

POLYNOMIALS - CHAPTER 2

EXERCISE 2A

Answer 1:

(i) $x^5 - 2x^3 + x + \sqrt{3}$ is an expression having only non-negative integral powers of x . So, it is a polynomial. Also, the highest power of x is 5, so, it is a polynomial of degree 5.

(ii) $y^3 + \sqrt{3}y$ is an expression having only non-negative integral powers of y . So, it is a polynomial. Also, the highest power of y is 3, so, it is a polynomial of degree 3.

(iii) $t^2 - \frac{2}{5}t + \sqrt{5}$ is an expression having only non-negative integral powers of t . So, it is a polynomial. Also, the highest power of t is 2, so, it is a polynomial of degree 2.

(iv) $x^{100} - 1$ is an expression having only non-negative integral power of x . So, it is a polynomial. Also, the highest power of x is 100, so, it is a polynomial of degree 100.

(v) $\frac{1}{\sqrt{2}}x^2 - \sqrt{2}x + 2$ is an expression having only non-negative integral powers of x . So, it is a polynomial. Also, the highest power of x is 2, so, it is a polynomial of degree 2.

(vi) $x^{-2} + 2x^{-1} + 3$ is an expression having negative integral powers of x . So, it is not a polynomial.

(vii) Clearly, 1 is a constant polynomial of degree 0.

(viii) Clearly, $-\frac{3}{5}$ is a constant polynomial of degree 0.

(ix) $\frac{x^2}{2} - 2x^2 = \frac{x^2}{2} - 2x^2$

This is an expression having negative integral power of x i.e. -2 . So, it is not a polynomial.

(x) $\sqrt[3]{2}x^2 - 8$ is an expression having only non-negative integral power of x . So, it is a polynomial. Also, the highest power of x is 2, so, it is a polynomial of degree 2.

(xi) $\frac{1}{2x^2} = \frac{1}{2}x^{-2}$ is an expression having negative integral power of x . So, it is not a polynomial.

(xii) $\frac{1}{\sqrt{5}}x^{\frac{1}{2}} + 1$

In this expression, the power of x is $\frac{1}{2}$ which is a fraction. Since it is an expression having fractional power of x , so, it is not a polynomial.

(xiii) $\frac{3}{5}x^2 - \frac{7}{3}x + 9$ is an expression having only non-negative integral powers of x . So, it is a polynomial. Also, the highest power of x is 2, so, it is a polynomial of degree 2.

(xiv) $x^4 - x^{\frac{3}{2}} + x - 3$

In this expression, one of the powers of x is $\frac{3}{2}$ which is a fraction. Since it is an expression having fractional power of x , so, it is not a polynomial.

$$(xv) 2x^3 + 3x^2 + \sqrt{x} - 1 = 2x^3 + 3x^2 + x^{\frac{1}{2}} - 1$$

In this expression, one of the powers of x is $\frac{1}{2}$ which is a fraction. Since it is an expression having fractional power of x , so, it is not a polynomial.

Answer 2:

- (i) $-7 + x$ is a polynomial with degree 1.
it is a linear polynomial.
- (ii) $6y$ is a polynomial with degree 1.
it is a linear polynomial.
- (iii) $-z^3$ is a polynomial with degree 3.
it is a cubic polynomial.
- (iv) $1 - y - y^3$ is a polynomial with degree 3.
it is a cubic polynomial.
- (v) $x - x^3 + x^4$ is a polynomial with degree 4.
it is a quartic polynomial.
- (vi) $1 + x + x^2$ is a polynomial with degree 2.
it is a quadratic polynomial.
- (vii) $-6x^2$ is a polynomial with degree 2.
it is a quadratic polynomial.
- (viii) -13 is a polynomial with degree 0.
it is a constant polynomial.
- (ix) $-p$ is a polynomial with degree 1.
it is a linear polynomial.

ANSWER 3. i) In $x + 3x^2 - 5x^3 + x^4$ the coefficient of x^3 is -5 .

ii) In $\sqrt{3} - 2\sqrt{2}x + 6x^2$ the coefficient of x is $-2\sqrt{2}$.

iii) $2x - 3 + x^3$ can be written as $x^3 + 0x^2 + 2x - 3$.
In $x^3 + 0x^2 + 2x - 3$ the coefficient of x^2 is 0 .

iv) In $\frac{3}{8}x^2 - \frac{2}{7}x + \frac{1}{6}$ the coefficient of x is $-\frac{2}{7}$.

v) In $\frac{\pi}{2}x^2 + 7x - \frac{2}{5}\pi$ the constant term is $-\frac{2}{5}\pi$.

ANSWER 4. i) $\frac{4x-5x^2+6x^3}{2x}$

We can write it separately as

$$= \frac{4x}{2x} - \frac{5x^2}{2x} + \frac{6x^3}{2x}$$

On further simplification we get

$$= 2 - \frac{5}{2}x + 3x^2$$

The degree of given expression is 2.

ii) $y^2(y - y^3)$

By multiplying the terms

We get

$$= y^3 - y^5$$

The degree of the given expression is 5.

iii) $(3x-2)(2x^3 + 3x^2)$

By multiplying the terms

We get

$$6x^4 + 9x^3 - 4x^3 - 6x^2$$

On further simplification

$$= 6x^4 + 5x^3 - 6x^2$$

The degree of the given expression is 4.

iv) $-\frac{1}{2}x + 3$

The degree of the given expression is 1.

v) -8

The given expression is a constant polynomial of degree is zero .

vi) $x^{-2}(x^4 + x^2)$

By taking common terms out

$$= x^{-2} \cdot x^2(x^2 + 1)$$

On further simplification

$$=x^{-2+2}(x^2+1)$$

So we get

$$=x^0(x^2+1) \\ =x^2+1$$

The degree of the expression is 2.

ANSWER 5. i) Example of a monomial of degree 5 is $4x^5$.

ii) Example of a binomial of degree 8 is $x - 4x^8$.

iii) Example of a trinomial of degree 4 is $1 + 3x + x^4$.

iv) Example of a monomial of degree 0 is 1.

ANSWER 6. i) $x - 2x^2 + 8 + 5x^3$ in standard form is written as $5x^3 - 2x^2 + x + 8$.

ii) $\frac{2}{3} + 4y^2 - 3y + 2y^3$ in standard form is written as $2y^3 + 4y^2 - 3y + \frac{2}{3}$.

iii) $6x^3 + 2x - x^5 - 3x^2$ in standard form is written as $-x^5 + 6x^3 - 3x^2 + 2x$.

iv) $2 + t - 3t^3 + t^4 - t^2$ in standard form is written as $t^4 - 3t^3 - t^2 + t + 2$.

