P we need E (Radial Component) & E_1 (tangential component) of electric field at P.

$$E_r = \frac{-dV}{dr}.$$ (When we travel in the radial direction).

$$E_t = -\frac{dV}{rd\theta}$$ (When we travel in the tangential direction).

$$V_p = \frac{kP\cos\theta}{r^2}$$

$$E_r = \frac{-d}{dr}\left(\frac{kP\cos\theta}{r^2}\right) = \frac{2kP\cos\theta}{r^3}$$

$$E_t = \frac{-d}{rd\theta}\left(\frac{kP\cos\theta}{r^2}\right) = \frac{-kP}{r^3}\frac{d}{d\theta}\cos\theta = \frac{kP\sin\theta}{r^3}$$

$$E_{net} = \sqrt{E_r^2 + E_t^2} = \sqrt{\left(\frac{kP}{r^3}\right)^2\left[4\cos^2\theta + \sin^2\theta\right]}$$

$$E_{net} = \sqrt{\left(\frac{kP}{r^3}\right)^2\left[1+3\cos^2\theta\right]}$$

$$E_{net} = \frac{kP}{r^3}\sqrt{1+3\cos^2\theta}$$

$$\tan\alpha = \frac{E_t}{E_r} = \frac{\frac{kP}{r^3}\sin\theta}{2\frac{kP\cos\theta}{r^3}} = \frac{\tan\theta}{2} \qquad \alpha = \tan^{-1}\left[\frac{\tan\theta}{2}\right]$$

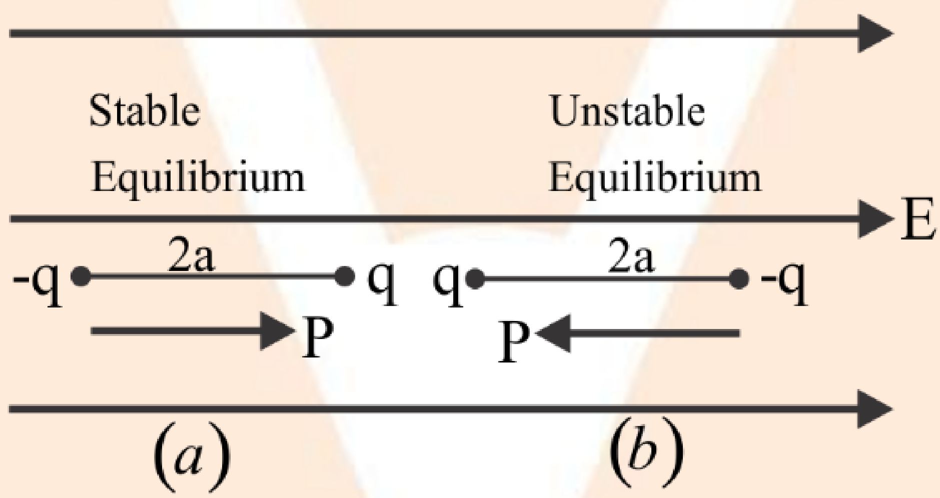
(Note: α is the angle with the radial direction)

7.2 Equilibrium of Dipole

We know that, for any equilibrium net torque and net force on a particle (or system) should be zero.

We already discussed when a dipole is placed in an uniform electric field net force on dipole is always zero. But net torque will be zero only when $\theta = 0^o$ or 180^0.

When $\theta = 0^o$ i.e., dipole is placed along the electric field it is said to be in stable equilibrium, because after turning it through a small angle, dipole tries to align itself again in the direction of electric field.



Stable equilibrium **Unstable equilibrium**

When $\theta = 180^o$ i.e., dipole is placed opposite to electric field, it is said to be in unstable equilibrium.

