NCERT Exemplar Solutions For Grade 11

Physics

Chapter 15 - Waves

Multiple Choice Questions – I

1. Water waves produced by a motor boat sailing in water are

a) neither longitudinal nor transverse.

b) both longitudinal and transverse.

c) only longitudinal.

d) only transverse.

Ans: The correct answer is option (b) both longitudinal and transverse.

Motor boat produces water waves on surface as well as inside the water. Hence, waves are both, transverse and longitudinal.

2. Sound waves of wavelength λ travelling in a medium with a speed of $v\ m/s$ enter into another medium where its speed is $2v\ m/s$. Wavelength of sound waves in the second medium is

a) λ

b) $\frac{\lambda}{2}$

c) 2λ

d) 4λ

Ans: The correct answer is option (c) 2λ

We know that when wave passes from one medium to another, its velocity and wavelength changes but its frequency remains the same.

We know that, $v = f\lambda$

Therefore, we have

$\Rightarrow f = \frac{v}{\lambda} ....(i)$

When sound wave enters into another medium, we get

And, $f = \frac{2v}{\lambda'} .....(ii)$

From (i) and (ii), we get

$\Rightarrow \frac{v}{\lambda} = \frac{2v}{\lambda'}$

$\Rightarrow \frac{1}{\lambda} = \frac{2}{\lambda'}$

$\Rightarrow \lambda' = 2\lambda$

3. Speed of sound wave in air

a) is independent of temperature.

b) increases with pressure.

c) increases with increase in humidity.

d) decreases with increase in humidity.

Ans: The correct answer is option (c) increases with increase in humidity.

We know that speed of sound wave (longitudinal wave) in air is given by $v = \sqrt{\frac{\lambda P}{\rho}}$.

The density of water vapours is less than the air. Thus, on increasing the humidity, density of the medium decreases and this in turn increases the speed of sound in air because $v\alpha \frac{1}{\sqrt{\rho}}$.

4. Change in temperature of the medium changes

a) frequency of sound waves.

b) amplitude of sound waves.

c) wavelength of sound waves.

d) loudness of sound waves.

Ans: The correct answer is option (c) wavelength of sound waves.

We know that, $v_t = v_0(1+.61t)$ or $v_t \alpha \sqrt{t}$

Therefore, on increasing the temperature, speed also increases because frequency doesn't change during propagation of wave by formula $V = v\lambda$.

Hence, velocity V and wavelength λ both increases.

5. With propagation of longitudinal waves through a medium, the quantity transmitted is

a) matter.

b) energy.

c) energy and matter.

d) energy, matter and momentum.

Ans: The correct answer is option (b) energy.

We know that during propagation of longitudinal waves through a medium, only energy is transmitted from one point to another. Matter doesn't change its own position as it only vibrates about its mean position.

6. Which of the following statements are true for wave motion?

a) Mechanical transverse waves can propagate through all mediums.

b) Longitudinal waves can propagate through solids only.

c) Mechanical transverse waves can propagate through solids only.

d) Longitudinal waves can propagate through vacuum.

Ans: The correct answer is option (c) Mechanical transverse waves can propagate through solids only.

Transverse waves can't propagate in gases. Hence, option (a) is incorrect.

Longitudinal waves can propagate through gases. Hence, option (b) is incorrect.

Mechanical transverse waves can propagate through solids and on surface of liquid also. Hence, option (c) is correct.

Longitudinal waves cannot propagate through vacuum. Hence, option (d) is incorrect.

7. A sound wave is passing through air column in the form of compression and rarefaction. In consecutive compressions and rarefactions,

a) density remains constant.

b) Boyle's law is obeyed.

c) bulk modulus of air oscillates.

d) there is no transfer of heat.

Ans: The correct answer is option (d) there is no transfer of heat.

We know that density of medium particles is maximum and minimum at compression and rarefaction point. Therefore option (a) is incorrect.

The temperature of medium increases because density changes very rapidly. Therefore, option (b) is incorrect.

Option (c) is incorrect because bulk modules of air remain constant.

We know that the time of compressions and rarefactions is very small. Thus, heat doesn't transfer. Hence, option (d) is the correct answer.

8. Equation of a plane progressive wave is given by $y = 0.6\sin 2\pi\left(t + \frac{x}{2}\right)$. On reflection from a denser medium its amplitude becomes $\frac{2}{3}$ of the amplitude of the incident wave. The equation of the reflected wave is

a) $y = 0.6\sin 2\pi\left(t + \frac{x}{2}\right)$

b) $y = -0.4\sin 2\pi\left(t + \frac{x}{2}\right)$

c) $y = 0.4\sin 2\pi\left(t + \frac{x}{2}\right)$

d) $y = -0.4\sin 2\pi\left(t - \frac{x}{2}\right)$

Ans: The correct answer is option (b) $y = -0.4\sin 2\pi\left(t + \frac{x}{2}\right)$

After reflection, wave changes by phase $180°$

Given equation: $y = 0.6\sin 2\pi\left(t + \frac{x}{2}\right)$

After reflection, it becomes

$\Rightarrow y_r = \frac{2}{3}\sin 2\pi\left(\pi + t + \frac{x}{2}\right)$

$\Rightarrow y_r = -0.4\sin 2\pi\left(t + \frac{x}{2}\right)$

9. A string of mass $2.5kg$ is under a tension of $200N$. The length of the stretched string is $20.0m$. If the transverse jerk is struck at one end of the string, the disturbance will reach the other end in

a) one second

b) 0.5 second

c) 2 seconds

d) data given is insufficient.