EXERCISE-2B

ANSWER1)

$$p(x) = 5 - 4x + 2x^2$$

$$(i) \quad p(0) = 5 - 4 \times 0 + 2 \times 0^2 = 5$$

$$(ii) \quad p(3) = 5 - 4 \times 3 + 2 \times 3^2 \\ = 5 - 12 + 18 \\ = 23 - 12 = 11$$

$$(iii) \quad p(-2) = 5 - 4(-2) + 2(-2)^2 \\ = 5 + 8 + 8 = 21$$

Answer 2) $p(y) = 4 + 3y - y^2 + 5y^2$

$$(i) \quad p(0) = 4 + 3 \times 0 - 0^2 + 5 \times 0^2 \\ = 4 + 0 - 0 + 0 = 4$$

$$(ii) \quad p(2) = 4 + 3 \times 2 - 2^2 + 5 \times 2^2 \\ = 4 + 6 - 4 + 40 \\ = 10 - 4 + 40 = 46$$

$$(iii) \quad p(-1) = 4 + 3(-1) - (-1)^2 + 5(-1)^2 = 4 - 3 - 1 + 5 = 5$$

Answer 3) $f(t) = 4t^2 - 3t + 6$

$$(i) \quad f(0) = 4 \times 0^2 - 3 \times 0 + 6 \\ = 0 - 0 + 6 = 6$$

$$(ii) \quad f(4) = 4(4)^2 - 3 \times 4 + 6 \\ = 64 - 12 + 6 = 58$$

$$(iii) \quad f(-5) = 4(-5)^2 - 3(-5) + 6 \\ = 100 + 15 + 6 = 121$$

Answer 4)

$$p(x) = x^3 - 3x^2 + 2x$$

Thus, we have

$$p(0) = 0^3 - 3(0)^2 + 2(0) = 0$$

$$p(1) = 1^3 - 3(1)^2 + 2(1) = 1 - 3 + 2 = 0$$

$$p(2) = 2^3 - 3(2)^2 + 2(2) = 8 - 12 + 4 = 0$$

Hence, 0, 1 and 2 are the zeros of the polynomial $p(x) = x^3 - 3x^2 + 2x$.

Answer 5)

$$p(x) = x^3 + x^2 - 9x - 9$$

Thus, we have

$$p(0) = 0^3 + 0^2 - 9(0) - 9 = -9$$

$$p(3) = 3^3 + 3^2 - 9(3) - 9 = 27 + 9 - 27 - 9 = 0$$

$$p(-3) = (-3)^3 + (-3)^2 - 9(-3) - 9 = -27 + 9 + 27 - 9 = 0$$

$$p(-1) = (-1)^3 + (-1)^2 - 9(-1) - 9 = -1 + 1 + 9 - 9 = 0$$

Hence, 0, 3 and -3 are the zeros of $p(x)$.

Now, 0 is not a zero of $p(x)$ since $p(0) \neq 0$.

Answer 6)

i) $p(x) = x - 4$

Then, $p(4) = 4 - 4 = 0$

\Rightarrow 4 is a zero of the polynomial $p(x)$.

ii) $q(x) = x + 3$

Then, $q(-3) = -3 + 3 = 0$

\Rightarrow -3 is not a zero of the polynomial $p(x)$.

iii) $p(x) = 2 - 5x$

Then,

$\Rightarrow \frac{2}{5}$ is a zero of the polynomial $p(x)$.

iv) $p(y) = 2y + 1$

Then,

is $\frac{-1}{2}$ a zero of the polynomial $p(y)$.

Answer 7)

i) $p(x) = x^2 - 3x + 2$

$$p(x) = (x - 1)(x - 2)$$

Then, $p(1) = (1 - 1)(1 - 2) = 0 \times -1 = 0$

\Rightarrow 1 is a zero of the polynomial $p(x)$.

Also, $p(2) = (2 - 1)(2 - 2) = 1 \times 0 = 0$

\Rightarrow 2 is a zero of the polynomial $p(x)$.

Hence, 1 and 2 are the zeroes of the polynomial $p(x)$.

ii) $q(x) = x^2 + x - 6$

Then, $q(2) = 2^2 + 2 - 6$

$= 4 + 2 - 6$

$= 6 - 6 = 0$

\Rightarrow 2 is a zero of the polynomial $p(x)$.

Also, $q(-3) = (-3)^2 - 3 - 6$

$= 9 - 3 - 6 = 0$

\Rightarrow -3 is a zero of the polynomial $p(x)$.

Hence, 2 and -3 are the zeroes of the polynomial $p(x)$.

iii) $r(x) = x^2 - 3x$

Then, $p(0) = 0^2 - 3 \times 0 = 0$

$r(3) = (3)^2 - 3 \times 3 = 9 - 9 = 0$

\Rightarrow 0 and 3 are the zeroes of the polynomial $p(x)$.

Answer 8)

(i) $p(x) = 0$

$\Rightarrow x - 5 = 0$

$\Rightarrow x = 5$

\Rightarrow 5 is the zero of the polynomial $p(x)$.

(ii) $q(x) = 0$

\Rightarrow is a zero of the polynomial $p(y)$.

$\Rightarrow x + 4 = 0$

$\Rightarrow x = -4$

\Rightarrow -4 is the zero of the polynomial $q(x)$.

(iii) $r(x) = 2x + 5$

Now, $r(x) = 0$

$\Rightarrow 2x + 5 = 0$

$\Rightarrow 2x = -5$

$$\Rightarrow x = -\frac{5}{2}$$

$\therefore -\frac{5}{2}$ is a zero of the polynomial $r(x)$.

(iv) $f(x) = 0$

$$\Rightarrow 3x + 1 = 0$$

$$\Rightarrow 3x = -1$$

$$\Rightarrow x = \frac{-1}{3}$$

$\Rightarrow x = \frac{-1}{3}$ is the zero of the polynomial $f(x)$.

(v) $g(x) = 0$

$$\Rightarrow 5 - 4x = 0$$

$$\Rightarrow -4x = -5$$

$$\Rightarrow x = \frac{5}{4}$$

$\Rightarrow x = \frac{5}{4}$ is the zero of the polynomial $g(x)$.

(vi) $h(x) = 6x - 2$

Now, $h(x) = 0$

$$\Rightarrow 6x - 2 = 0$$

$$\Rightarrow 6x = 2$$

$$\Rightarrow x = \frac{2}{6} = \frac{1}{3}$$

$\therefore \frac{1}{3}$ is a zero of the polynomial $h(x)$.

(vii) $p(x) = 0$

$$\Rightarrow ax = 0$$

$$\Rightarrow x = 0$$

$\Rightarrow 0$ is the zero of the polynomial $p(x)$.

(viii) $q(x) = 0$

$$\Rightarrow 4x = 0$$

$$\Rightarrow x = 0$$

⇒ 0 is the zero of the polynomial $q(x)$.

$$9) f(x) = 2x^3 - 5x^2 + ax + b$$

Now, 2 is a zero of $f(x)$.

$$\Rightarrow f(2) = 0$$

$$\Rightarrow 2(2)^3 - 5(2)^2 + a(2) + b = 0$$

$$\Rightarrow 16 - 20 + 2a + b = 0$$

$$\Rightarrow 2a + b - 4 = 0 \dots(i)$$

Also, 0 is a zero of $f(x)$.

$$\Rightarrow f(0) = 0$$

$$\Rightarrow 2(0)^3 - 5(0)^2 + a(0) + b = 0$$

$$\Rightarrow 0 - 0 + 0 + b = 0$$

$$\Rightarrow b = 0$$

Substituting $b = 0$ in (i), we get

$$2a + 0 - 4 = 0$$

$$\Rightarrow 2a = 4$$

$$\Rightarrow a = 2$$

Thus, $a = 2$ and $b = 0$

EXERCISE 2C

ANSWER 1. We can write (x^4+1) as $(x^4+0x^3+0x^2+0x+1)$

$$\begin{array}{r}
 x-1 \overline{) x^4+0x^3+0x^2+0x+1} \quad (x^3+x^2+x+1) \\
 \underline{x^4-x^3} \\
 -x^3+0x^2+0x+1 \\
 \underline{-x^3+x^2} \\
 x^2+0x+1 \\
 \underline{x^2-x} \\
 -x+1 \\
 \underline{-x+1} \\
 2
 \end{array}$$

quotient $= (x^3+x^2+x+1)$ and remainder $= 2$.

