CHAPTER-6- Determinants

Exercise-6A

Q1

We know that If A be $k \times k$ matrix then $|pA| = p^k|A|$.

We also have $p = 4, k = 2\ and\ |A| = 5$.

$$|4A| = 4^2 \times 5 = 16 \times 5 = 80$$

Q2

We know that If A be $k \times k$ matrix then $|pA| = p^k|A|$.

We also have $p = 3, k = 3$.

$$|3A| = 3^3 \times |A|$$

$$= 27|A|.$$

Comparing above with $k|A|$ gives $k = 27$.

Q3

We know that If A be $k \times k$ matrix then $|pA| = p^k|A|$.

We also have $p = 2, k = 3\ and\ |A| = 4$.

$$|2A| = 2^3 \times 4 = 8 \times 4 = 32$$

Q4

We know that A_ij is arrived by deleting i^{th} row and j^{th} column, the determinant of left matrix is called cofactor with multiplied by $(-1)^{(i+j)}$.

Its given : $i = 3\ and\ j = 2.$

$A_{32}=(-1)^{(3+2)}(2 \times 4-6 \times 5)=-1 \times (-22)=22$

$a_{32}=5$

$a_{32}A_{32}= 5 \times 22=110$

Q5

We know that the calculation can be done in two diverse ways either by removal the common things and formerly calculating the determinants or simply take determinant.

Let us remove $x + 1$ from second row.

$$(x-1) \times \begin{vmatrix} x^2 - x + 1 & x - 1 \\ 1 & 1 \end{vmatrix}$$

$$= (x+1) \times (x^2 - x + 1 - (x-1))$$

$$= (x+1) \times (x2 - 2x + 2)$$

$$= x3 - 2x2 + 2x + x2 - 2x + 2$$

$$= x3 - x2 + 2\ .$$

Q6

$$\begin{vmatrix} a+ib & c+id \\ -c+id & a-ib \end{vmatrix}$$

$$= ((a+ib)(a-ib)) - ((-c+id)(c+id))$$

$$= (a2+b2) - (-c2 - d2)$$

$$= a2+b2 + c2+d2(\because i \times i = -1)$$

Q7

We are given that

$$\begin{vmatrix} 3x & 7 \\ -2 & 4 \end{vmatrix} = \begin{vmatrix} 8 & 6 \\ 7 & 4 \end{vmatrix}$$

By matrix solution we get

$$\Rightarrow 12x + 14 = 32 - 4$$

$$\Rightarrow 12x = -10 - 14$$

$$\Rightarrow 12x = -24$$

$$\Rightarrow x = -2$$

Q8

We are given that

$$\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix}$$

By matrix solution we get

$$\Rightarrow 2x^2 - 40 = 18 + 14$$

$\Rightarrow\ 2x^2 = 72$

$\Rightarrow\ x^2 = 36$

$\Rightarrow\ x = \pm 6.$

Q9

We are given that

$$\begin{vmatrix} 2x & x+3 \\ 2(x+1) & x+1 \end{vmatrix} = \begin{vmatrix} 1 & 5 \\ 3 & 3 \end{vmatrix}$$

By matrix solution we get

$\Rightarrow 2x^2 + 2x - 2(x^2 + 4x + 3) = -12$

$\Rightarrow\ -6x - 6 = -12$

$\Rightarrow\ -6x = -6$

$\Rightarrow\ x = 1.$

Q10

We have given $A = \begin{pmatrix} 3 & 4 \\ 1 & 2 \end{pmatrix}$.

Find the determinant of A and then multiply it by 3

$$|A| = \begin{vmatrix} 1 & 5 \\ 3 & 3 \end{vmatrix} = 2$$

$$3|A| = 3 \times 2 = 6$$

Q11

We are given $2\begin{vmatrix} 7 & -2 \\ -10 & 5 \end{vmatrix} = 2(35 - 20) = 30$

Q12

We are given $\begin{vmatrix} \sqrt{6} & \sqrt{5} \\ \sqrt{20} & \sqrt{24} \end{vmatrix} = (\sqrt{6}\sqrt{24} - \sqrt{5}\sqrt{20}) = \sqrt{144} -$

$\sqrt{100} = 12 - 10 = 2$

Q13

We are given $\begin{vmatrix} 2\cos\theta & -2\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix} = (2\cos^2\alpha + 2\sin^2\alpha) = 2$

Q14

We are given $\begin{vmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{vmatrix} = (\cos^2\alpha + \sin^2\alpha) = 1$

Q15

We are given $\begin{vmatrix} \sin 60^0 & \cos 60^0 \\ -\sin 30^0 & \cos 30^0 \end{vmatrix} = \sin 60^0 \cos 30^0 +$

$\cos 60^0 \sin 30^0$

$$= \frac{\sqrt{3}}{2}\frac{\sqrt{3}}{2} + \frac{1}{2}\frac{1}{2} = \frac{3}{4} + \frac{1}{4} = 1$$

Q16

We are given $\begin{vmatrix} \cos 65^0 & \sin 65^0 \\ \sin 25^0 & \cos 25^0 \end{vmatrix} = \cos 65^0 \cos 25^0 - \sin 65^0 \sin 25^0$

$$= \cos(65 + 25)^0 = \cos 90^0 = 0$$

Q17

We are given $\begin{vmatrix} \cos 15^0 & \sin 15^0 \\ \sin 75^0 & \cos 75^0 \end{vmatrix} = \cos 15^0 \cos 75^0 - \sin 15^0 \sin 75^0$

$$= \cos(15 + 75)^0 = \cos 90^0 = 0$$

Q18

We are given $\begin{vmatrix} 0 & 2 & 0 \\ 2 & 3 & 4 \\ 4 & 5 & 6 \end{vmatrix} = 0(18 - 20) - 2(12 - 16) + 0(10 - 12) = 8$

Q19

We are given $|A| = \begin{vmatrix} 41 & 1 & 5 \\ 79 & 7 & 9 \\ 29 & 5 & 3 \end{vmatrix}$.

In the given determinant $C_1 = 8 \times C_3$

Thus $|A| = 0$ as 8 times of column 3 is same as column 1.

Thus our matrix is singular.

Q20

We are given $\begin{pmatrix} 3-2x & x+1 \\ 2 & 4 \end{pmatrix}$ as an singular matrix. Thus

$$\begin{vmatrix} 3-2x & x+1 \\ 2 & 4 \end{vmatrix} = 0$$

$$(3-2x)4 - 2(x+1) = 0$$

Therefore $x = 1$

Q21

We are given $\begin{vmatrix} 14 & 9 \\ -8 & -7 \end{vmatrix} == 14 \times (-7) - 9 \times (-8) = -26$

Q22

We are given $\begin{vmatrix} \sqrt{3} & \sqrt{5} \\ -\sqrt{5} & 3\sqrt{3} \end{vmatrix} = \left(3\sqrt{3}\sqrt{3} + \sqrt{5}\sqrt{5}\right) = 9 + 5 = 14$

Exercise:6B

Q1

We are given $|A| = \begin{vmatrix} 67 & 19 & 21 \\ 39 & 13 & 14 \\ 81 & 24 & 26 \end{vmatrix}$.