Ans: The correct answer is option (b) 0.5 second

Mass of string $(M) = 2.5kg$

Length of the stretched string $(l) = 20m$

Mass per unit length = $\frac{M}{l}$

$= \frac{2.5}{20}$

$= 0.125kg/m$

We know that $v = \sqrt{\frac{T}{\mu}}$. Therefore, we get

$$\Rightarrow v = \sqrt{\frac{200}{0.125}}$$

$$\Rightarrow v = \sqrt{1600}$$

$$\Rightarrow v = 40m/s$$

We know that time = distance / speed. Therefore, we get

$$\Rightarrow t = \frac{20m}{40m/s}$$

$$\Rightarrow t = \frac{1}{2}\text{sec}$$

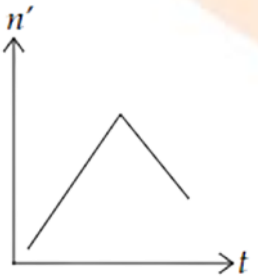
$$\Rightarrow t = 0.5\text{sec}$$

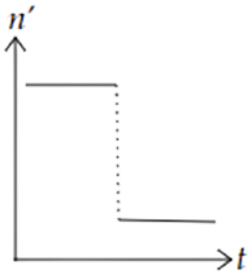
10. A train whistling at constant frequency is moving towards a station at a constant speed V. The train goes past a stationary observer on the station. The frequency n' of the sound as heard by the observer is plotted as a function of time t (Fig 15.1). Identify the expected curve.

a)

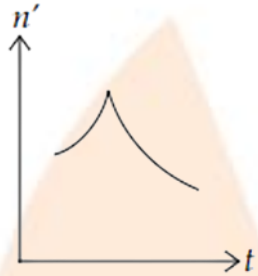


b)





c)



d)

Ans: The correct answer is option (c).

We know that when observer is at rest and the source of sound is moving towards observer, then observed frequency is given by, $n' = \left(\frac{v}{v - v_s} \right) n_o$, where n_o is the original frequency of source of sound.

v is speed of sound in medium

v_s is speed of source

$\therefore n' > n_o$

When source is moving away from the observer,

$\Rightarrow n' = \left(\frac{v}{v + v_s} \right) n_o$

$\therefore n'' < n_o$

Hence, the frequencies in both the cases are same and $n' > n''$.

Multiple Choice Question – II More than one option

11. A transverse harmonic wave on a string is described by

$$y(x,t) = 3.0\sin\left(36t + 0.018x + \frac{\pi}{4}\right)$$

where x and y are in cm and t is in s. The positive direction of x is from left to right.

a) The wave is travelling from right to left.

b) The speed of the wave is $20 m/s$.

c) Frequency of the wave is $5.7 Hz$.

d) The least distance between two successive crests in the wave is $2.5 cm$.

Ans: The correct options are (a), (b) and (c).

The standard equation of a wave propagated from left to right i.e., in $+ve$ direction is given by $y = a\sin(\omega t - kx + \phi)$

Given equation: $y = 3.0\sin\left(36t + 0.018x + \frac{\pi}{4}\right)$

a) In the given equation, x is in positive sign. Thus, the given wave is travelling from right to left verifies option (a).

b) From the equation, we have $\omega = 36$

$\Rightarrow 2\pi f = 36$

$\Rightarrow f = \frac{36}{2\pi}$

$\Rightarrow f = \frac{18}{\pi}.....(i)$

$\Rightarrow f = \frac{18}{3.14}$

$\Rightarrow f = 5.7Hz$

From the equation, we have $k = 0.018$

$\Rightarrow \frac{2\pi}{\lambda} = 0.018$

$\Rightarrow \frac{2\pi}{0.018} = \lambda$

We know that velocity, $v = f\lambda$. Therefore, we get

$\Rightarrow v = \frac{18}{\pi} \times \frac{2\pi}{0.018}$ $\left(f = \frac{18}{\pi} .....(i) \right)$

$\Rightarrow v = \frac{18}{0.018}$

$\Rightarrow v = 2000cm/s$

$\Rightarrow v = 20m/s$

This verifies option (b).

c) From the equation, we have $\omega = 36$

$\Rightarrow 2\pi f = 36$

$\Rightarrow f = \frac{36}{2\pi}$

$\Rightarrow f = \frac{18}{\pi}$

$\Rightarrow f = \frac{18}{3.14}$

$\Rightarrow f = 5.7Hz$

This verifies option (c).

d) Distance between two successive crests is given by, $\lambda = \frac{2\pi}{0.018}$

$\Rightarrow \lambda = \frac{\pi}{0.009}$

$\Rightarrow \lambda = \frac{3.14 \times 1000}{9}$

$\Rightarrow \lambda = \frac{3140}{9}$

$\Rightarrow \lambda = 348.8cm = 3.48m$

Hence, option (d) is incorrect.

12. The displacement of a string is given by $y(x,t) = 0.06\sin\left(\frac{2\pi x}{3}\right)\cos(120\pi t)$ where x and y are in m and t in s. The length of the string is $1.5m$ and its mass is $3.0 \times 10^{-2} kg$.

a) It represents a progressive wave of frequency $60Hz$.

b) It represents a stationary wave of frequency $60Hz$.

c) It is the result of superposition of two waves of wavelength $3m$, frequency $60Hz$ each travelling with a speed of $180m/s$ in opposite direction.

d) Amplitude of this wave is constant.

Ans: The correct options are (b) and (c).

We know that the standard equation of stationary wave is $y(x,t) = a\sin(kx)\cos(\omega t)$

Given equation: $y(x,t) = 0.06\sin\left(\frac{2\pi x}{3}\right)\cos(120\pi t)$

a) The given equation is stationary. Hence option (a) is incorrect.

b) On comparing standard and given equation, we get $\omega = 120\pi$

$\Rightarrow 2\pi f = 120\pi$

$\Rightarrow f = 60Hz$

This verifies option (b).

c) We know that $k = \frac{2\pi}{\lambda}$.

From the given equation, we have $k = \frac{2\pi}{3}$

$\Rightarrow \frac{2\pi}{\lambda} = \frac{2\pi}{3}$

$\Rightarrow \lambda = 3$

Speed, $v = f\lambda$

$\Rightarrow v = 60 \times 3m/s$

$\Rightarrow v = 180m/s$

This verifies option (c).

d) As the waves are stationary, so the amplitude of particles varies from 0 to 0.06 from nodes to antinodes. Thus, amplitude throughout the wave are not constant. Hence, the option is incorrect.

13. Speed of sound waves in a fluid depends upon

a) directly on density of the medium.

b) square of Bulk modulus of the medium.

c) inversely on the square root of density.

d) directly on the square root of bulk modulus of the medium.

Ans: The correct options are (c) and (d).

We know that speed of sound wave in fluid of bulk modules k and density ρ = $v = \sqrt{\frac{k}{\rho}}$.

Therefore, $v = \sqrt{k}$ (If ρ is constant)

And, $v = \sqrt{\frac{1}{\rho}}$ (If k is constant)

Hence, verifies the options (c) and (d).

14. During propagation of a plane progressive mechanical wave

a) all the particles are vibrating in the same phase.