$\tau = 0 \quad \tau_{max} = pE \qquad\qquad \tau = 0$

$W = 0 \quad W = pE \qquad\qquad W_{max} = 2pE$

$U_{min} = -pE \quad U = 0 \qquad\qquad U_{max} = pE$

Gauss's Law

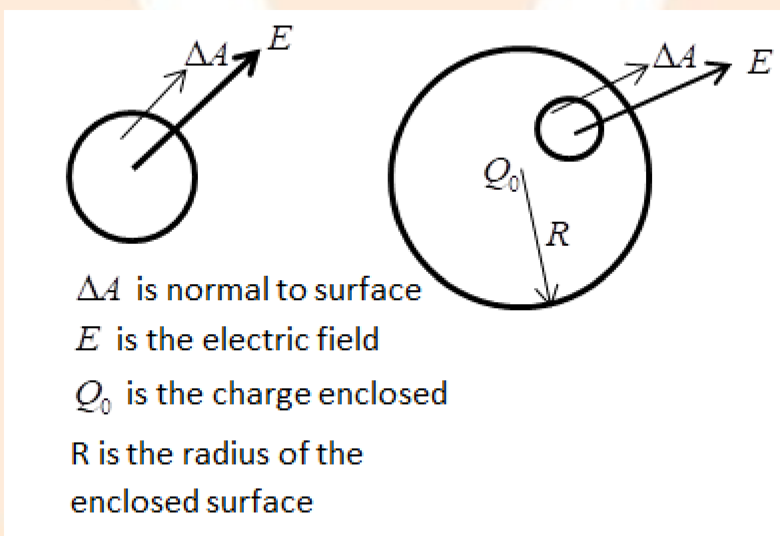
1. Electric Flux

1.1 Definition

Electric flux is defined as proportional to number of field lines crossing or cutting any area of cross section in space.

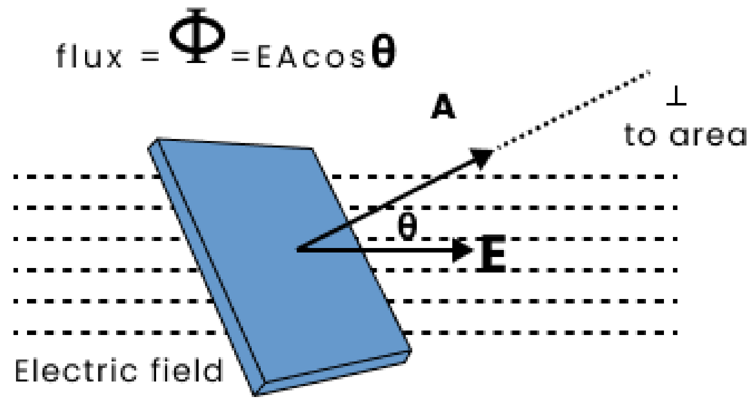
'The number of field lines passing through perpendicular unit area will be proportional to the magnitude of Electric Field there (Theory of Field Lines)

$$\frac{N}{A_{\perp}} \propto E \Rightarrow N \propto E_{\perp}$$



$\therefore$ Electric Flux, $\Phi_{A_{\perp}} = EA_{\perp}$

As θ increases, flux through area A decreases. If we draw a vector of magnitude A along the positive normal, it is called the **area vector**, $\vec{A}$ corresponding to the area A.



$\therefore$ Electric Flux, $\Phi_A = EA\cos\theta = \vec{E}\cdot\vec{A}$

(Assuming Electric Field is uniform over whole area)

Note:

If Electric field is not constant over the area of cross section, then

$\Phi = \int_A \vec{E}\cdot d\vec{A}$

1.2 Unit and Dimension

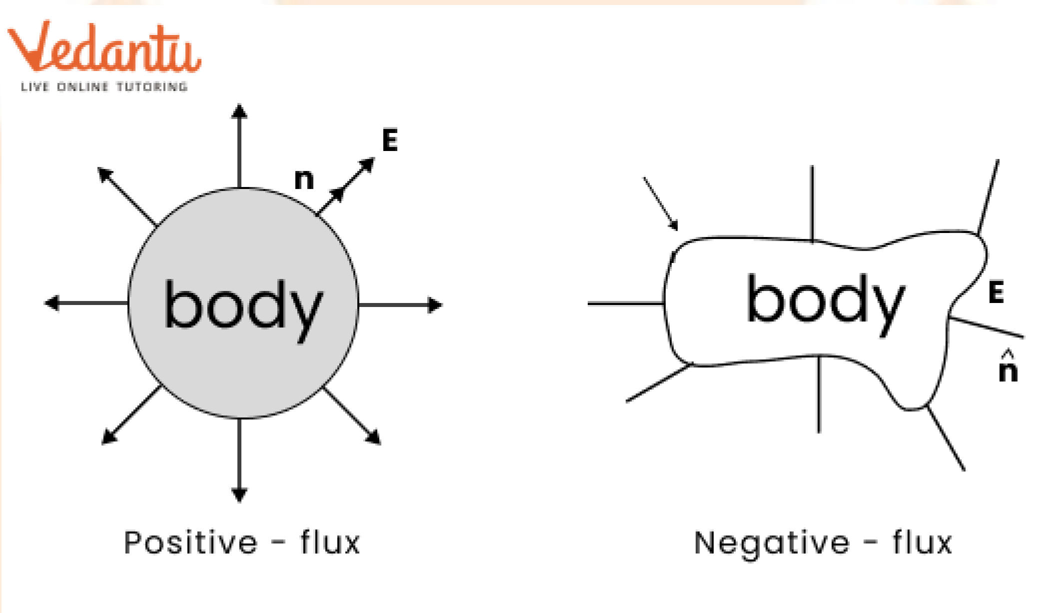
Flux is a scalar quantity.