We can evaluate it as

$$|A| = 67(13 \times 26 - 26 \times 14) - 19(39 \times 26 - 14 \times 81) + 21\,(39 \times 26 - 13 \times 81) = -43$$

Q2

We are given $|A| = \begin{vmatrix} 29 & 26 & 22 \\ 25 & 31 & 27 \\ 63 & 54 & 46 \end{vmatrix}$.

We can evaluate it as

$$|A| = 29(31 \times 46 - 54 \times 27) - 26(25 \times 46 - 63 \times 27) + 22\,(25 \times 54 - 63 \times 27) = 132$$

Q3

We are given $|A| = \begin{vmatrix} 102 & 18 & 36 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix} = 6 \times \begin{vmatrix} 17 & 3 & 6 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix} = 0$, as

Row1 and Row2 are identical.

Q4

We are given $|A| = \begin{vmatrix} 1^2 & 2^2 & 3^2 \\ 2^2 & 2^2 & 4^2 \\ 3^2 & 4^2 & 5^2 \end{vmatrix} = \begin{vmatrix} 1 & 4 & 9 \\ 4 & 4 & 16 \\ 9 & 16 & 25 \end{vmatrix} = 0$.

We can evaluate it as

$$|A| = 1(4 \times 25 - 16 \times 16) - 4(4 \times 25 - 9 \times 16)$$
$$+ 9\,(4 \times 16 - 9 \times 4) = -8$$

Q5

We are given $|A| = \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ bc & ca & ab \end{vmatrix}$

$= \begin{vmatrix} 0 & 0 & 1 \\ a-b & b-c & c \\ bc-ca & ca-ab & ab \end{vmatrix}$, as $C_1' = C_1 - C_2\,; C_2' = C_2 - C_3$.

$$= \begin{vmatrix} 0 & 0 & 1 \\ a-b & b-c & c \\ -c(a-b) & -a(b-c) & ab \end{vmatrix}$$

$= (a-b)(b-c)\begin{vmatrix} 0 & 0 & 1 \\ 1 & 1 & c \\ -c & -a & ab \end{vmatrix}$, as $C_1' = \frac{C_1}{a-b}\,; C_2' = \frac{C_2}{b-c}$.

$= (a-b)(b-c)(c-a)$

Q6

We are given $|A| = \begin{vmatrix} 1 & b+c & b^2+c^2 \\ 1 & c+a & c^2+a^2 \\ 1 & a+b & a^2+b^2 \end{vmatrix}$

$= \begin{vmatrix} 0 & b-c & b^2-a^2 \\ 0 & c-b & c^2-b^2 \\ 1 & a+b & a^2+b^2 \end{vmatrix}$, as $R_1' = R_1 - R_2$; $R_2' = R_2 - R_3$.

$= \begin{vmatrix} 0 & b-c & (b-a)(b+a) \\ 0 & c-b & (c-b)(c+b) \\ 1 & a+b & a^2+b^2 \end{vmatrix}$

$= (b-a)(c-b)\begin{vmatrix} 0 & 1 & (b+a) \\ 0 & 1 & (c+b) \\ 1 & a+b & a^2+b^2 \end{vmatrix}$, as $R_1' = \frac{R_1}{a-b}$; $R_2' = \frac{R_2}{b-c}$.

$= (a-b)(b-c)(c-a)$

Q7

We are given $|A| = \begin{vmatrix} 1 & 1+p & 1+p+q \\ 2 & 3+2p & 1+3p+2q \\ 3 & 6+3p & 1+6p+3q \end{vmatrix}$

$= \begin{vmatrix} -1 & -2-p & -2p-q \\ -1 & -3-p & -3p-q \\ 3 & 6+3p & 1+6p+3q \end{vmatrix}$, as $R_1' = R_1 - R_2$; $R_2' = R_2 - R_3$.

$$= \begin{vmatrix} 0 & 1 & p \\ -1 & -3-p & -3p-q \\ 3 & 6+3p & 1+6p+3q \end{vmatrix}, \text{as } R_1' = R_1 - R_2$$

$$= \left(\frac{1}{2}\right) \begin{vmatrix} 0 & 1 & p \\ -2 & -6-2p & -6p-2q \\ 3 & 6+3p & 1+6p+3q \end{vmatrix}, \text{ as } R_2' = 2R_2.$$

$$= \left(\frac{1}{2}\right) \begin{vmatrix} 0 & 1 & p \\ 1 & p & 1+q \\ 3 & 6+3p & 1+6p+3q \end{vmatrix}, \text{ as } R_2' = R_3 + R_2.$$

$$= \left(\frac{1}{2}\right)(0 + 3(1+q) - (1+6p+3q) + p(6+3p-3p))$$

$$= 1$$

Q8

We are given $|A| = \begin{vmatrix} a+x & y & z \\ x & a+y & z \\ x & y & a+z \end{vmatrix}$

$$= \begin{vmatrix} a & -a & 0 \\ 0 & a & -a \\ x & y & a+z \end{vmatrix}, \text{ as } R_1' = R_1 - R_2 \; ; R_2' = R_2 - R_3.$$

$$= a^2 \begin{vmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ x & y & a+z \end{vmatrix}$$

$$= a^2(a + z - (-y) - (-x))$$

$= a^2(a + x + y + z)$

Q9

We are given $|A| = \begin{vmatrix} x & a & a \\ a & x & a \\ a & a & x \end{vmatrix}$

$= \begin{vmatrix} x+2a & x+2a & x+2a \\ a & x & a \\ a & a & x \end{vmatrix}$, as $R_1' = R_1 + R_2 + R_3$.

$= (x + 2a) \begin{vmatrix} 1 & 1 & 1 \\ a & x & a \\ a & a & x \end{vmatrix}$ as $R_1' = \frac{R_1}{x+2a}$

$= (x + 2a) \begin{vmatrix} 1 & 1 & 1 \\ 0 & x-a & a-x \\ a & a & x \end{vmatrix}$ as $R_2' = R_2 - R_3$

$= (x + 2a) \begin{vmatrix} 1 & 1 & 1 \\ 0 & x-a & -(x-a) \\ a & a & x \end{vmatrix}$

$= (x + 2a)(x - a) \begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & -1 \\ a & a & x \end{vmatrix}$

$= (x + 2a)(x - a)^2$

Q10

We are given $|A| = \begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix}$

$= \begin{vmatrix} 5x+4 & 5x+4 & 5x+4 \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix}$, as $R_1' = R_1 + R_2 + R_3$.