By verification:

$$f(x) = x^4 + 1$$

By substituting 1 in the place of x

$$f(1) = 1^4 + 1$$

$$f(1) = 1 + 1$$

so we get

$$f(1) = 2, \text{ which is remainder.}$$

ANSWER 2. $\overline{(x+2)2x^4-6x^3+2x^2-x+2(2x^3-10x^2+22x-45)}$
 $2x^4+4x^3$

$$\begin{array}{r}
 - \\
 \underline{-10x^3+2x^2} \\
 -10x^3-20x^2 \\
 \underline{+ } \\
 22x^2-x \\
 \\
 22x^2-44x \\
 \underline{- } \\
 -45x+2 \\
 -45x-90 \\
 \underline{+ } \\
 92
 \end{array}$$

We know that ,

$$(x+2)(2x^3-10x^2+22x-45) + 92$$

So we get ,

$$= 2x^4 - 10x^3 + 22x^2 - 45x + 4x^3 - 20x^2 + 44x - 90 + 92$$

$$= 2x^4 - 6x^3 + 2x^2 - x + 2$$

$$= p(x)$$

Therefore ,the division algorithm is verified.

ANSWER 3. Given,

$$p(x) = x^3 - 6x^2 + 9x + 3$$

To find the value of x ,

Consider,

$$g(x) = 0$$

$$x - 1 = 0$$

So we get

$$x = 1$$

According to the remainder theorem,

$p(x)$ divided by $(x-1)$ obtains the remainder as $g(1)$.

Calculating $g(1)$

$$= 1^3 - 6(1)^2 + 9(1) + 3$$

On further simplification

$$= 1 - 6 + 9 + 3$$

So we get

$$= (-5) + 12$$

$$= 7$$

Therefore, the remainder of the given expression is 7.

ANSWER 4. Given,

$$p(x) = 2x^3 - 7x^2 + 9x - 13$$

To find the value of x ,

Consider,

$$g(x) = 0$$

$$x - 3 = 0$$

so we get

$$x = 3$$

According to the remainder theorem,

$p(x)$ divided by $(x-3)$ obtains the remainder as $g(3)$.

Calculating $g(3)$

$$= 2(3)^3 - 7(3)^2 + 9(3) - 13$$

On further simplification
 $= 2(27) - 7(9) + 27 - 13$

So we get
 $= 54 - 63 + 27 - 13$
 $= 5$

Therefore, the remainder of the given expression is 5.

ANSWER 5. Given,
 $p(x) = 3x^4 - 6x^2 - 8x - 2$

To find the value of x ,
Consider,
 $g(x) = 0$

$$x - 2 = 0$$

so we get
 $x = 2$

According to the remainder theorem,
 $p(x)$ divided by $(x-2)$ obtains the remainder as $g(2)$.

Calculating $g(2)$
 $= 3(2)^4 - 6(2)^2 - 8(2) - 2$

On further simplification
 $= 3(16) - 6(4) - 16 - 2$

So we get
 $= 48 - 24 - 16 - 2$
 $= 6$

Therefore, the remainder of the given expression is 6.

ANSWER 6. Given,
 $p(x) = 2x^3 - 9x^2 + x + 15$

To find the value of x ,
Consider,
 $g(x) = 0$

$$2x - 3 = 0$$

so we get

$$2x = 3$$

By dividing

$$x = \frac{3}{2}$$

According to the remainder theorem,
 $p(x)$ divided by $(2x-3)$ obtains the remainder as $g(\frac{3}{2})$.

Calculating $g(\frac{3}{2})$

$$= 2(\frac{3}{2})^3 - 9(\frac{3}{2})^2 + (\frac{3}{2}) + 15$$

On further simplification

$$= 2(\frac{27}{8}) - 9(\frac{9}{4}) + (\frac{3}{2}) + 15$$

$$= \frac{27}{4} - \frac{81}{4} + \frac{3}{2} + 15$$

$$= \frac{27-81+6+60}{4}$$

So we get

$$= \frac{12}{4}$$

$$= 3$$

Therefore, the remainder of the given expression is 3.

ANSWER 7. Given,

$$p(x) = x^3 - 2x^2 - 8x - 1$$

To find the value of x ,

Consider,

$$g(x) = 0$$

$$x + 1 = 0$$

so we get

$$x = -1$$

According to the remainder theorem,
 $p(x)$ divided by $(x+1)$ obtains the remainder as $g(-1)$.

Calculating $g(-1)$

$$= (-1)^3 - 2(-1)^2 - 8(-1) - 1$$

On further simplification

$$= -1 - 2 + 8 - 1$$

So we get

$$= -3 + 7$$

$$= 4$$

Therefore, the remainder of the given expression is 4.

ANSWER 8. Given,

$$p(x) = 2x^3 + x^2 - 15x - 12$$

To find the value of x ,

Consider,

$$g(x) = 0$$

$$x + 2 = 0$$

So we get

$$x = -2$$

According to the remainder theorem,

$p(x)$ divided by $(x + 2)$ obtains the remainder as $g(-2)$.

Calculating $g(-2)$

$$= 2(-2)^3 + (-2)^2 - 15(-2) - 12$$

On further simplification

$$= -16 + 4 + 30 - 12$$

So we get

$$= -12 + 18$$

$$= 6$$

Therefore, the remainder of the given expression is 6.

ANSWER 9. Given,

$$p(x) = 6x^3 + 13x^2 + 3$$

To find the value of x ,

Consider,

$$g(x) = 0$$

$$3x+2 = 0$$

So we get

$$x = \left(\frac{-2}{3}\right)$$

According to the remainder theorem,
 $p(x)$ divided by $(3x+ 2)$ obtains the remainder as $g\left(\frac{-2}{3}\right)$.

$$\begin{aligned} \text{Calculating } g\left(\frac{-2}{3}\right) \\ = 6\left(\frac{-2}{3}\right)^3 + 13\left(\frac{-2}{3}\right)^2 + 3 \end{aligned}$$

$$\begin{aligned} \text{On further simplification} \\ = 6\left(\frac{-8}{27}\right) + 13\left(\frac{4}{9}\right) + 3 \end{aligned}$$

$$= \frac{-48}{27} + \frac{52}{9} + 3$$

$$= \frac{-16}{9} + \frac{52}{9} + 3$$

So we get

$$= \frac{63}{9}$$

$$= 7$$

Therefore , the remainder of the given expression is 7.

ANSWER 10. Given,

$$p(x) = x^3 - 6x^2 + 2x - 4$$

To find the value of x ,

Consider,

$$g(x) = 0$$

$$1 - \frac{3}{2}x = 0$$

So we get

$$x = \frac{2}{3}$$

According to the remainder theorem,

$p(x)$ divided by $\left(1 - \frac{3}{2}x\right)$ obtains the remainder as $g\left(\frac{2}{3}\right)$.

Calculating $g\left(\frac{2}{3}\right)$

$$= \left(\frac{2}{3}\right)^3 - 6\left(\frac{2}{3}\right)^2 + 2\left(\frac{2}{3}\right) - 4$$

On further simplification

$$= \frac{8}{27} - 6\left(\frac{4}{9}\right) + \frac{4}{3} - 4$$

$$= \frac{8}{27} - \frac{8}{3} + \frac{4}{3} - 4$$

$$= \frac{8-72+36-108}{27}$$

So we get

$$= \frac{-136}{27}$$

Therefore, the remainder of the given expression is $\left(\frac{-136}{27}\right)$.