S.I. unit : (volt $\times$m) or $\frac{N\cdot m^2}{C}$

It's Dimensional formula: $\left(ML^3\,T^{-3}\,A^{-1}\right)$

1.3 Types of Flux

For a closed body outward flux is taken to be positive, while inward flux is taken to be negative.



2. Gauss's Law

2.1 Definition

According to Gauss's law, total electric flux through a closed surface enclosing a charge is $\frac{1}{\varepsilon_0}$ times the magnitude of the charge enclosed.

i.e., $\phi_{\text{not}} = \frac{1}{\varepsilon_0}(Q_{\text{cc}})$

i.e., $\oint \vec{E} \cdot d\vec{A} = \frac{Q_{en}}{E_0}$.

Note:

Gauss's law is only applicable for a closed surface.

2.2 Gaussian Surface

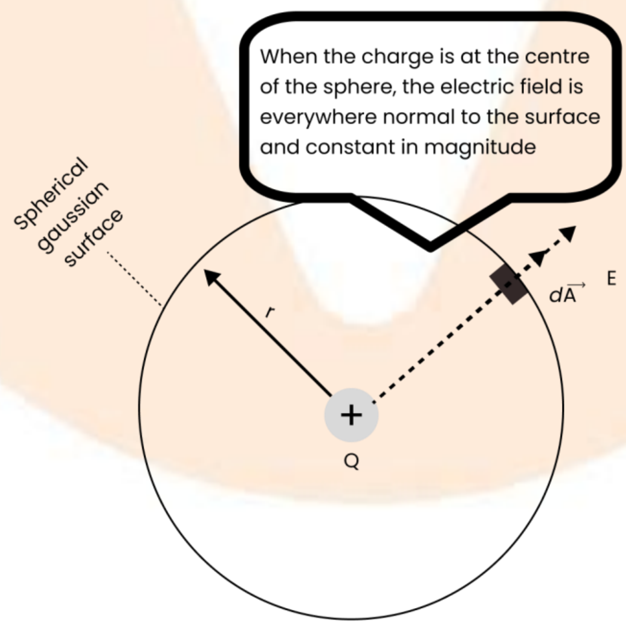
The closed surface on which Gauss law is applicable is defined as a Gaussian surface.

Note:

- Gaussian surface can be of any shape \& size, only condition is that it should be closed.
- Gaussian surface is hypothetical in nature. It does not have a physical existence.

2.3 Deriving Gauss's Law from Coulomb's Law

Let’s take a spherical gaussian surface with charge '$+Q$' kept at the centre.



Total electric flux of point charge

We know field lines for a +ve charge are always radially outward.

Angle between $d\vec{A}$ and $\vec{E}$ is zero.

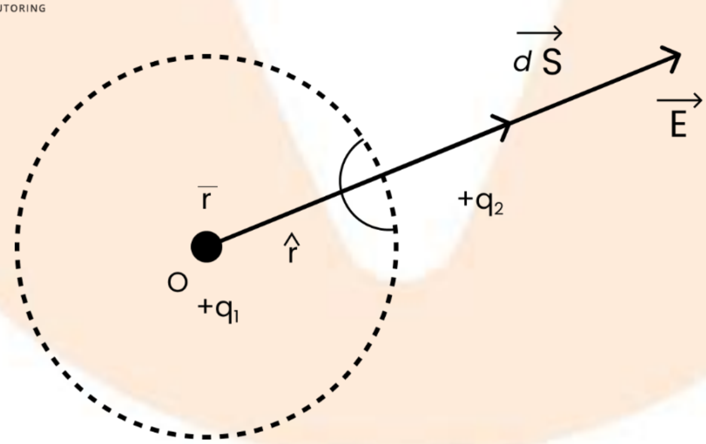
$$E = \frac{kQ}{r^2} = \frac{Q}{4\pi \in_0 r^2}$$

Hence Net flux $= Q/\varepsilon_0$.