$= (5x+4)\begin{vmatrix} 1 & 1 & 1 \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix}$ as $R_1' = \frac{R_1}{5x+4}$

$= (5x+4)\begin{vmatrix} 1 & 1 & 1 \\ 0 & -x+4 & x-4 \\ 2x & 2x & x+4 \end{vmatrix}$ as $R_2' = R_2 - R_3$

$= (5x+4)(x-4)\begin{vmatrix} 1 & 1 & 1 \\ 0 & -1 & 1 \\ 2x & 2x & x+4 \end{vmatrix}$

$= (5x+4)(x-4)^2$

Q11

We are given $|A| = \begin{vmatrix} x+\lambda & 2x & 2x \\ 2x & x+\lambda & 2x \\ 2x & 2x & x+\lambda \end{vmatrix}$

$= \begin{vmatrix} 5x+\lambda & 5x+\lambda & 5x+\lambda \\ 2x & x+\lambda & 2x \\ 2x & 2x & x+\lambda \end{vmatrix}$, as $R_1' = R_1 + R_2 + R_3$.

$$= (5x+\lambda)\begin{vmatrix} 1 & 1 & 1 \\ 2x & x+\lambda & 2x \\ 2x & 2x & x+\lambda \end{vmatrix} \text{ as } R_1' = \frac{R_1}{5x+\lambda}$$

$$= (5x+\lambda)\begin{vmatrix} 1 & 1 & 1 \\ 0 & -x+\lambda & x-\lambda \\ 2x & 2x & x+\lambda \end{vmatrix} \text{as } R_2' = R_2 - R_3$$

$$= (5x+\lambda)(x-\lambda)\begin{vmatrix} 1 & 1 & 1 \\ 0 & -1 & 1 \\ 2x & 2x & x+\lambda \end{vmatrix}$$

$$= (5x+\lambda)(x-\lambda)^2$$

Q12

We are given $|A| = \begin{vmatrix} a^2+2a & 2a+1 & 1 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix}$

$$= \begin{vmatrix} a^2-1 & a-1 & 0 \\ 2a-2 & a-1 & 0 \\ 3 & 3 & 1 \end{vmatrix}, \text{ as } R_1' = R_1 - R_2; R_2' = R_2 - R_3.$$

$$= \begin{vmatrix} (a-1)(a+1) & a-1 & 0 \\ 2(a-1) & a-1 & 0 \\ 3 & 3 & 1 \end{vmatrix}$$

$$= (a-1)^2\begin{vmatrix} (a+1) & 1 & 0 \\ 2 & 1 & 0 \\ 3 & 3 & 1 \end{vmatrix} \text{ as } R_1' = \frac{R_1}{a-1}; R_2' = \frac{R_2}{a-1}$$

$$= (a-1)^3$$

Q13

We are given $|A| = \begin{vmatrix} x & x+y & x+2y \\ x+2y & x & x+y \\ x+y & x+2y & x \end{vmatrix}$

$= \begin{vmatrix} 3(x+y) & 3(x+y) & 3(x+y) \\ x+2y & x & x+y \\ x+y & x+2y & x \end{vmatrix}$, as $R_1' = R_1 + R_2 + R_3$.

$= 3(x+y)\begin{vmatrix} 1 & 1 & 1 \\ x+2y & x & x+y \\ x+y & x+2y & x \end{vmatrix}$ as $R_1' = \frac{R_1}{3(x+y)}$

$= 3(x+y)\begin{vmatrix} 1 & 1 & 1 \\ y & -2y & y \\ x+y & x+2y & x \end{vmatrix}$ as $R_2' = R_2 - R_3$

$= 3y(x+y)\begin{vmatrix} 1 & 1 & 1 \\ 1 & -2 & 1 \\ x+y & x+2y & x \end{vmatrix}$

$= 3y(x+y)\begin{vmatrix} 0 & 3 & 0 \\ 1 & -2 & 1 \\ x+y & x+2y & x \end{vmatrix}$ as $R_1' = R_1 - R_2$

$= 9y^2(x+y)$

Q14

We are given $|A| = \begin{vmatrix} 3x & -x+y & -x+y \\ x-y & 3y & z-y \\ x-z & y-z & 3z \end{vmatrix}$

$= \begin{vmatrix} x+y+z & -x+y & -x+y \\ x+y+z & 3y & z-y \\ x+y+z & y-z & 3z \end{vmatrix}$, as $C_1' = C_1 + C_2 + C_3$.

$= (x+y+z)\begin{vmatrix} 1 & -x+y & -x+y \\ 1 & 3y & z-y \\ 1 & y-z & 3z \end{vmatrix}$ as $C_1' = \frac{C_1}{(x+y+z)}$

$= (x+y+z)\begin{vmatrix} 1 & -x+y & -x+y \\ -x-2y & 2y+z & y-z \\ -x+z & z-y & 3z \end{vmatrix}$ as $C_1' = C_1 - C_2; C_2' = C_2 - C_3$

$= (x+y+z)(3xy+3yz+3xz)$

$= 3(x+y+z)(xy+yz+xz)$

Q15

We are given $|A| = \begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ x^3 & y^3 & z^3 \end{vmatrix}$

$= xyz\begin{vmatrix} 1 & 1 & 1 \\ x & z & z \\ x^2 & y^2 & z^2 \end{vmatrix}$, as $C_1' = \frac{C_1}{x}$; $C_2' = \frac{C_2}{y}$; $C_3' = \frac{C_3}{z}$.

$$= xyz\begin{vmatrix} 0 & 0 & 1 \\ x-y & y-z & z \\ x^2-y^2 & y^2-z^2 & z^2 \end{vmatrix} \text{ as } C_1' = C_1 - C_2; C_2' = C_2 - C_3$$

$$= xyz(x-y)(y-z)\begin{vmatrix} 0 & 0 & 1 \\ 1 & 1 & z \\ x+y & y+z & z^2 \end{vmatrix} \text{as } C_1' = \frac{C_1}{x-y}; C_2' = \frac{C_2}{y-z}$$

$$= xyz(x-y)(y-z)(z-x)$$

Q16

We are given $|A| = \begin{vmatrix} b+c & a-b & a \\ c+a & b-c & b \\ a+b & c-a & c \end{vmatrix}$

$$= \begin{vmatrix} 2(a+b+c) & 0 & a+b+c \\ c+a & b-c & b \\ a+b & c-a & c \end{vmatrix}, \text{ as } R_1' = R_1 + R_2 + R_3.$$

$$= (a+b+c)\begin{vmatrix} 2 & 0 & 1 \\ c+a & b-c & b \\ a+b & c-a & c \end{vmatrix} \text{ as } R_1' = \frac{R_1}{a+b+c}$$

$$= (a+b+c)(ab+bc+ac-a^2-b^2-c^2)$$

$$= 3abc - a^3 - b^3 - c^3$$

Q17

We are given $|A| = \begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$

$= \begin{vmatrix} 2(b+c) & 2(a+c) & 2(a+b) \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$, as $R_1' = R_1 + R_2 + R_3$.