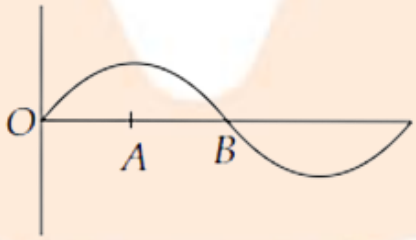
b) amplitude of all the particles is equal.

c) particles of the medium executes S.H.M.

d) wave velocity depends upon the nature of the medium.

Ans: The correct options are (b), (c) and (d).

Each particle between any two successive crests and troughs is in different mode of vibration or in different phase. Hence, option (a) is incorrect.



We know that during propagation of mechanical wave each particle displaces from zero to maxima i.e., up to amplitude. Thus, amplitude of each particle is equal.

Hence, verifies option (b).

For progressive wave, medium particles oscillates about their mean position in which restoring force $F\alpha(-y)$. Therefore, motion of medium particles is simple harmonic motion.

Hence, verifies option (c).

We know that wave velocity is given by $v=\sqrt{\frac{k}{\rho}}$.

We can see that v depends on k and ρ. Also, k and ρ are different for different medium. Thus, wave velocity depends upon nature of the medium. Hence, verifies the option (d).

15. The transverse displacement of a string (clamped at its both ends) is given by y $y(x,t)=0.06\sin\left(\frac{2\pi x}{3}\right)\cos(120\pi t)$. All the points on the string between two consecutive nodes vibrate with

a) same frequency

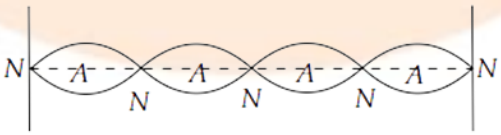
b) same phase

c) same energy

d) different amplitude.

Ans: The correct options are (a), (b) and (d).

We know that the standard equation of stationary wave is $y(x,t)=a\sin(kx)\cos(\omega t)$



Given equation: $y(x,t)=0.06\sin\left(\frac{2\pi x}{3}\right)\cos(120\pi t)$

Thus, given equation is a stationary wave.

a) The frequencies of all the particles are same. Hence, verifies the option (a).

b) The particles between any two consecutive nodes vibrates either upside or downside having same phase $120\pi t$ at a time. Hence, verifies the option (b).

c) We know that amplitude of different particles are different between two nodes. Also, we know that energy, $E\alpha A^2$. Therefore, particles have different energies. Hence, option (c) is incorrect.

d) Option (c) verifies that option (d) is correct

16. A train, standing in a station yard, blows a whistle of frequency $400Hz$ in still air. The wind starts blowing in the direction from the yard to the station with a speed of $10m/s$. Given that the speed of sound in still air is $340m/s$,

a) the frequency of sound as heard by an observer standing on the platform is $400Hz$.

b) the speed of sound for the observer standing on the platform is $350m/s$.

c) the frequency of sound as heard by the observer standing on the platform will increase.

d) the frequency of sound as heard by the observer standing on the platform will decrease.

Ans: The correct options are (a) and (b).

Frequency of whistle is still air, $f = 400Hz$

Velocity of wind from source of listener, $v_w = 10m/s$

Speed of sound in still air, $v_s = 340m/s$

Given that, listener is standing on platform.

Therefore, speed of sound w.r.t. the listener = $v_s + v_w = 340 + 10 = 350 m/s$

Hence, verifies option (b).

The distance between listener and source doesn't change. Thus, frequency of sound doesn't change. Therefore, he heard $f = 400 Hz$.

Hence, option (c) and (d) are incorrect.

17. Which of the following statements are true for a stationary wave?

a) Every particle has a fixed amplitude which is different from the amplitude of its nearest particle.

b) All the particles cross their mean position at the same time.

c) All the particles are oscillating with same amplitude.

d) There is no net transfer of energy across any plane.

e) There are some particles which are always at rest.

Ans: The correct options are (a), (b), (d) and (e).

We know that the standard equation of stationary wave is $y(x,t) = a\sin(kx)\cos(\omega t)$

In stationary waves, the particles between the two nodes vibrate with different amplitude. The amplitude increases from node to antinodes from zero to maximum and then from antinodes to nodes amplitude decreases from maximum to zero. The amplitude of a particle will remain constant, but amplitude will varies with λ.

$\therefore k = \frac{2\pi}{\lambda}$. This verifies option (a) and rejects option (c).

The particles present at nodes have amplitude zero. Hence, verifies option (e).

We know that particles at nodes are at rest. Therefore, there is no energy transfer. This verifies option (d).

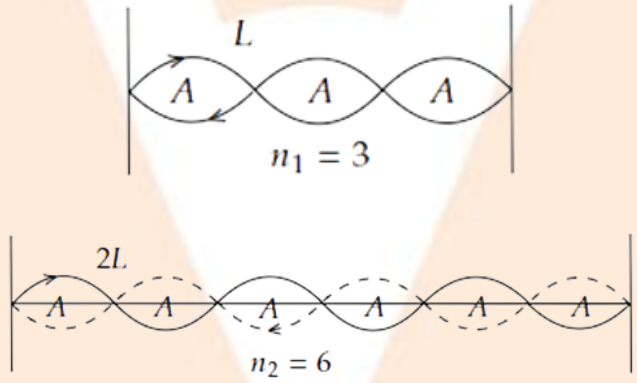
Particles between the two nodes are in the same phase, it means motion of particles will be either upward or downward. Thus, particles will cross mean position at the same time. This verifies the option (b).

Very Short Answer Type Questions

18. A sonometer wire is vibrating in resonance with a tuning fork. Keeping the tension applied same, the length of the wire is doubled. Under what conditions would the tuning fork still be is resonance with the wire?

Ans: When a wire of length L vibrates its resonant frequency in nth node after stretching it by a tension T is given by $v = \frac{n}{2L}\sqrt{\frac{T}{m}}$, where m is mass per unit length of stretched wire.