Although we derived gauss law for a spherical surface it is valid for any shape of gaussian surface and for any charge kept anywhere inside the surface.

2.4 Coulomb's Law from Gauss's Law

We choose an imaginary sphere (Gaussian surface) of radius r centred on the charge $+q$. Due to symmetry, E must have the same magnitude at any point on the surface, and $\vec{E}$ points radially outward, parallel to $\overrightarrow{dA}$. Hence we write the integral in Gauss's law as:



$$\phi_{net} = \oint \vec{E} \cdot \overrightarrow{dA} = \oint EdA = E\oint dA = E\left(4\pi r^2\right)$$

$Q_{\text{enclosed}} = q$

Thus, $E\left(4\pi r^2\right) = \frac{q}{\varepsilon_0}$ or $E = \frac{q}{4\pi\varepsilon_0 r^2}$

From the definition of the electric field, the force on a point charge q_0 located at a distance r from the charge q is $F = q_0 E$. Therefore,

$$F = \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r^2}$$

which is Coulomb's law.

3. Applications of Gauss's Law

Using Gauss's law to derive ‘E’ due to various charge distributions.

3.1 Electric Field Due to a Line Charge

Consider an infinite line which has a linear charge density λ. Using Gauss's law, let us find the electric field at a distance 'r' from the line charge.

The cylindrical symmetry tells us that the field strength will be the same at all points at a fixed distance r from the line. Thus, if the charges are positive. The field lines are directed radially outwards, perpendicular to the line charge.

The appropriate choice of Gaussian surface is a cylinder of radius r and length L. On the flat end faces, S_2 and $S_3, \vec{E}$ is perpendicular $d\bar{S}$, which means flux is zero on them. On the curved surface $S_1, \vec{E}$ is parallel to $d\bar{S}$, so that $\vec{E} \cdot \overrightarrow{dS} = EdS$. The charge enclosed by the cylinder is $Q = \lambda L$. Applying Gauss's law to the curved surface, we have

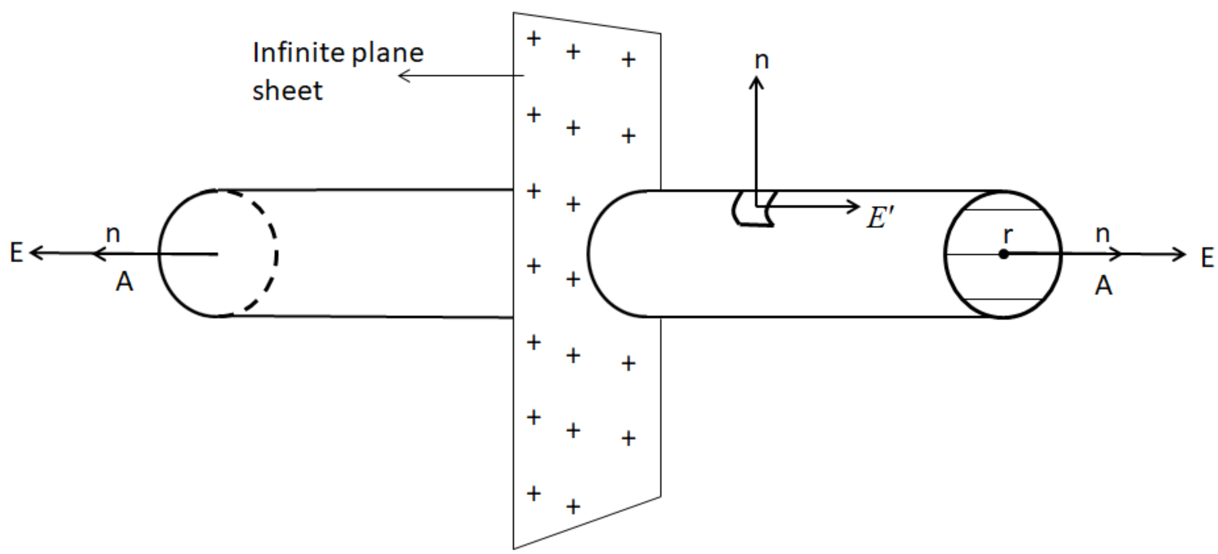
$$E\oint dS = E(2\pi rL) = \frac{\lambda L}{\varepsilon_0} \text{ or } E = \frac{\lambda}{2\pi\varepsilon_0 r} = \frac{2k\lambda}{r}$$

Note:

This is the field at a distance r from the line. It is directed away from the line if the charge is positive and towards the line if the charge is negative.