$= 2\begin{vmatrix} (b+c) & (a+c) & (a+b) \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$ as $R_1' = \frac{R_1}{2}$

$= 2\begin{vmatrix} c & 0 & a \\ b-c & a & -a \\ c & c & a+b \end{vmatrix}$ as $R_1' = R_1 - R_2; R_2' = R_2 - R_3$

$= 2abc$

Q18

We are given $|A| = \begin{vmatrix} a & a+2b & a+2b+3c \\ 3a & 4a+6b & 5a+7b+9c \\ 6a & 9a+12b & 11a+15b+18c \end{vmatrix}$

$= \left(\frac{1}{3}\right)\begin{vmatrix} 3a & 3a+6b & 3a+6b+9c \\ 3a & 4a+6b & 5a+7b+9c \\ 6a & 9a+12b & 11a+15b+18c \end{vmatrix}$, as $R_1' = 3R_1$.

$= \left(\frac{1}{3}\right)\begin{vmatrix} 0 & -a & -2a-b \\ 3a & 4a+6b & 5a+7b+9c \\ 6a & 9a+12b & 11a+15b+18c \end{vmatrix}$ as $R_1' = R_1 - R_2$

$$= \left(\frac{1}{6}\right)\begin{vmatrix} 0 & -a & -2a-b \\ 6a & 8a+12b & 10a+14b+18c \\ 6a & 9a+12b & 11a+15b+18c \end{vmatrix} \text{ as } R_2' = 2R_2$$

$$= \left(\frac{1}{6}\right)\begin{vmatrix} 0 & -a & -2a-b \\ 0 & -a & -a-b \\ 6a & 9a+12b & 11a+15b+18c \end{vmatrix} \text{ as } R_2' = R_2 - R_3$$

$$= \left(\frac{1}{6}\right)(-6a^3)$$

$$= -a^3$$

Q19

We are given $|A| = \begin{vmatrix} a+b+c & -c & -b \\ -c & a+b+c & -a \\ -b & -a & a+b+c \end{vmatrix}$

$$= \begin{vmatrix} a+b & a+b & -(a+b) \\ -c & a+b+c & -a \\ -b & -a & a+b+c \end{vmatrix}, \text{ as } R_1' = R_1 + R_2.$$

$$= (a+b)\begin{vmatrix} 1 & 1 & -1 \\ -c & a+b+c & -a \\ -b & -a & a+b+c \end{vmatrix}, \text{ as } R_1' = \frac{R_1}{a+b}$$

$$= (a+b)\begin{vmatrix} 1 & 1 & -1 \\ -c-b & b+c & b+c \\ -b & -a & a+b+c \end{vmatrix}, \text{ as } R_2' = R_2 + R_3$$

$$= (a+b)(b+c)\begin{vmatrix} 1 & 1 & -1 \\ -1 & 1 & 1 \\ -b & -a & a+b+c \end{vmatrix}, \text{ as } R_2' = \frac{R_2}{b+c}$$

$= 2(a+b)(b+c)(c+a)$

Q20

We are given $|A| = \begin{vmatrix} a & b & ax+by \\ b & c & bx+cy \\ ax+by & bx+cy & 0 \end{vmatrix}$

$= \frac{1}{xy}\begin{vmatrix} ax & bx & ax^2+bxy \\ by & cy & bxy+cy^2 \\ ax+by & bx+cy & 0 \end{vmatrix}$, as $R_1' = xR_1; R_2' = yR_2$.

$= \frac{1}{xy}\begin{vmatrix} ax & bx & ax^2+bxy \\ by & cy & bxy+cy^2 \\ ax+by & bx+cy & 0 \end{vmatrix}$, as $R_1' = R_1 + R_2 + R_3$

$= \frac{1}{xy}(ax^2+2bxy+cy^2)(b^2xy+bcy^2-acxy-bcy^2)$

$= (b^2-ac)(ax^2+2bxy+cy^2)$

Q21

We are given $|A| = \begin{vmatrix} a^2 & b^2 & c^2 \\ (a+1)^2 & (b+1)^2 & (c+1)^2 \\ (a-1)^2 & (b-1)^2 & (c-1)^2 \end{vmatrix}$

$= \begin{vmatrix} a^2 & b^2 & c^2 \\ a^2+2a+1 & b^2+2b+1 & c^2+2c+1 \\ a^2-2a+1 & b^2-2b+1 & c^2-2c+1 \end{vmatrix}$, as $R_2' = R_2 - R_3$

$$= 4\begin{vmatrix} a^2 & b^2 & c^2 \\ a & b & c \\ a^2-2a+1 & b^2-2b+1 & c^2-2c+1 \end{vmatrix}, \text{ as } R_2' = \frac{R_2}{4}$$

$$= 4\begin{vmatrix} a^2 & a & a^2-2a+1 \\ b^2 & b & b^2-2b+1 \\ c^2 & c & c^2-2c+1 \end{vmatrix} \text{ by interchanging all rows and column}$$

$$= 4\begin{vmatrix} a^2-b^2 & a-b & (a^2-b^2)-2(a-b) \\ b^2-c^2 & b-c & (b^2-c^2)-2(b-c) \\ c^2 & c & c^2-2c+1 \end{vmatrix} \text{ as } R_1' = R_1 -$$

R_2 ; $R_2' = R_2 - R_3$

$$= 4(a-b)(b-c)\begin{vmatrix} a+b & 1 & (a+b)-2 \\ b+c & 1 & (b+c)-2 \\ c^2 & c & c^2-2c+1 \end{vmatrix} \text{ as } R_1' = \frac{R_1}{a-b} ; R_2' =$$

$\frac{R_2}{b-c}$

$$= 4(a-b)(b-c)\begin{vmatrix} a-c & 0 & a-c \\ b+c & 1 & (b+c)-2 \\ c^2 & c & c^2-2c+1 \end{vmatrix} \text{ as } R_1' = R_1 - R_2$$

$$= 4(a-b)(b-c)(a-c)\begin{vmatrix} 1 & 0 & 1 \\ b+c & 1 & (b+c)-2 \\ c^2 & c & c^2-2c+1 \end{vmatrix} \text{ as } R_1' = \frac{R_1}{a-c}$$

$$= 4(a-b)(b-c)(a-c)$$

Q22

We are given $|A| = \begin{vmatrix} (x-2)^2 & (x-1)^2 & x^2 \\ (x-1)^2 & x^2 & (x+1)^2 \\ x^2 & (x+1)^2 & (x+2)^2 \end{vmatrix}$

$$= \begin{vmatrix} x^2-4x+4 & x^2-2x+1 & x^2 \\ x^2-2x+1 & x^2 & x^2+2x+1 \\ x^2 & x^2+2x+1 & x^2+4x+4 \end{vmatrix}$$

$$= \begin{vmatrix} -2x+3 & -2x+1 & -2x+1 \\ -2x+1 & -2x-1 & -2x-3 \\ x^2 & x^2+2x+1 & x^2+4x+4 \end{vmatrix} \text{ as } R_1' = R_1 - R_2 ; R_2' = R_2 - R_3$$