ANSWER 11. Given,

$$p(x) = 2x^3 + 3x^2 - 11x - 3$$

To find the value of x ,

Consider,

$$g(x) = 0$$

$$x + \frac{1}{2} = 0$$

So we get

$$x = \frac{-1}{2}$$

According to the remainder theorem,

$p(x)$ divided by $\left(x + \frac{1}{2}\right)$ obtains the remainder as $g\left(\frac{-1}{2}\right)$.

Calculating $g\left(\frac{-1}{2}\right)$

$$= 2\left(\frac{-1}{2}\right)^3 + 3\left(\frac{-1}{2}\right)^2 - 11\left(\frac{-1}{2}\right) - 3$$

On further simplification

$$= 2\left(\frac{-1}{8}\right) + 3\left(\frac{1}{4}\right) + \frac{11}{2} - 3$$

$$= -\frac{1}{4} + \frac{3}{4} + \frac{11}{2} - 3$$

$$= \frac{-1+3+22-12}{2}$$

So we get

$$= \frac{12}{2}$$

$$= 6$$

Therefore, the remainder of the given expression is 6.

ANSWER 12. Given,

$$p(x) = x^3 - ax^2 + 6x - a$$

To find the value of x,

Consider,

$$g(x) = 0$$

$$x - a = 0$$

So we get

$$x = a$$

According to the remainder theorem,

p(x) divided by (x - a) obtains the remainder as g(a).

Calculating g(a),

$$= a^3 - a(a)^2 + 6a - a$$

On further simplification

$$= a^3 - a^3 + 5a$$

So we get

$$= 5a$$

Therefore, the remainder of the given expression is 5a.

ANSWER 13. Consider $p(x) = (2x^3 + x^2 - ax + 2)$ and $q(x) = (2x^3 - 3x^2 - 3x + a)$

When p(x) and q(x) are divided by (x-2) the remainder obtained is p(2) and q(2).

To find a,

Let us take

$$p(2) = q(2)$$

$$2x^3 + x^2 - ax + 2 = 2x^3 - 3x^2 - 3x + a$$

By substituting 2 in the place of x

$$\Rightarrow 2(2)^3 + (2)^2 - a(2) + 2 = 2(2)^3 - 3(2)^2 - 3(2) + a$$

On further calculation:

$$\Rightarrow 2(8) + 4 - 2a + 2 = 2(8) - 3(4) - 6 + a$$

$$\Rightarrow 16 + 4 - 2a + 2 = 16 - 12 - 6 + a$$

So we get

$$\Rightarrow 22 - 2a = -2 + a$$

$$\Rightarrow 22 + 2 = 2a + a$$

$$\Rightarrow 24 = 3a$$

By dividing

$$a = 8$$

Thus, the value of a is 8.

ANSWER 14. Given,

$$p(x) = x^4 - 2x^3 + 3x^2 - ax + b$$

Consider $(x-1) = 0$ where $x = 1$ and the remainder is 5

$$p(1) = 5$$

by substituting 1 in the place of x

$$1^4 - 2(1)^3 + 3(1)^2 - a + b = 5$$

So we get

$$2 - a + b = 5 \dots\dots\dots(1)$$

Consider $(x+1) = 0$ where $x = -1$ and the remainder is 19 .

$$p(-1) = 19$$

by substituting (-1) in the place of x

$$(-1)^4 - 2(-1)^3 + 3(-1)^2 - a + b = 19$$

So we get

$$1 + 2 + 3 - a + b = 19$$

$$6 - a + b = 19 \dots\dots\dots(2)$$

By adding equation (1) and (2)

$$8 + 2b = 24$$

$$2b = 24 - 8$$

By dividing 16 by 2 we get

$$b = 8 \dots\dots\dots(3)$$

Now applying (3) in (1)

$$2 - a + 8 = 5$$

So we get

$$10 - a = 5$$

$$a = 5$$

Substituting the value of a and b in p(x) when divided by (x-2)

$$p(x) = x^4 - 2x^3 + 3x^2 - ax + b$$

by substituting 2 in the place of x

$$p(2) = 2^4 - 2(2)^3 + 3(2)^2 - (5)(2) + 8$$

On further calculation:

$$p(2) = 16 - 16 + 12 - 10 + 8$$

$$p(2) = 10$$

Therefore , the remainder when p(x) is divided by (x-2) is 10 .

ANSWER 15. Consider,

$$g(x) = 0$$

which means

$$x - 2 = 0$$

$$x = 2$$

Now applying $x = 2$ in $p(x)$, we obtain

$$p(x) = x^3 - 5x^2 + 4x - 3$$

By substituting the value 2 in the place of x

$$p(2) = 2(2)^3 - 5(2)^2 + 4(2) - 3$$

On further calculation:

$$p(2) = 8 - 20 + 8 - 3$$

So we get

$$p(2) = -4 - 3$$

$$p(2) = -7 \neq 0$$

Therefore, it is proved that $p(x)$ is not a multiple of $g(x)$.

ANSWER 16. Consider,

$$g(x) = 0$$

$$2x + 1 = 0$$

So we get

$$2x = -1$$

$$x = \frac{-1}{2}$$

Now apply $x = \frac{-1}{2}$ in $p(x)$

$$p(x) = 2x^3 - 11x^2 - 4x + 5$$

By substituting $\frac{-1}{2}$ in the place of x

$$p\left(\frac{-1}{2}\right) = 2\left(\frac{-1}{2}\right)^3 - 11\left(\frac{-1}{2}\right)^2 - 4\left(\frac{-1}{2}\right) + 5$$

So we get

$$p\left(\frac{-1}{2}\right) = \frac{-1}{4} - \frac{11}{2} + 7$$

$$p\left(\frac{-1}{2}\right) = \frac{-1-11+28}{4}$$

by dividing 16 by 4

$$p\left(\frac{-1}{2}\right) = \left(\frac{16}{4}\right)$$

So we get

$$p\left(\frac{-1}{2}\right) = 4 \neq 0$$

Hence, it is shown that $g(x)$ is not a factor of $p(x)$.



EXERCISE 2D

Answer 1

$$g(x) = x - 2$$

$$\Rightarrow x = 2$$

$$\text{Then, } p(x) = x^3 - 8 = 2^3 - 8 = 0 \text{ (given } x=2)$$

Yes, $g(x)$ is factor of $p(x)$

Answer 2

$$g(x) = x - 3$$

$$\Rightarrow x = 3$$

$$\begin{aligned} \text{Then, } p(x) &= 2x^3 + 7x^2 - 24x - 45 = 2(3^3) + 7(3^2) - 24 \times 3 - 45 \\ &\Rightarrow 54 + 63 - 72 - 45 = 0 \end{aligned}$$

Yes, $g(x)$ is factor of $p(x)$

Answer 3

$$g(x) = x - 1$$

$$\Rightarrow x = 1$$

$$\begin{aligned} \text{Then, } p(x) &= 2x^4 + 9x^3 + 6x^2 - 11x - 6 = 2 \times 1^4 + 9 \times 1^3 + 6 \times 1^2 - 11 \times 1 - 6 \\ &= 2 + 9 + 6 - 11 - 6 \\ &= 0 \end{aligned}$$