We have two cases:



$$\Rightarrow v_1 = \frac{n_1}{2L_1}\sqrt{\frac{T_1}{m_1}}$$

$$\Rightarrow v_2 = \frac{n_2}{2L_2}\sqrt{\frac{T_2}{m_2}}$$

We are given that,

$T_1 = T_2 = T$

$L_2 = 2L_1$

As wire is same. Therefore, $m_1 = m_2 = m$

Now, on substituting the values, we get

$$\Rightarrow v_1 = \frac{n_1}{2L_1}\sqrt{\frac{T}{m}}$$

$$\Rightarrow v_2 = \frac{n_2}{2\times 2L_1}\sqrt{\frac{T}{m}}$$

$$\Rightarrow \frac{v_1}{v_2} = \frac{\frac{n_1}{2L_1}\sqrt{\frac{T}{m}}}{\frac{n_2}{2\times 2L_1}\sqrt{\frac{T}{m}}}$$

$$\Rightarrow \frac{v_1}{v_2} = \frac{\frac{n_1}{1}}{\frac{n_2}{2}}$$

$$\Rightarrow \frac{v_1}{v_2} = \frac{2n_1}{n_2}$$

It is given that tuning fork is same. Thus, in both the harmonics n_1 and n_2, frequency of resonance is same. Therefore, $v_1 = v_2 = v$

$$\Rightarrow \frac{v}{v} = \frac{2n_1}{n_2}$$

$$\Rightarrow 1 = \frac{2n_1}{n_2}$$

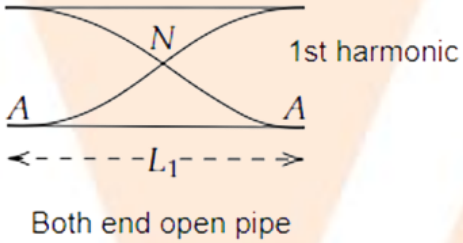
$$\Rightarrow n_2 = 2n_1$$

Hence, when length of wire is doubled, the number of harmonics also doubled for same resonant frequency, tension and mass per unit length.

19. An organ pipe of length L open at both ends is found to vibrate in its first harmonic when sounded with a tuning fork of $480Hz$. What should be the length of a pipe closed at one end, so that it also vibrates in its first harmonic with the same tuning fork?

Ans: Given that, medium, frequency and the number of harmonic in open and closed pipes are same. Thus, the number of nodes and λ in both the cases will also be same.

In both end open pipe, we have



Both end open pipe

$$\Rightarrow L_1 = \frac{2 \times \lambda_1}{4}$$

$$\Rightarrow L_1 = \frac{\lambda_1}{2}$$

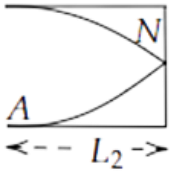
$$\Rightarrow \lambda_1 = 2L_1$$

We know that $v = \frac{c}{\lambda}$. Therefore, we get

$$\Rightarrow v_1 = \frac{c}{\lambda_1}$$

$$\Rightarrow v_1 = \frac{c}{2L_1} .....(i)$$

In one end open pipe, we have



One end close pipe

$\Rightarrow L_2 = \frac{1\times\lambda_2}{4}$

$\Rightarrow \lambda_2 = 4L_2$

And, $v_2 = \frac{c}{4L_2}.....(ii)$

Given that, medium and tuning fork in both the cases are same. Therefore, we have $v_1 = v_2$ and $c_1 = c_2 = c$.

From equation (i) and (ii), we get

$\Rightarrow \frac{c}{2L_1} = \frac{c}{4L_2}$

$\Rightarrow L_1 = 2L_2$

$\Rightarrow L_2 = \frac{L_1}{2}$

Hence, the length of one end closed pipe will be half of the both end open pipe.

20. A tuning fork A, marked $512Hz$, produces 5 beats per second, where sounded with another unmarked tuning fork B. If B is loaded with wax the number of beats is again 5 per second. What is the frequency of the tuning fork B when not loaded?

Ans: Given that, $v_A = 512$

On loading, frequencies of B decreased

So, in first case $v_1 = 5$

$\Rightarrow v_B = v_A \pm 5$

When B is loaded, frequency of B becomes,

$\Rightarrow v_B = 512 \pm 5$

We can write it as,

$\Rightarrow v_B = 507$ or $v_B = 517$

On load in frequency of B decreased 507 to lower value of number of beats will increase so $v_B \neq 507$

If $v_B = 517$ then its frequency decrease $10Hz$ by then the number of beats will also be the same as $512 - 507 = 5$.

Thus, the frequency of the tuning fork B when not loaded is $517Hz$.

21. The displacement of an elastic wave is given by the function $y = 3\sin\omega t + 4\cos\omega t$. where y is in cm and t is in second. Calculate the resultant amplitude.

Ans: We have, $y = 3\sin\omega t + 4\cos\omega t.....(i)$

Let $3 = a\cos\varphi.....(ii)$ and $4 = a\sin\phi.....(iii)$

Thus, equation (i) becomes $y = a\cos\varphi\sin\omega t + a\sin\phi\cos\omega t$

$\Rightarrow y = a\sin(\omega t + \phi)$

From equation (ii) and (iii), we get

$\Rightarrow \tan\phi = \frac{4}{3}$

$\Rightarrow \phi = \tan^{-1}\frac{4}{3}$

On squaring and adding equation (ii) and (iii), we get

$\Rightarrow a^2\cos^2\varphi + a^2\sin^2\phi = 3^2 + 4^2$

$\Rightarrow a^2\left(\cos^2\varphi + \sin^2\phi\right) = 9 + 16$

$\Rightarrow a^2(1) = 25$

$\Rightarrow a^2 = 25$

$\Rightarrow a = 5cm$

Hence, resultant amplitude is $5cm$.

22. A sitar wire is replaced by another wire of same length and material but of three times the earlier radius. If the tension in the wire remains the same, by what factor will the frequency change?

Ans: The wire is stretched at both the ends, so frequency of stretched wire is,

$v = \frac{n}{2L}\sqrt{\frac{T}{m}}$

As the number of harmonic n, length L and tension T are kept same in both cases.

$\therefore v\alpha \frac{1}{\sqrt{m}}$

$\therefore \frac{v_1}{v_2} = \frac{\sqrt{m_2}}{\sqrt{m_1}}.....(i)$

Mass per unit length (m) is given by,

$= \frac{mass\ of\ wire}{length}$

$$m = \frac{\left(\pi r^2 l\right)\rho}{l}$$

$$m = \pi r^2 \rho$$

As the material of wire is same. Therefore, we get

$$\Rightarrow \frac{v_1}{v_2} = \sqrt{\frac{\pi r_2^2 \rho}{\pi r_1^2 \rho}}$$

Given that, sitar wire is replaced by another wire of three times the earlier radius.

$$\Rightarrow \frac{v_1}{v_2} = \sqrt{\frac{\left(3r\right)^2}{r^2}}$$

$$\Rightarrow \frac{v_1}{v_2} = \frac{3r}{r}$$

$$\Rightarrow \frac{v_1}{v_2} = \frac{3}{1}$$

$$\therefore v_2 = \frac{1}{3} v_1$$

Thus, the frequency of sitar is reduced by $\frac{1}{2}$ of previous value.