3.2 Electric Field Due to a Plane Sheet of Charge

Consider a large plane sheet of charge with surface charge density (charge per unit area) **$\sigma $**. We have to find the electric field **${\text{E}}$** at a point **${\text{P}}$** in front of the sheet.



Note:

If the charge is positive, the field is away from the plane.

To calculate the field E at P. Choose a cylinder of area of cross-section A through the point P as the Gaussian surface. The flux due to the electric field of the plane sheet of charge passes only through the two circular caps of the cylinder.

According to Gauss law $\oint \vec{E} \cdot d\bar{S} = q_{\text{in}} / \varepsilon_0$

$$\underset{\text{I circular surface}}{\int \vec{E}.\overline{dS}} + \underset{\text{II circular surface}}{\int \vec{E}.\overline{dS}} + \underset{\text{cylindricalr surface}}{\int \vec{E}.\overrightarrow{dS}} = \frac{\sigma A}{\varepsilon_0}$$

or $\mathrm{EA} + \mathrm{EA} + 0 = \frac{\sigma \mathrm{A}}{\varepsilon_0}$

or $E = \frac{\sigma}{2\varepsilon_0}$

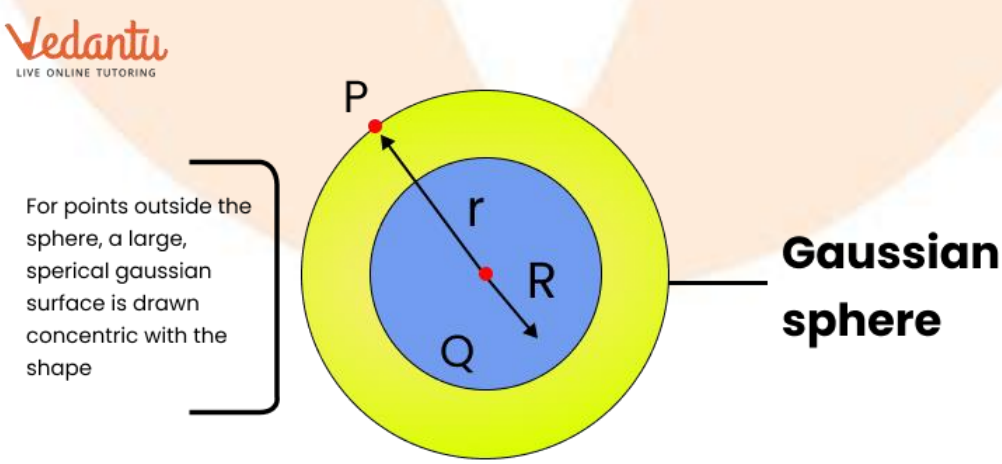
Note:

We see that the field is uniform and does not depend on the distance from the charge sheet. This is true as long as the sheet is large as compared to its distance from P.

3.3 Uniform Spherical Charge Distribution

3.3.1 Outside the Sphere

P is a point outside the sphere at a distance r from the centre.



According to Gauss law, $\oint \vec{E} \cdot d\vec{s} = \frac{Q}{E_D}$ or $E\left(4\pi r^2\right) = \frac{Q}{\varepsilon_D}$

Electric field at P (Outside sphere)

$$E_{out} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{r^2} = \frac{\sigma R^2}{\varepsilon_0 r^2} \text{ and}$$

$$V_{out} = -\int_{\infty}^{r} \vec{E} d\vec{r} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{r} = \frac{\sigma R^2}{\varepsilon_0 r}$$

Note:

$Q = 0 \times A$

$= \sigma \times 4\pi R^2$

3.3.2 At the surface of sphere

At surface $r = R$

So, $E_e = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{R^2} = \frac{\sigma}{\varepsilon_0}$ and $V_\varepsilon = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{R} = \frac{\sigma R}{\varepsilon_0}$

3.3.3 Inside the Sphere

Inside the conducting charged sphere electric field is zero and potential remains constant everywhere and equals to the potential at the surface.