Yes, $g(x)$ is factor of $p(x)$

Answer 4

$$g(x) = x + 2$$

$$\Rightarrow x = -2$$

$$\begin{aligned} \text{Then, } p(x) &= x^4 - x^2 - 12 = (-2)^4 - (-2)^2 - 12 \\ &= 16 - 4 - 12 = 0 \end{aligned}$$

Yes, $g(x)$ is factor of $p(x)$

Answer 5

$$g(x) = x + 3$$

$$\Rightarrow x = -3$$

$$\begin{aligned} \text{Then, } p(x) &= 69 + 11x - x^2 + x^3 = 69 + 11(-3) - (-3)^2 + (-3)^3 \\ &\Rightarrow 69 - 33 - 9 - 27 = 0 \end{aligned}$$

Yes, $g(x)$ is factor of $p(x)$

Answer 6

$$g(x) = x + 5$$

$$\Rightarrow x = -5$$

$$\begin{aligned} \text{Then, } p(x) &= 2x^3 + 9x^2 - 11x - 30 = 2(-5)^3 + 9(-5)^2 - 11(-5) - 30 \\ &= -250 + 45 + 55 - 30 \\ &= -180 \end{aligned}$$

Yes, $g(x)$ is not factor of $p(x)$

Answer 7

$$g(x) = 2x - 3$$

$$\Rightarrow x = \frac{3}{2}$$

$$\begin{aligned} \text{Then, } p(x) &= 2x^4 + x^3 - 8x^2 - x + 6 = 2\left(\frac{3}{2}\right)^4 + \left(\frac{3}{2}\right)^3 - 8\left(\frac{3}{2}\right)^2 - \frac{3}{2} + 6 \\ &= \left(2 \frac{81}{16}\right) + \frac{27}{8} - \left(8 \frac{9}{4}\right) - \frac{3}{2} + 6 \\ &= \frac{81}{8} + \frac{27}{8} - \frac{144}{8} - \frac{12}{8} + \frac{48}{8} \\ &= 0 \end{aligned}$$

Yes, $g(x)$ is factor of $p(x)$

Answer 8

$$g(x) = 3x - 2$$

$$\Rightarrow x = \frac{2}{3}$$

$$\begin{aligned} \text{Then, } p(x) &= 3x^3 + x^2 - 20x + 12 = 3\left(\frac{2}{3}\right)^3 + \left(\frac{2}{3}\right)^2 - 20\left(\frac{2}{3}\right) + 12 \\ &= \left(3 \frac{8}{27}\right) + \frac{4}{9} - \left(20 \frac{2}{3}\right) + 12 \\ &= \frac{8}{9} + \frac{4}{9} - \frac{120}{9} + \frac{108}{9} \\ &= 0 \end{aligned}$$

Yes, $g(x)$ is factor of $p(x)$

Answer 9

$$g(x) = x = \sqrt{2}$$

$$\Rightarrow x = \sqrt{2}$$

$$\begin{aligned} \text{Then, } p(x) &= 7x^2 - 4\sqrt{2}x - 6 = 7(\sqrt{2})^2 - 4(\sqrt{2} \times \sqrt{2}) - 6 \\ &\Rightarrow (7 \times 2) - (4 \times 2) - 6 = 14 - 8 - 6 = 0 \end{aligned}$$

Answer 10

$$g(x) = x + \sqrt{2}$$

$$\Rightarrow x = -\sqrt{2}$$

$$\begin{aligned} \text{Then, } p(x) &= 2\sqrt{2}x^2 + 5x + \sqrt{2} = 2\sqrt{2}(-\sqrt{2})^2 + 5 \times (-\sqrt{2}) + \sqrt{2} \\ &= 4\sqrt{2} - 5\sqrt{2} + \sqrt{2} \\ &= 0 \end{aligned}$$

Answer 11

$$\text{Let } g(p) = (p^{10} - 1) \text{ and}$$

$$\text{And } h(p) = (p^{11} - 1)$$

Let $f(p) = (p-1)$
 then, $\Rightarrow p - 1 = 0$
 $\Rightarrow p = 1$

Now, $g(1) = [(p^{10} - 1)] = (1^{10} - 1)$
 $= (1-1) = 0$

Hence, $f(p-1)$ is factor of $g(p)$

$h(p) = (p^{11} - 1)$

$h(1) = [(p^{11} - 1)] = (1^{11} - 1) = (1 - 1) = 0$

hence, $f(p-1)$ is also factor of $h(p)$

Answer12

Here $f(x) = x-1$

$\Rightarrow x = 1$

Now, given $p(x) = 2x^3 + 9x^2 + x + k$
 $\Rightarrow = 2(1)^3 + 9(1)^2 + 1 + k$
 $k = -2 - 9 - 1 = -12$

Answer13

Here $f(x) = x - 4$

$\Rightarrow x = 4$

Now, given $p(x) = 2x^3 - 3x^2 - 18x + a$
 $\Rightarrow = 2(4^3) - 3(4^2) - 18(4) + a$
 $= 2 \times 64 - 48 - 72 + a$
 $a = -8$

Answer14

Here, $f(x) = x + 1$

$\Rightarrow x = -1$

Now, given $p(x) = ax^3 + x^2 - 2x + 4a - 9$
 $= a(-1)^3 + (-1)^2 - 2(-1) + 4a - 9$
 $a - 4a = 1 + 2 - 9$
 $\Rightarrow 3a = 6$
 $\Rightarrow a = 2$

Answer15

Here, $f(x) = x + 2a$

$\Rightarrow x = -2a$

Now, given $p(x) = x^5 - 4a^2x^3 + 2x + 2a + 3$
 $= (-2a)^5 - 4a^2(-2a)^3 + 2(-2a) + 2a + 3$
 $= -32a^5 + 32a^5 - 4a + 2a + 3$
 $2a = 3$
 $a = \frac{3}{2}$

Answer16

Here, $f(x) = 2x-1$

$$\Rightarrow x = \frac{1}{2}$$

$$\begin{aligned} \text{Now, given } p(x) &= 8x^4 + 4x^3 - 16x^2 + 10x + m \\ &= 8\left(\frac{1}{2}\right)^4 + 4\left(\frac{1}{2}\right)^3 - 16\left(\frac{1}{2}\right)^2 + 10\left(\frac{1}{2}\right) + m \\ -m &= \left(8 \times \frac{1}{16}\right) + \left(4 \times \frac{1}{8}\right) - \left(16 \times \frac{1}{4}\right) + \left(10 \times \frac{1}{2}\right) \\ -m &\Rightarrow \frac{1}{2} + \frac{1}{2} - 4 + 5 = \frac{4}{2} \\ m &= -2 \end{aligned}$$

Answer17

$$\text{Here, } f(x) = x+3$$

$$\Rightarrow x = -3$$

$$\begin{aligned} \text{Now, given } p(x) &= x^4 - x^3 - 11x^2 - x + a \\ -a &= (-3)^4 - (-3)^3 - 11(-3)^2 - (-3) \\ -a &\Rightarrow 81 + 27 - 99 + 3 = 12 \\ a &= -12 \end{aligned}$$

Answer18

$$\begin{aligned} \text{Here, } f(x) &= x^2 + 2x - 3 \\ &= (x^2 + 3x - x - 3) = (x+3)(x-1) \end{aligned}$$

Now, $p(x)$ will be divisible by $f(x)$ only when it is divisible by $(x-1)$ as well as by $(x+3)$