23. At what temperatures (in $^\circ C$) will the speed of sound in air be 3 times its value at $0^\circ C$?

Ans: We know that $v \alpha \sqrt{T}$

$$\Rightarrow \frac{v_T}{v_0} = \sqrt{\frac{T}{T_0}}$$

Given that, $v_T = 3v_0$

$\therefore \frac{3\nu_0}{\nu_0} = \sqrt{\frac{T}{273+0}}$

$\Rightarrow \sqrt{T} = 3\sqrt{273}$

$\Rightarrow T = 9 \times 273$

$\Rightarrow T = 2457K$

$\Rightarrow T = 2457 - 273$

$\Rightarrow T = 2184^{\circ}C$

24. When two waves of almost equal frequencies n_1 and n_2 reach at a point simultaneously, what is the time interval between successive maxima?

Ans: Given that, the two frequencies are almost equal, $n_1 = n_2$

For formation of the beats, the frequencies must be $n_2 > n_1$

Number of frequency per second in maxima = $n = n_2 - n_1$

Thus, time period of beats or maxima = $\frac{1}{n} = \frac{1}{n_2 - n_1}$ sec

Short Answer Type Questions

25. A steel wire has a length of $12m$ and a mass of $2.10kg$. What will be the speed of a transverse wave on this wire when a tension of $2.06 \times 10^4 N$ is applied?

Ans: We have, length of steel wire, $l = 12m$

Total mass of steel wire, $M = 2.10kg$

Tension, $T = 2.06 \times 10^4 N$

We know that $m = \frac{M}{l}$. Therefore, we get

$\Rightarrow m = \frac{2.10}{12}$

We know that $v = \sqrt{\frac{T}{m}}$. Therefore, we get

$\Rightarrow v = \sqrt{\frac{2.06 \times 10^4 \times 12}{2.10}}$

$\Rightarrow v = \sqrt{\frac{1236 \times 10^4}{105}}$

$\Rightarrow v = \sqrt{11.77} \times 10^2 m/s$

$\Rightarrow v = 3.43 \times 10^2 m/s$

$\Rightarrow v = 343.0 m/s$

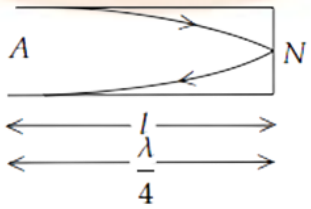
26. A pipe $20cm$ long is closed at one end. Which harmonic mode of the pipe is resonantly excited by a source of $1237.5Hz$?(sound velocity in air = $330ms^{-1}$)

Ans: We have, length of the pipe, $l = 20cm = 0.2m$

And, $v = 1237.5Hz$

Sound velocity in air, $v = 330ms^{-1}$

$l = \frac{\lambda}{4}$ or $\lambda = 4l$



For fundamental frequency, $v_1 = \frac{v}{\lambda} = \frac{v}{4l}$. Therefore, we get

$$\Rightarrow v_1 = \frac{330}{4 \times 20 \times 10^{-2}}$$

$$\Rightarrow v_1 = 412.5Hz$$

$$v(given) = 1237.5Hz$$

$$\Rightarrow \frac{v(g)}{v_1} = \frac{1237.5Hz}{412.5Hz}$$

$$\Rightarrow \frac{v(g)}{v_1} = \frac{3}{1}$$

Hence, the frequency in $Ist, IInd, IIIrd, ...$ harmonic are in the ratio $1:2:3:4...$ in one end open pipe.

27. A train standing at the outer signal of a railway station blows a whistle of frequency $400Hz$ still air. The train begins to move with a speed of $10ms^{-1}$ towards the platform. What is the frequency of the sound for an observer standing on the platform? (sound velocity in air = $330ms^{-1}$)

Ans: Frequency of train standing at the outer signal, $v_o = 400Hz$

Speed of train moving towards platform, $v_s = 10ms^{-1}$

Velocity of sound in air, $v_a = 330ms^{-1}$

Apparent frequency of the sound for an observer standing on platform,

$$\Rightarrow v' = \frac{v_a}{(v_a - v_s)} v_o$$

$$\Rightarrow v' = \frac{330 \times 400}{(330 - 10)} Hz$$

$\Rightarrow v' = \frac{330 \times 400}{320} Hz$

$\Rightarrow v' = \frac{825}{2} Hz$

$\Rightarrow v' = 412.5 Hz$

28. The wave pattern on a stretched string is shown in Fig. 15.2 . Interpret what kind of wave this is and find its wavelength.

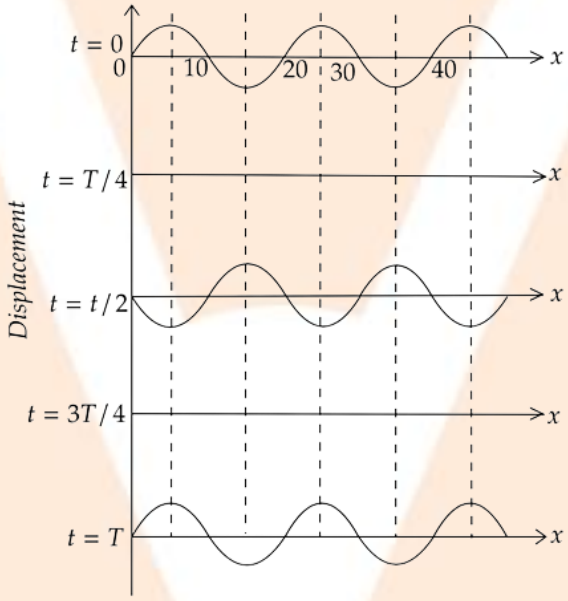


Fig. 15.2

Ans: All particles are at rest at $t = \frac{T}{4}$ and $t = \frac{3T}{4}$, which is in stationary wave when the particle crosses its mean position.