$$= \begin{vmatrix} 2 & 2 & 2 \\ -2x+1 & -2x-1 & -2x-3 \\ x^2 & x^2+2x+1 & x^2+4x+4 \end{vmatrix} \text{ as } R_1' = R_1 - R_2$$

$$= 2\begin{vmatrix} 1 & 1 & 1 \\ -2x+1 & -2x-1 & -2x-3 \\ x^2 & x^2+2x+1 & x^2+4x+4 \end{vmatrix} \text{ as } R_1' = \frac{R_1}{2}$$

$$= 2\begin{vmatrix} 1 & -2x+1 & x^2 \\ 1 & -2x-1 & x^2+2x+1 \\ 1 & -2x-3 & x^2+4x+4 \end{vmatrix}$$ as interchange of rows and columns

$$= 2\begin{vmatrix} 0 & 2 & -2x-1 \\ 0 & 2 & -2x-3 \\ 1 & -2x-3 & x^2+4x+4 \end{vmatrix} \text{ as } R_1' = R_1 - R_2 ; R_2' = R_2 - R_3$$

$$= 2\begin{vmatrix} 0 & 0 & 2 \\ 0 & 2 & -2x-3 \\ 1 & -2x-3 & x^2+4x+4 \end{vmatrix} \text{ as } R_1' = R_1 - R_2$$

$$= -8$$

Q23

We are given $|A| = \begin{vmatrix} (m+n)^2 & l^2 & mn \\ (n+l)^2 & m^2 & ln \\ (l+m)^2 & n^2 & lm \end{vmatrix}$

$$= \frac{1}{2}\begin{vmatrix} m^2+2mn+n^2 & l^2 & 2mn \\ n^2+2nl+l^2 & m^2 & 2ln \\ l^2+2ml+m^2 & n^2 & 2lm \end{vmatrix} \text{ as } C_3' = 2C_3$$

$$= \frac{1}{2}\begin{vmatrix} m^2+n^2 & l^2 & 2mn \\ n^2+l^2 & m^2 & 2ln \\ l^2+m^2 & n^2 & 2lm \end{vmatrix} \text{ as } C_3' = C_1 - C_3$$

$$= \left(\frac{1}{2}\right)\begin{vmatrix} l^2+m^2+n^2 & l^2 & 2mn \\ l^2+m^2+n^2 & m^2 & 2ln \\ l^2+m^2+n^2 & n^2 & 2lm \end{vmatrix} \text{ as } C_1' = C_1 + C_2$$

$$= \left(\frac{1}{2}\right)(l^2+m^2+n^2)\begin{vmatrix} 1 & l^2 & 2mn \\ 1 & m^2 & 2ln \\ 1 & n^2 & 2lm \end{vmatrix} \text{ as } C_1' = \frac{C_1}{l^2+m^2+n^2}$$

$$= \left(\frac{1}{2}\right)(l^2+m^2+n^2)\begin{vmatrix} 1 & 1 & 1 \\ l^2 & m^2 & n^2 \\ 2mn & 2ln & 2lm \end{vmatrix}$$ by interchanging the rows with columns

$$= \left(\frac{1}{2}\right)(l^2 + m^2 + n^2)\begin{vmatrix} 0 & 0 & 1 \\ l^2 - m^2 & m^2 - n^2 & n^2 \\ -2n(l-m) & -2l(m-n) & 2lm \end{vmatrix} \text{ as } C_1' =$$

$C_1 - C_2$ &$C_2' = C_2 - C_3$

$$= (l^2 + m^2 + n^2)(l-m)(m-n)\begin{vmatrix} 0 & 0 & 1 \\ l+m & m+n & n^2 \\ -n & -l & lm \end{vmatrix} \text{ as } C_1' =$$

$\frac{C_1}{l-m}$ &$R_2' = \frac{C_2}{l-m}$

$$= (l^2 + m^2 + n^2)(l - m)(m - n)(n - l)(l + m + n)$$

Q24

We have given $|A| = \begin{vmatrix} (b+c)^2 & a^2 & bc \\ (c+a)^2 & b^2 & ca \\ (a+b)^2 & c^2 & ab \end{vmatrix}$

$$= \frac{1}{2}\begin{vmatrix} b^2+c^2+2bc & a^2 & 2bc \\ c^2+a^2+2ac & b^2 & 2ca \\ a^2+b^2+2ab & c^2 & 2ab \end{vmatrix} [C_3' = 2C_3]$$

$$= \frac{1}{2}\begin{vmatrix} b^2+c^2 & a^2 & 2bc \\ c^2+a^2 & b^2 & 2ca \\ a^2+b^2 & c^2 & 2ab \end{vmatrix} [C_1' = C_1 - C_3]$$

$$= \frac{1}{2}\begin{vmatrix} a^2+b^2+c^2 & a^2 & 2bc \\ a^2+b^2+c^2 & b^2 & 2ca \\ a^2+b^2+c^2 & c^2 & 2ab \end{vmatrix} [C_1' = C_1 + C_2]$$

$$= \frac{1}{2}(a^2+b^2+c^2)\begin{vmatrix} 1 & a^2 & 2bc \\ 1 & b^2 & 2ca \\ 1 & c^2 & 2ab \end{vmatrix}$$ [$C_1' = C_1/(a^2 + b^2 + c^2)$]

$$= \frac{1}{2}(a^2+b^2+c^2)\begin{vmatrix} 1 & 1 & 1 \\ a^2 & b^2 & c^2 \\ 2bc & 2ca & 2ab \end{vmatrix}$$ [transforming row and column]

$$= \frac{1}{2}(a^2+b^2+c^2)\begin{vmatrix} 1 & 1 & 1 \\ a^2-b^2 & b^2-c^2 & c^2 \\ -2c(a-b) & -2a(b-c) & 2ab \end{vmatrix}$$

[$C_1' = C_1 - C_2$ & $C_2' = C_2 - C_3$]

$$= (a^2+b^2+c^2)(a-b)(b-c)\begin{vmatrix} 0 & 0 & 1 \\ a+b & b+c & c^2 \\ -c & -a & ab \end{vmatrix}$$

[$C_1' = C_1/(a - b)$ & $C_2' = C_2/(b - c)$]

$$= (a^2 + b^2 + c^2)(a - b)(b - c)(c - a)(a + b + c)$$

Q25

We have given $|A| = \begin{vmatrix} b^2+c^2 & a^2 & a^2 \\ b^2 & c^2+a^2 & b^2 \\ c^2 & c^2 & a^2+b^2 \end{vmatrix}$

$$= \begin{vmatrix} 2(b^2+c^2) & 2(c^2+a^2) & 2(a^2+b^2) \\ b^2 & c^2+a^2 & b^2 \\ c^2 & c^2 & a^2+b^2 \end{vmatrix}$$ [$R_1' = R_1 + R_2 + R_3$]

$$= 2\begin{vmatrix} (b^2+c^2) & (c^2+a^2) & (a^2+b^2) \\ b^2 & c^2+a^2 & b^2 \\ c^2 & c^2 & a^2+b^2 \end{vmatrix} \left[R_1' = \frac{R_1}{2}\right]$$

$$= 2\begin{vmatrix} c^2 & 0 & a^2 \\ b^2 & c^2+a^2 & b^2 \\ c^2 & c^2 & a^2+b^2 \end{vmatrix} [R_1' = R_1 - R_2]$$

$$= 4a^2b^2c^2$$

Q26

We have to show

$$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = \left(1+a^2+b^2\right)^3.$$