Now, $(x-1=0 \Rightarrow x=1)$ and $(x+3 \Rightarrow x=-3)$

By the factor theorem, $p(x)$ will be divisible by $f(x)$, if $p(1) = 0$ and $p(-3) = 0$

$$p(x) = x^3 - 3x^2 - 13x + 15$$

$$p(1) = (1)^3 - 3(1)^2 - 13(1) + 15 = 1 - 3 - 13 + 15 = 0$$

$$p(-3) = (-3)^3 - 3(-3)^2 - 13(-3) + 15 = -27 - 27 + 39 + 15 = 0$$

Answer19

$$p(x) = x^3 + ax^2 + bx + 6, g(x) = x-2 \text{ and } h(x) = x-3, \text{ then,}$$

$$g(x) = 0 \Rightarrow x-2 = 0 \Rightarrow x = 2$$

$$h(x) = 0 \Rightarrow x-3 = 0 \Rightarrow x = 3$$

$$(x-2) \text{ is factor of } p(x) \Rightarrow p(2) = 0$$

$$\begin{aligned} \text{Now, } p(2) = 0 &\Rightarrow [(2)^3 + a(2)^2 + b(2) + 6] = 8 + 4a + 2b + 6 \\ &\Rightarrow 4a + 2b = -14 \dots\dots(1) \end{aligned}$$

Since, it is given that factor $(x-3)$ leaves the remainder 3

$$\begin{aligned} \text{Now, } p(3) = 3 &\Rightarrow [(3)^3 + a(3)^2 + b(3) + 6] = 27 + 9a + 3b + 6 \\ &\Rightarrow 9a + 3b = -30 \dots\dots(2) \end{aligned}$$

Solving both equation,

$$4a + 2b = -14 \dots\dots(\text{divide each term by } 2)$$

$$9a + 3b = -30 \dots\dots(\text{divide each term by } 3)$$

$$\text{We get, } 2a + b = -7 \dots\dots(3)$$

$$3a + b = -10 \dots\dots(4)$$

On solving (3) and (4) we get, $a = -3$ and $b = -1$

Answer20

Here, $f(x) = x-1$ and $x-2$

$\Rightarrow x = 1$ and $x = 2$

Now, $p(x)$ will be divisible by both $(x-1)$ and $(x-2)$

$$p(x) = x^3 - 10x^2 + ax + b$$

$$p(1) = (1)^3 - 10(1)^2 + a(1) + b$$

$$= (1) - 10 + a + b$$

$$a + b = 9 \dots\dots\dots(1)$$

$$p(2) = (2)^3 - 10(2)^2 + a(2) + b$$

$$= 8 - 40 + 2a + b$$

$$2a + b = 32 \dots\dots\dots(2)$$

Solving (1) and (2)

$$a + b = 9$$

$$2a + b = 32$$

By subtracting, we get

$$\Rightarrow a = 32 - 9 = 23$$

And another equation is $\Rightarrow b = 9 - 23 = -14$

Answer 21

$$p(x) = (x^4 + ax^3 - 7x^2 - 8x + b)$$

$$g(x) = (x+2)$$

$$h(x) = (x+3)$$

since $g(x)$ and $h(x)$ both are exactly divisible, these are the factor of $p(x)$

so,

$$p(x) = p(-2) = (-2^4) + a(-2^3) - 7(-2^2) - 8(-2) + b = 0$$

$$p(x) = 16 + a(-8) - 7(4) + 16 + b = 0$$

$$p(x) = 8a - b = 4 \dots\dots(1)$$

and now,

$$p(x) = p(-3) = (-3^4) + a(-3^3) - 7(-3^2) - 8(-3) + b = 0$$

$$p(x) = 81 - 27a - 63 + 24 + b = 0$$

$$p(x) = 27a - b = 42 \dots\dots(2)$$

on solving equation (1) and (2) we get,

$$8a - b = 4$$

$$27a - b = 42$$

On subtracting equation, we get

$$\Rightarrow 19a = 38$$

$$\Rightarrow a = \frac{38}{19} = 2$$

$$b = 8a - 4 = 8(2) - 4 = 12$$

$$a = 2 \text{ and } b = 12$$

Answer 22

$$\text{Let } g(x) = (x-2)$$

$$\Rightarrow x = 2$$

$$\text{And } h(x) = \left(x - \frac{1}{2}\right)$$

$$\Rightarrow x = \frac{1}{2}$$

And $p(x) = px^2 + 5x + r$

$$\begin{aligned} \text{Put the value } p(2) = 0 &\Rightarrow p(2)^2 + 5(2) + r \\ &\Rightarrow 4p + r = -10 \dots\dots\dots(1) \end{aligned}$$

$$\begin{aligned} P\left(\frac{1}{2}\right) = 0 &\Rightarrow p\left(\frac{1}{2}\right)^2 + 5\left(\frac{1}{2}\right) + r \\ &\Rightarrow \frac{p}{4} + \frac{5}{2} + r \\ &\Rightarrow \frac{p}{4} + r = -\frac{5}{2} \\ &\Rightarrow p + 4r = -10 \dots\dots\dots(2) \end{aligned}$$

Solving equation 1 and 2

$$4p + r = -10$$

$$p + 4r = -10$$

hence, $4p + r = p + 4r$

$$\Rightarrow 3p = 3r$$

$$\Rightarrow p = r$$

Answer23

$$\begin{aligned} \text{Here, } f(x) &= x^2 - 3x + 2 \\ &= (x^2 - 2x - x + 2) = (x - 2)(x - 1) \end{aligned}$$

Now, $p(x)$ will be divisible by $f(x)$ only when it is divisible by $(x-2)$ as well as by $(x-1)$

Now, $(x - 2 = 0 \Rightarrow x = 2)$ and $(x-1 \Rightarrow x = 1)$

By the factor theorem, $p(x)$ will be divisible by $f(x)$, if $p(2) = 0$ and $p(1) = 0$

$$p(x) = 2x^4 - 5x^3 - 2x^2 - x + 2$$

$$p(2) = 2(2)^4 - 5(2)^3 + 2(2)^2 - (2) + 2 = 32 - 40 + 8 - 2 + 2 = 0$$

$$p(-3) = 2(1)^4 - 5(1)^3 + 2(1)^2 - (1) + 2 = 2 - 5 + 2 - 1 + 2 = 0$$

Answer24

$$\text{Here, } f(x) = x - 2$$

$$\Rightarrow x = 2$$

$$\text{And } p(x) = 2x^4 - 5x^3 + 2x^2 - x - 3 = 2(2)^4 - 5(2)^3 + 2(2)^2 - 2 - 3$$

$$\Rightarrow 32 - 40 + 8 - 2 - 3 = 5$$

Hence, 5 be added to exactly divisible.