Thus, the graph shows stationary wave. The wave at $x = 10, 20, 30, 40 cm$ there are nodes and distance between successive nodes is $\frac{\lambda}{2}$.

$\therefore \frac{\lambda}{2} = (30 - 20)$

$\Rightarrow \lambda = 20cm$

29. The pattern of standing waves formed on a stretched string at two instants of time are shown in Fig. 15.3. The velocity of two waves superimposing to form stationary waves is $360ms^{-1}$ and their frequencies are $256Hz$.

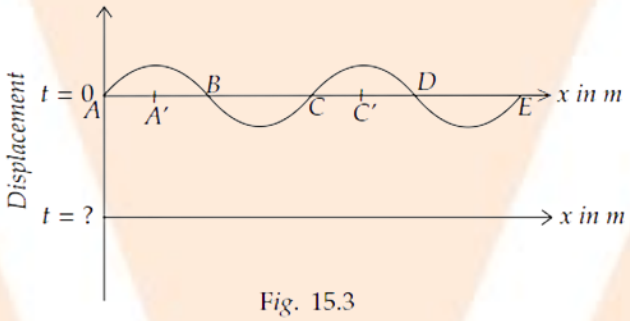


Fig. 15.3

a) Calculate the time at which the second curve is plotted.

b) Mark nodes and antinodes on the curve.

c) Calculate the distance between A′ and C′.

Ans: Given, Velocity $v = 360ms^{-1}$

Frequency of wave, $f = 256Hz$

$\therefore T = \frac{1}{f}$

$\Rightarrow T = \frac{1}{256}\text{sec}$

$\Rightarrow T = 0.00390$

$\Rightarrow T = 3.9 \times 10^{-3}\text{ sec}$

a) In stationary wave, when particle's mean position changes after $\frac{T}{4}$, the time is calculated as

b) Point doesn't vibrate i.e., their displacement is zero always. So nodes are at A,B,C,D and E. The point A' and C' are at maximum displacement. So, the antinodes are A' and C'.

c) Distance between A' and C' is given by, $\lambda = \frac{v}{f}$

$\Rightarrow \lambda = \frac{360}{256}$

$\Rightarrow \lambda = 1.41m$

30. A tuning fork vibrating with a frequency of $512Hz$ is kept close to the open end of a tube filled with water (Fig. 15.4). The water level in the tube is gradually lowered. When the water level is $17cm$ below the open end, maximum intensity of sound is heard. If the room temperature is $20^{\circ}C$, calculate

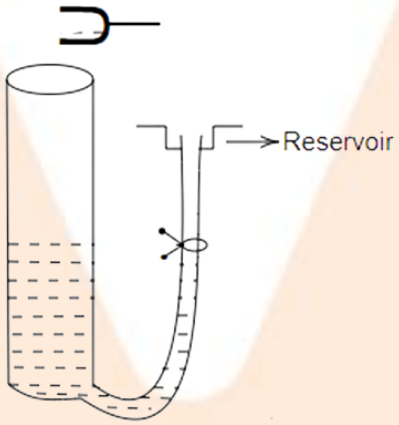


Fig. 15.4

a) speed of sound in air at room temperature

Ans: Pipe partially filled with water behaves like a one end open organ pipe. In 1st harmonic, one anti-node and one node at water level formed so, first harmonic or at maximum intensity is heard at $L = 17cm$. So,

$L = \frac{\lambda}{4}$

$\Rightarrow \lambda = 4L$

$\Rightarrow \lambda = 4 \times 17$

$\Rightarrow \lambda = 68cm$

$\Rightarrow \lambda = 0.68m$

Frequency of tuning fork, $f = 512Hz$

Velocity of sound in air, $v = \lambda f$. Therefore, we get

$\Rightarrow v = 0.68 \times 512 m/s$

$\Rightarrow v = 348.16 m/s$

Thus, velocity of sound in air at room temperature $20^{\circ}C$ is $348.16 m/s$

b) speed of sound in air at $0^{\circ}C$

Ans: We know that, $v \alpha \sqrt{T}$

$\Rightarrow \frac{v_0}{v_T} = \sqrt{\frac{T_0}{T}}$

$T_0 = 273K$

$v_T = v_{20} = 348.16$

$T = 20 + 273 = 293K$

$\Rightarrow v_0 = v_T \sqrt{\frac{T_0}{T_T}}$

$\Rightarrow v_0 = 348.16 \sqrt{\frac{273}{293}}$

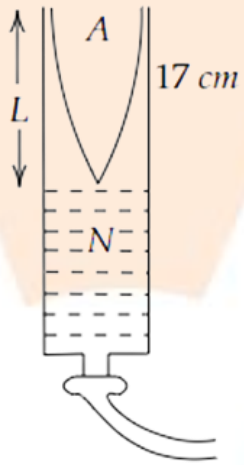
$\Rightarrow v_0 = 348.16 \sqrt{0.9317}$

$\Rightarrow v_0 = 348.16 \times 0.96526$

$\Rightarrow v_0 = 336 m/s$

c) if the water in the tube is replaced with mercury, will there be any difference in your observations?

Ans: The sound is reflected into air column by water and mercury to form stationary wave. Mercury is denser than water, thus refection is more in mercury than water. So, intensity of sound heard will be larger but reading doesn't change as the medium in tube (air) and tuning fork are same.

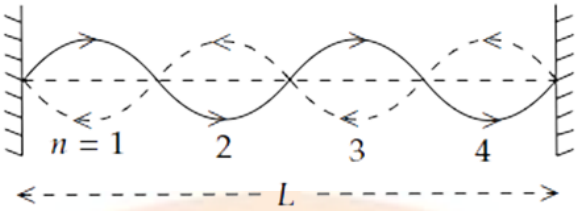


31. Show that when a string fixed at its two ends vibrates in 1 loop, 2 loops, 3 loops and 4 loops, the frequencies are in the ratio 1:2:3:4.

Ans: Let the number of loops in the string be n

Length of each loop = $\frac{\lambda}{2}$

$\therefore L = \frac{n\lambda}{2}$ or $\lambda = \frac{2L}{n}$



We know that velocity, $\upsilon = \lambda v$. Therefore, we get

$\Rightarrow \frac{\upsilon}{v} = \lambda$

$\Rightarrow \frac{\upsilon}{v} = \frac{2L}{n}$

So, $v = \frac{n}{2L}.\upsilon$

We know that $\upsilon = \sqrt{\frac{T}{m}}$ for stretch string.