Operating $R1 \to R1 + bR3, R2 \to R2 - aR3$

$$\begin{vmatrix} 1+a^2-b^2+2b^2 & 2ab-2ab & -2b+b-a^2b-b^3 \\ 2ab-2ab & 1-a^2+b^2+2a^2 & 2a-a+a^3+ab^2 \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix}$$

$$= \begin{vmatrix} 1+a^2+b^2 & 0 & -b-a^2b-b^3 \\ 0 & 1+a^2+b^2 & a+a^3+ab^2 \\ 2b & -2a & 1-a^2+b^2 \end{vmatrix}$$

$$= \begin{vmatrix} 1+a^2+b^2 & 0 & -b(1+a^2+b^2) \\ 0 & 1+a^2+b^2 & a(1+a^2+b^2) \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix}$$

Taking $(1 + a^2 + b^2)$ from R1 and R2

$$= (1+a^2+b^2)^2 \begin{vmatrix} 1 & 0 & -b \\ 0 & 1 & a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix}$$

$$Operating\ R3 \rightarrow R3 - 2bR1 + 2aR2$$

$$= (1+a^2+b^2)^2 \begin{vmatrix} 1 & 0 & -b \\ 0 & 1 & a \\ 0 & 0 & 1+a^2+b^2 \end{vmatrix}$$

$$Taking\ (1 + a^2 + b^2)\ from\ R3$$

$$= (1+a^2+b^2)^3 \begin{vmatrix} 1 & 0 & -b \\ 0 & 1 & a \\ 0 & 0 & 1 \end{vmatrix}$$

$$Expanding\ with\ respect\ to\ C1$$

$$= (1+a^2+b^2)^3$$

Q27

We have to show

$$\begin{vmatrix} a & b-c & c+b \\ a+c & b & c-a \\ a-b & a+b & c \end{vmatrix} = (a+b+c)(a^2+b^2+c^2).$$

$$Operating\ C1 \rightarrow aC1$$

$$\frac{1}{a}\begin{vmatrix} a^2 & b-c & c+b \\ a^2+ac & b & c-a \\ a^2-ab & a+b & c \end{vmatrix}$$

$$Operating\ C1 \rightarrow C1 + bC2 + cC3$$

$$=\frac{1}{a}\begin{vmatrix} a^2+b^2-bc+c^2+bc & b-c & c+b \\ a^2+ac+b^2+c^2-ac & b & c-a \\ a^2-ab+ab+b^2+c^2 & a+b & c \end{vmatrix}$$

$$=\frac{1}{a}\begin{vmatrix} a^2+b^2+c^2 & b-c & c+b \\ a^2+b^2+c^2 & b & c-a \\ a^2+b^2+c^2 & a+b & c \end{vmatrix}$$

Taking $(a^2+b^2+c^2)$ common from C

$$=\frac{1}{a}(a^2+b^2+c^2)\begin{vmatrix} 1 & b-c & c+b \\ 1 & b & c-a \\ 1 & a+b & c \end{vmatrix}$$

Operating $R_1 \rightarrow R_1-R_3$, $R_2 \rightarrow R_2-R$

$$=\frac{1}{a}(a^2+b^2+c^2)\begin{vmatrix} 0 & -c-a & b \\ 0 & -a & -a \\ 1 & a+b & c \end{vmatrix}$$

Operating $C_2 \rightarrow C_2-C_3$

$$=\frac{1}{a}(a^2+b^2+c^2)\begin{vmatrix} 0 & -(a+b+c) & b \\ 0 & 0 & -a \\ 1 & (a+b+c) & c \end{vmatrix}$$

Taking $(a+b+c)$ common from C_2

$$=\frac{1}{a}(a^2+b^2+c^2)(a+b+c)\begin{vmatrix} 0 & -1 & b \\ 0 & 0 & -a \\ 1 & 1 & c \end{vmatrix}$$

Expanding with respect to C_1

$$= \frac{1}{a}(a^2+b^2+c^2)(a+b+c)\times 1\times (0-(-a))$$

$$= \frac{1}{a}(a^2+b^2+c^2)(a+b+c)\ (a)$$

$$= (a^2+b^2+c^2)(a+b+c)$$

Q28

We have the determinant

$$\begin{vmatrix} b^2c^2 & bc & b+c \\ c^2a^2 & ca & c+a \\ a^2b^2 & ab & a+b \end{vmatrix} = 0.$$

Expanding with R1

$=b^2c^2(a^2c+abc-abc-a^2b)-bc(a^3c^2+a^2bc^2-a^2b^2c-a^3b^2)+(b+c)(a^3bc^2-a^3b^2c)$

$=a^2b^3c^2-a^2b^3c^2-a^3bc^2-a^2b^3c^2+a^2b^3c^2+a^3b^3c+a^3b^2c^2-a^3b^3c+a^3bc^3-a^3b^2c^2$

$=0$

Q29

To prove the given relation

$$\begin{vmatrix} (b+c)^2 & ab & ca \\ ab & (a+c)^2 & bc \\ ac & bc & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3.$$

Let us solve LHL

$$= \begin{vmatrix} b^2+c^2+2bc & ab & ac \\ ab & a^2+c^2+2ac & bc \\ ac & bc & a^2+b^2+2ab \end{vmatrix}$$

Operating $R_1 \to aR_1$, $R_2 \to bR_2$, $R_3 \to cR_3$

$$= \frac{1}{abc} \begin{vmatrix} a(b^2+c^2+2bc) & a^2b & a^2c \\ ab^2 & b(a^2+c^2+2ac) & b^2c \\ ac^2 & bc^2 & c(a^2+b^2+2ab) \end{vmatrix}$$

Taking a, b, c common from C_1, C_2, C_3 respectively

$$= \frac{abc}{abc} \begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (a+c)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix}$$

Operating $R_1 \to R_1 - R_3$, $R_2 \to R_2 - R_3$

$$= \begin{vmatrix} (b+c)^2-a^2 & 0 & a^2 \\ 0 & (a+c)^2-b^2 & b^2 \\ c^2-(a+b)^2 & c^2-(a+b)^2 & (a+b)^2 \end{vmatrix}$$

$$= \begin{vmatrix} (b+c+a)(b+c-a) & 0 & a^2 \\ 0 & (a+c+b)(a+c-b) & b^2 \\ (c-a-b)(c+a+b) & (c-a-b)(c+a+b) & (a+b)^2 \end{vmatrix}$$