Answer25

When, the given polynomial is divided by a quadratic polynomial, then the remainder is a linear expression, say $(ax+b)$

$$\text{Let, } p(x) = (x^4 + 2x^3 - 2x^2 + 4x + 6) - (ax+b) \text{ and } f(x) = x^2 + 2x - 3$$

$$\text{Then, } p(x) = (x^4 + 2x^3 - 2x^2 + (4 - a)x + (6 + b))$$

$$f(x) = x^2 + 2x - 3$$

$$\text{then, } = (x^2 + 3x - x - 3) = (x + 3)(x - 1)$$

Now, $p(x)$ will be divisible by $f(x)$ only when it is divisible by $(x-1)$ as well as by $(x+3)$

Now, $(x - 1 = 0 \Rightarrow x = 1)$ and $(x+3 \Rightarrow x = -3)$

By the factor theorem, $p(x)$ will be divisible by $f(x)$, if $p(1) = 0$ and $p(-3) = 0$

$$p(x) = x^4 + 2x^3 - 2x^2 + (4 - a)x + (6 + b)$$

$$p(1) = (1)^4 + 2(1)^3 - 2(1)^2 + (4 - a)(1) + (6 + b) = 1 + 2 - 2 + 4 + 6 - a + b$$

$$\Rightarrow a + b = 11 \dots\dots(1)$$

$$p(-3) = (-3)^4 + 2(-3)^3 - 2(-3)^2 + (4-a)(-3) + (6+b) = 81 - 54 - 18 - 12 + 3a + 6 + b$$
$$\Rightarrow 3a - b = -3 \dots (2)$$

Solve 1 and 2 equations

$$a = 11 - b$$

$$\text{then, } 3(11-b) - b = -3$$

$$\Rightarrow -4b = -33 - 3 = -36$$

$$\Rightarrow b = 9$$

$$\text{And } a \Rightarrow 11 - b = 11 - 9 = 2$$

Hence, the required expression is $(2x - 9)$

Answer 26

$$\text{Let } f(x) = (x+a)$$

$$\Rightarrow x = -a$$

Then, $p(x) = x^n + a^n$, where n is positive odd integer

$$\text{Now, } p(-a) = 0$$

$$p(-a) \Rightarrow (-a)^n + a^n = [(-1)^n a^n + a^n] = [(-1)^n + 1] a^n$$

$$\Rightarrow (-1 + 1) a^n = 0 \dots [\because n \text{ being odd, } (-1)^n = -1]$$

Hence proved.

MULTIPLE CHOICE QUESTION**Answer 1:**

(c) $\sqrt{2}x^2 - \sqrt{3}x + 6$

Clearly, $\sqrt{2}x^2 - \sqrt{3}x + 6$ is a polynomial in one variable because it has only non-negative integral powers of x .

Answer 2:

(d) $x^2 + 2\frac{x^{\frac{3}{2}}}{\sqrt{x}} + 6$

We have:

$$\begin{aligned}x^2 + 2\frac{x^{\frac{3}{2}}}{\sqrt{x}} + 6 &= x^2 + 2x^{\frac{3}{2}}x^{-\frac{1}{2}} + 6 \\ &= x^2 + 2x + 6\end{aligned}$$

It a polynomial because it has only non-negative integral powers of x .

Answer 3:

(c) y

y is a polynomial because it has a non-negative integral power 1.

Answer 4:

(d) -4

-4 is a constant polynomial of degree zero.

Answer 5:

(d) 0

0 is a polynomial whose degree is not defined.

Answer 6:

(d) $x^2 + 5x + 4$

$x^2 + 5x + 4$ is a polynomial of degree 2. So, it is a quadratic polynomial.

Answer 7:

(b) $x + 1$

Clearly, $x + 1$ is a polynomial of degree 1. So, it is a linear polynomial.

Answer 8:

(b) $x^2 + 4$

Clearly, $x^2 + 4$ is an expression having two non-zero terms. So, it is a binomial.

Answer 9:

(d) 0

$\sqrt{3}$ is a constant term, so it is a polynomial of degree 0.

Answer 10:

(c) not defined

Degree of the zero polynomial is not defined.

Answer 11:

(d) not defined

Zero of the zero polynomial is not defined.

Answer 12:

(d) 8

Let:

$$p(x) = (x + 4)$$

$$\therefore p(-x) = (-x) + 4 = -x + 4$$

Thus, we have:

$$p(x) + p(-x) = \{(x + 4) + (-x + 4)\}$$

$$= 4 + 4$$

$$= 8$$

Answer 13:

(b) 1

$$p(x) = x^2 - 2\sqrt{2}x + 1$$

$$\begin{aligned} \therefore p(2\sqrt{2}) &= (2\sqrt{2})^2 - 2\sqrt{2} \times (2\sqrt{2}) + 1 \\ &= 8 - 8 + 1 \\ &= 1 \end{aligned}$$

Answer 14:

$$p(x) = 5x - 4x^2 + 3$$

Putting $x = -1$ in $p(x)$, we get

$$p(-1) = 5 \times (-1) - 4 \times (-1)^2 + 3 = -5 - 4 + 3 = -6$$

Hence, the correct answer is option (d).

Answer 15:

$$\text{Let } f(x) = x^{51} + 51$$

By remainder theorem, when $f(x)$ is divided by $(x + 1)$, then the remainder = $f(-1)$.

Putting $x = -1$ in $f(x)$, we get

$$f(-1) = (-1)^{51} + 51 = -1 + 51 = 50$$

\therefore Remainder = 50

Thus, the remainder when $(x^{51} + 51)$ is divided by $(x + 1)$ is 50.

Hence, the correct answer is option (d).

Answer 16:

(c) 2

$(x+1)$ is a factor of $2x^2+kx$.

So, -1 is a zero of $2x^2+kx$.

$$\text{Thus, we have: } 2 \times (-1)^2 + k \times (-1) = 0 \Rightarrow k = 2$$

Answer 17:

(d) 21

$$x-2=0 \Rightarrow x=2$$

By the remainder theorem, we know that when $p(x)$ is divided by $(x - 2)$, the remainder is $p(2)$.

Thus, we have:

$$p(2) = 2^4 + 2 \times 2^3 - 3 \times 2^2 + 2 - 1 = 21$$

Answer 18:

(d) 4

$$x + 2 = 0 \Rightarrow x = -2$$

By the remainder theorem, we know that when $p(x)$ is divided by $(x + 2)$, the remainder is $p(-2)$.

Now, we have:

$$p(-2) = (-2)^3 - 3 \times (-2)^2 + 4 \times (-2) + 32 = 4$$

Answer 19:

(c) -2

$$2x - 1 = 0 \Rightarrow x = \frac{1}{2}$$

By the remainder theorem, we know that when $p(x)$ is divided by $(2x - 1)$, the remainder is $p\left(\frac{1}{2}\right)$.

Now, we have:

$$p\left(\frac{1}{2}\right) = 4 \times \left(\frac{1}{2}\right)^3 - 12 \times \left(\frac{1}{2}\right)^2 + 11 \times \left(\frac{1}{2}\right) - 5 = -2$$

Answer 20:

By remainder theorem, when $p(x) = x^3 - ax^2 + x$ is divided by $(x - a)$, then the remainder = $p(a)$.

Putting $x = a$ in $p(x)$, we get

$$p(a) = a^3 - a \times a^2 + a = a^3 - a^3 + a = a$$

\therefore Remainder = a

Hence, the correct answer is option (b).

Answer 21:

(c) $-a$

$$x + a = 0 \Rightarrow x = -a$$

By the remainder theorem, we know that when $p(x)$ is divided by $(x + a)$, the remainder is $p(-a)$.

Thus, we have:

$$p(-a) = (-a)^3 + a \times (-a)^2 + 2 \times (-a) + a$$

$$= -a^3 + a^3 - 2a + a = -a$$

Answer 22:

(c) $x^3 - 2x^2 - x - 2$

Let:

$$f(x) = x^3 - 2x^2 - x - 2$$

By the factor theorem, $(x + 1)$ will be a factor of $f(x)$ if $f(-1) = 0$.