$\Rightarrow v = \frac{n}{2L}\sqrt{\frac{T}{m}}$

For $n = 1$, $v_1 = \frac{1}{2L}\sqrt{\frac{T}{m}} = v_0$

For $n = 2$, $v_2 = \frac{2}{2L}\sqrt{\frac{T}{m}} = 2v_0$

For $n = 3$, $v_3 = \frac{3}{2L}\sqrt{\frac{T}{m}} = 3v_0$

For $n = 4$, $v_4 = \frac{4}{2L}\sqrt{\frac{T}{m}} = 4v_0$

Therefore, $v_1 : v_2 : v_3 : v_4 = 1:2:3:4$

Long Answer Type Questions

32. The earth has a radius of $6400km$. The inner core of $1000km$ radius is solid. Outside it, there is a region from $1000km$ to a radius of $3500km$ which is in molten state. Then again from $3500km$ to $6400km$ the earth is solid. Only longitudinal (P) waves can travel inside a liquid. Assume that the P wave has a speed of $8km\ s^{-1}$ in solid parts and of $5km\ s^{-1}$ in liquid parts of the earth. An earthquake occurs at some place close to the surface of the earth. Calculate the time after which it will be recorded in a seismometer at a diametrically opposite point on the earth if wave travels along diameter?

Ans: We have,

$r_1 = 1000km$

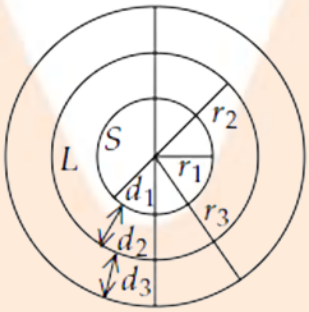
$r_2 = 3500km$

$r_3 = 6400km$

$d_1 = 1000km$

$d_2 = 3500 - 1000 = 2500km$

$d_3 = 6400 - 3500 = 2900km$



Solid distance diametrically = $2(d_1 + d_3)$

$= 2(1000 + 2900)$

$= 2 \times 3900km$

Thus, time taken by wave produced by earthquake in solid part = $\frac{3900\times 2}{8}\sec$

Liquid part along diametrically = $2d_2 = 2\times 2500$

Thus, time taken by seismic wave in liquid part = $\frac{2500\times 2}{5}\sec$

Total time = $\frac{3900\times 2}{8}+\frac{2500\times 2}{5}$

$=2\left(\frac{3900}{8}+\frac{2500}{5}\right)$

$=2\left(487.5+500\right)$

$=2\times 987.5$

$=1975\sec$

$=32\min 55\sec$

Therefore, required answer is $32\min 55\sec$.

33. If c is r.m.s. speed of molecules in a gas and v is the speed of sound waves in the gas, show that $\frac{c}{v}$ is constant and independent of temperature for all diatomic gases.

Ans: We know that for molecules, $c=\sqrt{\frac{3P}{\rho}}$.

We know that $\frac{p}{\rho}=\frac{PT}{M}$. Therefore, we get

$\Rightarrow \frac{P}{\rho}=\frac{RT/V}{M/V}$

$$\Rightarrow \frac{P}{\rho} = \frac{RT}{M}$$

Thus, we can write the equation for the molecules as,

$$\Rightarrow c = \sqrt{\frac{3RT}{M}}$$

Where M is the molar mass of gas.

For sound waves, we have

$$\Rightarrow v = \sqrt{\frac{\gamma P}{\rho}}$$

We know that $PV = nRT$. For $n = 1$, we have

$$\Rightarrow P = \frac{RT}{V}$$

Thus, we get

$$\Rightarrow v = \sqrt{\frac{\gamma RT}{M}}$$

$$\Rightarrow \frac{c}{v} = \frac{\sqrt{\frac{3RT}{M}}}{\sqrt{\frac{\gamma RT}{M}}}$$

$$\Rightarrow \frac{c}{v} = \sqrt{\frac{3}{\gamma}}$$

We know that $\gamma = \frac{C_P}{C_v} = \frac{7}{5}$ is adiabatic constant for diatomic gas. Therefore, we get

$$\Rightarrow \frac{c}{v} = \sqrt{\frac{3}{\frac{7}{5}}}$$

$\Rightarrow \frac{c}{v} = \sqrt{\frac{3\times 5}{7}}$

$\Rightarrow \frac{c}{v} = \sqrt{\frac{15}{7}}$ = constant.

34. Given below are some functions of x and t to represent the displacement of an elastic wave.

a) $y = 5\cos(4x)\sin(20t)$

b) $y = 4\sin\left(5x - \frac{t}{2}\right) + 3\cos\left(5x - \frac{t}{2}\right)$

c) $y = 10\cos\left[(252-250)\pi t\right]\cos\left[(252+250)\pi t\right]$

d) $y = 100\cos(100\pi t + 0.5x)$

State which of these represent

a) a travelling wave along $-x$ direction

b) a stationary wave

c) beats

d) a travelling wave along $+x$ direction.

Given reasons for your answers.

Ans: a) A travelling wave along $(-x)$ direction must have $+kx$ i.e., in (iv)
$y = 100\cos(100\pi t + 0.5x)$

b) When a stationary wave is there, we have (i) $y = 5\cos(4x)\sin(20t)$

c) Beats involve $(v_1 + v_2)$ and $(v_1 - v_2)$. So, beats can be represented by (iii)
$y = 10\cos\left[(252-250)\pi t\right]\cos\left[(252+250)\pi t\right]$

d) $y = 4\sin\left(5x - \frac{t}{2}\right) + 3\cos\left(5x - \frac{t}{2}\right).....(i)$

Let $4 = a\cos\phi....(ii)$

And, $3 = a\sin\phi.....(iii)$

On squaring and adding equation (ii) and (iii), we get

$\Rightarrow a^2\sin^2\phi + a^2\cos^2\phi = 3^2 + 4^2$

$\Rightarrow a^2\left(\sin^2\phi + \cos^2\phi\right) = 9 + 16$

$\Rightarrow a^2 = 25$

$\Rightarrow a = 25$

On substituting (ii) and (iii) in equation (i), we get

$\Rightarrow y = a\cos\phi\sin\left(5x - \frac{t}{2}\right) + a\sin\phi\cos\left(5x - \frac{t}{2}\right)$

$\Rightarrow y = a\sin\left(5x - \frac{t}{2} + \phi\right)$

$\Rightarrow y = 5\sin\left(5x - \frac{t}{2} + \phi\right)$

We can see in the above written equation that the sign of $5x$ and $\frac{1}{2}t$ are opposite. Hence, the wave travels in $+x$ direction. Therefore, correct answer is (ii).