Taking (a+b+c) common from R_1, R_2

$$= (a+b+c)^2 \begin{vmatrix} b+c-a & 0 & a^2 \\ 0 & a+c-b & b^2 \\ c-a-b & c-a-b & (a+b)^2 \end{vmatrix}$$

Operating $R_3 \to R_3 - R_1 - R_2$

$$= (a+b+c)^2 \begin{vmatrix} b+c-a & 0 & a^2 \\ 0 & a+c-b & b^2 \\ -2b & -2a & a^2+b^2+2ab-a^2-b^2 \end{vmatrix}$$

$$= (a+b+c)^2 \begin{vmatrix} b+c-a & 0 & a^2 \\ 0 & a+c-b & b^2 \\ -2b & -2a & 2ab \end{vmatrix}$$

Operating $C_1 \to aC_1$, $C_2 \to bC_2$

$$\frac{(a+b+c)^2}{ab}\begin{vmatrix} a(b+c-a) & 0 & a^2 \\ 0 & b(a+c-b) & b^2 \\ -2ab & -2ab & 2ab \end{vmatrix}$$

Operating $C_1 \to C_1+C_3$, $C_2 \to C_2+C_3$

$$= \frac{(a+b+c)^2}{ab}\begin{vmatrix} a(b+c) & a^2 & a^2 \\ b^2 & b(a+c) & b^2 \\ 0 & 0 & 2ab \end{vmatrix}$$

Taking a, b, 2ab from R_1, R_2, R_3

$$= \frac{(a+b+c)^2 a.b.\,2ab}{ab}\begin{vmatrix} b+c & a & a \\ b & a+c & b \\ 0 & 0 & 1 \end{vmatrix}$$

Expanding with R_3

$$= 2ab(a+b+c)^2 \times 1 \times (ab+ac+bc+c^2-ab)$$

$$= 2ab(a+b+c)^2\big(c(a+b+c)\big)$$

$$= 2abc(a+b+c)^3$$

Q30

To prove the given relation as

$$\begin{vmatrix} b^2-ab & b-c & bc-ac \\ ab-a^2 & a-b & b^2-ab \\ bc-ac & c-a & ab-a^2 \end{vmatrix} = 0.$$

Let us compute first LHL

$$= \begin{vmatrix} b(b-a) & b-c & c(b-a) \\ a(b-a) & a-b & b(b-a) \\ c(b-a) & c-a & a(b-a) \end{vmatrix}$$

Taking (b-a) common from C_1, C_3

$$= (b-a)^2 \begin{vmatrix} b & b-c & c \\ a & a-b & b \\ c & c-a & a \end{vmatrix}$$

Operating $R_2 \rightarrow R_2 - R_1 + R_3$

$$= \begin{vmatrix} b & b-c-b+c & c \\ a & a-b-a+b & b \\ c & c-a-c+a & a \end{vmatrix}$$

$$= (b-a)^2 \begin{vmatrix} b & 0 & c \\ a & 0 & b \\ c & 0 & a \end{vmatrix}$$

= 0 as the whole column 2 is zero.

Q31

We want prove the given relation as

$$\begin{vmatrix} -a\left(b^2+c^2-a^2\right) & 2b^3 & 2c^3 \\ 2a^3 & -b\left(c^2+a^2-b^2\right) & 2c^3 \\ 2a^3 & ab^3 & -c\left(a^2+b^2+c^2\right) \end{vmatrix}$$

$$= (abc)\left(a^2+b^2+c^2\right)^3.$$

Taking a, b, c from C_1, C_2, C_3

$$= abc\begin{vmatrix} -b^2-c^2+a^2 & 2b^2 & 2c^2 \\ 2a^2 & b^2-c^2-a^2 & 2c^2 \\ 2a^2 & 2b^2 & -a^2-b^2+c^2) \end{vmatrix}$$

Operating $R_1 \rightarrow R_1 - R_3$, $R_2 \rightarrow R_2 - R_3$

$$= abc\begin{vmatrix} -b^2-c^2-a^2 & 0 & a^2+b^2+c^2 \\ 0 & -(a^2+b^2+c^2) & a^2+b^2+c^2 \\ 2a^2 & 2b^2 & -a^2-b^2+c^2 \end{vmatrix}$$

Taking $(a^2+b^2+c^2)$ common from R_1, R_2

$$= abc(a^2+b^2+c^2)^2\begin{vmatrix} -1 & 0 & 1 \\ 0 & -1 & 1 \\ 2a^2 & 2b^2 & -a^2-b^2+c^2 \end{vmatrix}$$

Operating $R_3 \rightarrow R_3 + R_1 + R_2$

$$= abc(a^2+b^2+c^2)^2\begin{vmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 2a^2 & 2b^2 & a^2+b^2+c^2 \end{vmatrix}$$

Taking $(a^2+b^2+c^2)$ common from C_3

$$= abc(a^2+b^2+c^2)^3\begin{vmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 2a^2 & 2b^2 & 1 \end{vmatrix}$$

Expanding with C_3

$= abc(a^2+b^2+c^2)^3 \times 1 \times (1-0)$

$= abc\ (a^2+b^2+c^2)^3$

Q32

To prove the given relation as

$$\begin{vmatrix} x-3 & x-4 & x-\alpha \\ x-2 & x-3 & x-\beta \\ x-1 & x-2 & x-\gamma \end{vmatrix} = 0,$$ where α, β, γ are in AP.

Given that α, β, γ are in an AP, which means $2\beta=\alpha+\gamma$

Operating $R_3 \rightarrow R_3-2R_2+R_1$

$$= \begin{vmatrix} x-3 & x-4 & x-\alpha \\ x-2 & x-3 & x-\beta \\ x-1-2x+4+x-3 & x-2-2x+6+x-4 & x-\gamma-2x+2\beta+x-\alpha \end{vmatrix}$$

$$= \begin{vmatrix} x-3 & x-4 & x-\alpha \\ x-2 & x-3 & x-\beta \\ 0 & 0 & -\gamma+2\beta-\alpha \end{vmatrix}$$ [we know that $2\beta=\alpha+\gamma$]

Operating $R_1 \rightarrow R_1-R_3$, $R_2 \rightarrow R_2-R_3$

$$= \begin{vmatrix} x-3 & x-4 & x-\alpha \\ x-2 & x-3 & x-\beta \\ 0 & 0 & -\gamma+\alpha+\gamma-\alpha \end{vmatrix}$$

$$= \begin{vmatrix} x-3 & x-4 & x-\alpha \\ x-2 & x-3 & x-\beta \\ 0 & 0 & 0 \end{vmatrix}$$

=0 as the row 3 is zero.