We have:

$$f(-1) = (-1)^3 - 2 \times (-1)^2 + (-1) + 2 = -1 - 2 - 1 + 2 = -2 \neq 0$$

Hence, $(x + 1)$ is not a factor of $f(x) = x^3 - 2x^2 - x - 2$

Now,

Let:

$$f(x) = x^3 - 2x^2 - x - 2$$

By the factor theorem, $(x + 1)$ will be a factor of $f(x)$ if $f(-1) = 0$.

We have:

$$f(-1) = (-1)^3 + 2 \times (-1)^2 + (-1) - 2 = -1 + 2 - 1 - 2 = -2 \neq 0$$

Hence, $(x + 1)$ is not a factor of $f(x) = x^3 - 2x^2 - x - 2$

Now,

Let:

$$f(x) = x^3 - 2x^2 - x - 2$$

By the factor theorem, $(x + 1)$ will be a factor of $f(x)$ if $f(-1) = 0$.

We have:

$$f(-1) = (-1)^3 + 2 \times (-1)^2 - (-1) - 2 = -1 + 2 + 1 - 2 = 0$$

Hence, $(x + 1)$ is a factor of $f(x) = x^3 - 2x^2 - x - 2$

Answer 23:

The zero of the polynomial $p(x)$ can be obtained by putting $p(x) = 0$.

$$p(x) = 0$$

$$\Rightarrow 2x + 5 = 0$$

$$\Rightarrow 2x = -5$$

$$\Rightarrow x = -\frac{5}{2}$$

Hence, the correct answer is option (b).

Answer 24:

The given polynomial is $p(x) = x^2 + x - 6$.

Putting $x = 2$ in $p(x)$, we get

$$p(2) = 2^2 + 2 - 6 = 4 + 2 - 6 = 0$$

Therefore, $x = 2$ is a zero of the polynomial $p(x)$.

Putting $x = -3$ in $p(x)$, we get

$$p(-3) = (-3)^2 - 3 - 6 = 9 - 9 = 0$$

Therefore, $x = -3$ is a zero of the polynomial $p(x)$.

Thus, 2 and -3 are the zeroes of the given polynomial $p(x)$.

Hence, the correct answer is option (c).

Answer 25:

The given polynomial is $p(x) = 2x^2 + 5x - 3$.

Putting $x = \frac{1}{2}$ in $p(x)$, we get

$$p\left(\frac{1}{2}\right) = 2 \times \left(\frac{1}{2}\right)^2 + 5 \times \frac{1}{2} - 3 = 3 - 3 = 0$$

Therefore, $x = \frac{1}{2}$ is a zero of the polynomial $p(x)$.

Putting $x = -3$ in $p(x)$, we get

$$p(-3) = 2 \times (-3)^2 + 5 \times (-3) - 3 = 18 - 15 - 3 = 0$$

Therefore, $x = -3$ is a zero of the polynomial $p(x)$.

Thus, $\frac{1}{2}$ and -3 are the zeroes of the given polynomial $p(x)$.

Hence, the correct answer is option (b).

Answer 26:

The given polynomial is $p(x) = 2x^2 + 7x - 4$.

Putting $x = \frac{1}{2}$ in $p(x)$, we get

$$p\left(\frac{1}{2}\right) = 2 \times \left(\frac{1}{2}\right)^2 + 7 \times \frac{1}{2} - 4 = 4 - 4 = 0$$

Therefore, $x = \frac{1}{2}$ is a zero of the polynomial $p(x)$.

Putting $x = -4$ in $p(x)$, we get

$$p(-4) = 2 \times (-4)^2 + 7 \times (-4) - 4 = 32 - 28 - 4 = 32 - 32 = 0$$

Therefore, $x = -4$ is a zero of the polynomial $p(x)$.

Thus, $\frac{1}{2}$ and -4 are the zeroes of the given polynomial $p(x)$.

Hence, the correct answer is option (c).

Answer 27:

(b) 5

 $(x+5)$ is a factor of $p(x) = x^3 - 20x + 5k$.

$$\therefore p(-5)=0 \Rightarrow (-5)^3 - 20 \times (-5) + 5k = 0 \Rightarrow -125 + 100 + 5k = 0 \Rightarrow 5k = 25 \Rightarrow k = 5$$

Answer 28:(b) $m = 7, n = -18$ Let: $p(x) = x^3 + 10x^2 + mx + n$

Now,

$$x+2=0 \Rightarrow x=-2$$

 $(x+2)$ is a factor of $p(x)$.So, we have $p(-2)=0$

$$\Rightarrow (-2)^3 + 10 \times (-2)^2 + m \times (-2) + n = 0$$

$$\Rightarrow -8 + 40 - 2m + n = 0$$

$$\Rightarrow 32 - 2m + n = 0$$

$$\Rightarrow 2m - n = 32 \quad \dots(i)$$

Now,

$$x-1=0 \Rightarrow x=1$$

Also,

 $(x-1)$ is a factor of $p(x)$.We have: $p(1) = 0$

$$\Rightarrow 1^3 + 10 \times 1^2 + m \times 1 + n = 0$$

$$\Rightarrow 1 + 10 + m + n = 0$$

$$\Rightarrow 11 + m + n = 0$$

$$\Rightarrow m + n = -11 \quad \dots(ii)$$

From (i) and (ii), we get: $3m = 21 \Rightarrow m = 7$ By substituting the value of m in (i), we get $n = -18$.

$$\therefore m = 7 \text{ and } n = -18$$

Answer 29:

(a) 1

Let: $p(x) = x^{100} + 2x^{99} + k$

Now,

$$x + 1 = 0 \Rightarrow x = -1$$

$$(x+1) \text{ is divisible by } p(x) \therefore p(-1)=0 \Rightarrow (-1)^{100} + 2 \times (-1)^{99} + k = 0$$

$$\Rightarrow 1 - 2 + k = 0$$

$$\Rightarrow -1 + k = 0$$

$$\Rightarrow k = 1$$

Answer 30:

(d) -3

$$\text{Let: } p(x) = 2x^3 - kx^2 + 3x + 10$$

Now,

$$x+2=0 \Rightarrow x=-2$$

$$p(x) \text{ is completely divisible by } (x+2) \therefore p(-2)=0 \Rightarrow 2 \times (-2)^3 - k \times (-2)^2 + 3 \times (-2) + 10 = 0$$

$$\Rightarrow -16 - 4k - 6 + 10 = 0$$

$$\Rightarrow -12 - 4k = 0$$

$$\Rightarrow k = -3$$

Answer 31:

(b) 0, 3

$$\text{Let: } p(x) = x^2 - 3x$$

Now, we have:

$$p(x)=0 \Rightarrow x^2 - 3x = 0$$

$$\Rightarrow x(x - 3) = 0$$

$$\Rightarrow x = 0 \text{ and } x - 3 = 0$$

$$\Rightarrow x = 0 \text{ and } x = 3$$

Answer 32:

(d) $\frac{1}{\sqrt{3}}$ and $-\frac{1}{\sqrt{3}}$

$$\text{Let: } p(x) = 3x^2 - 1$$

$$\text{To find the zeroes of } p(x), \text{ we have : } p(x) = 0 \Rightarrow 3x^2 - 1 = 0$$

$$\Rightarrow 3x^2 = 1 \Rightarrow x = \pm \frac{1}{\sqrt{3}} \Rightarrow x = \frac{1}{\sqrt{3}} \text{ and } x = -\frac{1}{\sqrt{3}}$$