35. In the given progressive wave $y = 5\sin\left(100\pi t - 0.4\pi x\right)$ where y and x are in m, t is in s. What is the

a) amplitude

b) wave length

c) frequency

d) wave velocity

e) particle velocity amplitude.

Ans: Wave is travelling in $+x$ direction because ωt and kx have opposite sign.

Standard equation is given by, $y = a\sin(\omega t - kx + \phi)$

We have, $y = 5\sin(100\pi t - 0.4\pi x)$

Therefore, on comparing we get $a = 5m$, $\omega = 100\pi$, $k = 0.4\pi$

a) Amplitude, $a = 5m$

b) Wavelength λ

We know that, $k = \frac{2\pi}{\lambda}$. Therefore, we get

$$\Rightarrow \lambda = \frac{2\pi}{k}$$

We have, $k = 0.4\pi$

$$\Rightarrow \lambda = \frac{2\pi}{0.4\pi}$$

$$\Rightarrow \lambda = 5m$$

c) Frequency v

We know that, $\omega = 2\pi v$. Therefore, we get

$$\Rightarrow v = \frac{\omega}{2\pi}$$

We have, $\omega = 100\pi$

$$\Rightarrow v = \frac{100\pi}{2\pi}$$

$\Rightarrow v = 50Hz$

d) Wave velocity υ

We know that, $\upsilon = \lambda v$. Therefore, we get

$\Rightarrow \upsilon = 5 \times 50$

$\Rightarrow \upsilon = 250m/s$

e) Particle velocity at a distance x from source in the direction of amplitude,

$\Rightarrow y = 5\sin(100\pi t - 0.4\pi x)$

$\Rightarrow \frac{dy}{dt} = 5 \times 100\pi \cos(100\pi t - 0.4\pi x)$

For maximum velocity, particle is at its mean position

$\Rightarrow \cos(100\pi t - 0.4\pi x) = 1$

$\Rightarrow 100\pi t - 0.4\pi x = 0$

$\therefore \left(\frac{dy}{dt}\right)_{\max} = 5 \times 100\pi \times 1$

Thus, $v_{\max}$ of medium particle = $500\pi m/s$.

36. For the harmonic travelling wave $y = 2\cos 2\pi(10t - 0.0080x + 3.5)$ where x and y are in cm and t is second. What is the phase difference between the oscillatory motion at two points separated by a distance of

a) $4m$

b) $0.5m$

c) $\frac{\lambda}{2}$

d) $\frac{3\lambda}{4}$ (at a given instant of time)

e) What is the phase difference between the oscillation of a particle located at $x = 100cm$, at $t = T\ s$ and $t = 5s$?

Ans: We have, $y = 2\cos 2\pi(10t - 0.0080x + 3.5)$

$\Rightarrow y = 2\cos(20\pi t - 0.016\pi x + 7\pi)$

Wave is propagated in $+x$ direction because ωt and kx have opposite sign.

We know that standard equation is, $y = a\cos(\omega t - kx + \phi)$.

Therefore, we get $a = 2$, $\omega = 20\pi$, $k = 0.016\pi$ and $\phi = 7\pi$.

We know that phase difference $\triangle\phi = \frac{2\pi}{\lambda} \times p$, where $\frac{2\pi}{\lambda} = k$.

So, we get $\triangle\phi = k \times p$

a) Given that, path difference $p = 4m$

$\Rightarrow p = 400cm$

$\Rightarrow \triangle\phi = k \times 400$

$\Rightarrow \triangle\phi = 0.016\pi \times 400$

$\Rightarrow \triangle\phi = 6.4\pi\ radian$

Hence, phase difference = $6.4\pi\ radian$

b) Given that, path difference $p = 0.5m$

$\Rightarrow p = 50cm$

$\Rightarrow \triangle\phi = k \times 50$

$\Rightarrow \triangle\phi = 0.016\pi \times 50$

$\Rightarrow \triangle\phi = 0.8\pi\ radian$

Hence, phase difference = $0.8\pi\ radian$

c) Given that, path difference, $p = \frac{\lambda}{2}$

We know that $\triangle\phi = \frac{2\pi}{\lambda} \times p$. Therefore, we get

$\Rightarrow \triangle\phi = \frac{2\pi}{\lambda} \times \frac{\lambda}{2}$

$\Rightarrow \triangle\phi = \pi\ radian$

Hence, phase difference = $\pi\ radian$

d) Given that, path difference $p = \frac{3\lambda}{4}$

We know that $\triangle\phi = \frac{2\pi}{\lambda} \times p$. Therefore, we get

$\Rightarrow \triangle\phi = \frac{2\pi}{\lambda} \times \frac{3\lambda}{4}$

$\Rightarrow \triangle\phi = \frac{3\lambda}{2} radian$

Hence, phase difference = $\frac{3\lambda}{2} radian$

e) We know that $T = \frac{2\pi}{\omega}$. On substituting the value of $\omega = 2\pi$, we get

$\Rightarrow T = \frac{2\pi}{20\pi}$

$\Rightarrow T = \frac{1}{10}\sec$

We have,

$x = 100cm$

$t = T$

At $x = 100cm$, $t = T$

$\Rightarrow \phi_1 = 20\pi T - 0.016\pi(100) + 7\pi$

$\Rightarrow \phi_1 = 20\pi \frac{1}{10} - 1.6\pi + 7\pi$

$\Rightarrow \phi_1 = 2\pi - 1.6\pi + 7\pi$

$\Rightarrow \phi_1 = 7.4\pi$

At $x = 100cm$, $t = 5s$

$\Rightarrow \phi_2 = 20\pi(5) - 0.016\pi(100) + 7\pi$

$\Rightarrow \phi_2 = 100\pi - 1.6\pi + 7\pi$

$\Rightarrow \phi_2 = 105.4\pi$

Therefore, phase difference = $\phi_2 - \phi_1$

$\Rightarrow 105.4\pi - 7.4\pi = 98\pi \; radian$