Q33

We want to show the relation as

$$\begin{vmatrix} (a+1)(a+2) & a+2 & 1 \\ (a+2)(a+3) & a+3 & 1 \\ (a+3)(a+4) & a+4 & 1 \end{vmatrix} = -2$$

Operating $R_1 \rightarrow R_1 - R_2$, $R_2 \rightarrow R_2 - R_3$

$$= \begin{vmatrix} (a+1)(a+2)-(a+2)(a+3) & a+2-a-3 & 0 \\ (a+2)(a+3)-(a+3)(a+4) & a+3-a-4 & 0 \\ (a+3)(a+4) & a+4 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} (a+2)(a+1-a-3) & -1 & 0 \\ (a+3)(a+2-a-4) & -1 & 0 \\ (a+3)(a+4) & a+4 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} -2(a+2) & -1 & 0 \\ -2(a+3) & -1 & 0 \\ (a+3)(a+4) & a+4 & 1 \end{vmatrix}$$

Expanding with C_3

$= (2(a+2) - 2(a+3))$

$= (2a+4-2a-6)$

$= -2$

Q34

We have given the $x \neq y \neq z$ and

$$\begin{vmatrix} x & x^3 & x^4-1 \\ y & y^3 & y^4-1 \\ z & z^3 & z^4-1 \end{vmatrix} = 0,$$

To show: $xyz\,(xy + yz + zx) = (x + y + z)$.

By properties of determinants, we can split the given determinant into 2 parts

$$0 = \begin{vmatrix} x & x^3 & x^4 \\ y & y^3 & y^4 \\ z & z^3 & z^4 \end{vmatrix} - \begin{vmatrix} x & x^3 & 1 \\ y & y^3 & 1 \\ z & z^3 & 1 \end{vmatrix}$$

Taking x, y, z common from R_1, R_2, R_3 respectively

$$0 = xyz\begin{vmatrix} 1 & x^2 & x^3 \\ 1 & y^2 & y^3 \\ 1 & z^2 & z^3 \end{vmatrix} - \begin{vmatrix} x & x^3 & 1 \\ y & y^3 & 1 \\ z & z^3 & 1 \end{vmatrix}$$

Operating $R_1 \rightarrow R_1 - R_3$, $R_2 \rightarrow R_2 - R_3$

$$0 = xyz\begin{vmatrix} 0 & x^2-z^2 & x^3-z^3 \\ 0 & y^2-z^2 & y^3-z^3 \\ 1 & z^2 & z^3 \end{vmatrix} - \begin{vmatrix} x-z & x^3-z^3 & 0 \\ y-z & y^3-z^3 & 0 \\ z & z^3 & 1 \end{vmatrix}$$

$$\begin{vmatrix} x-z & (x-z)(x^2+xz+z^2) & 0 \\ y-z & (y-z)(y^2+yx+z^2) & 0 \\ z & z^3 & 1 \end{vmatrix} = xyz\begin{vmatrix} 0 & (x-z)(x+z) & (x-z)(x^2+xz+z^2) \\ 0 & (y-z)(y+z) & (y-z)(y^2+yz+z^2) \\ 1 & z^2 & z^3 \end{vmatrix}$$

Taking (x-z) and (y-z) common from R_1, R_2

$$(x-z)(y-z)\begin{vmatrix} 1 & (x^2+xz+z^2) & 0 \\ 1 & (y^2+yz+z^2) & 0 \\ z & z^3 & 1 \end{vmatrix} = (x-z)(y-z)\begin{vmatrix} 0 & x+z & (x^2+xz+z^2) \\ 0 & y+z & (y^2+yz+z^2) \\ 1 & z^2 & z^3 \end{vmatrix}$$

Expanding with R_3

$y^2+yz+z^2-x^2-xz-z^2 = xyz(xy^2+xyz+xz^2+zy^2+yz^2+z^3-x^2y-xyz-yz^2-x^2z-xz^2-z^3)$

$(y-x)(y+x) +z(y-x) =xyz(xy^2+zy^2 -x^2y -x^2z)$

$(y-x)(x+y+z)=xyz(xy(y-x)+z(y^2-x^2))$

$(y-x)(x+y+z)= xyz(xy(y-x)+z(x+y)(y-x))$

$(y-x)(x+y+z) = xyz(xy(y-x)+(xz+yz)(y-x))$

$(y-x)(x+y+z)= xyz(y-x)(xy+xz+yz)$

$x+y+z = xyz(xy+xz+yz)$

Q35

To prove the relation as

$$\begin{vmatrix} 1 & a^2+bc & a^3 \\ 1 & b^2+ca & b^3 \\ 1 & c^2+ab & c^3 \end{vmatrix} = -(a-b)(b-c)(c-a)(a^2+b^2+c^2).$$

Operating $R_1 \rightarrow R_1-R_2$, $R_2 \rightarrow R_2-R_3$

$$= \begin{vmatrix} 0 & a^2+bc-b^2-ac & a^3-b^3 \\ 0 & b^2+ca-c^2-ab & b^3-c^3 \\ 1 & c^2+ab & c^3 \end{vmatrix}$$

$$= \begin{vmatrix} 0 & (a-b)(a+b)-c(a-b) & (a-b)(a^2+ab+b^2) \\ 0 & (b-c)(b+c)-a(b-c) & (b-c)(b^2+bc+c^2) \\ 1 & c^2+ab & c^3 \end{vmatrix}$$

Taking (a-b), (b-c) common from R_1, R_2 respectively

$$= (a-b)(b-c)\begin{vmatrix} 0 & a+b-c & a^2+ab+b^2 \\ 0 & b+c-a & b^2+bc+c^2 \\ 1 & c^2+ab & c^3 \end{vmatrix}$$

Operating $R_1 \rightarrow R_1 - R_2$

$$= (a-b)(b-c)\begin{vmatrix} 0 & 2a-2c & a^2+ab-bc-c^2 \\ 0 & b+c-a & b^2+bc+c^2 \\ 1 & c^2+ab & c^3 \end{vmatrix}$$

$$= (a-b)(b-c)\begin{vmatrix} 0 & 2(a-c) & (a+c)(a-c)+b(a-c) \\ 0 & b+c-a & b^2+bc+c^2 \\ 1 & c^2+ab & c^3 \end{vmatrix}$$

Taking (a-c) common from R_1

$$= (a-c)(a-b)(b-c)\begin{vmatrix} 0 & 2 & a+b+c \\ 0 & b+c-a & b^2+bc+c^2 \\ 1 & c^2+ab & c^3 \end{vmatrix}$$

Expanding with C_1

$= (a-c)(a-b)(b-c)\times(2b^2+2bc+2c^2-ab-b^2-bc-ac-bc-c^2+a^2+ab+ac)$

$=-(c-a)(b-c)(a-b)(a^2+b^2+c^2)$

Q36

We want to show

$$\begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix} = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

Operating $R_1 \rightarrow R_1 - R_2$, $R_2 \rightarrow R_2 - R_3$

$$\begin{vmatrix} 0 & a-b & bc-ac \\ 0 & b-c & ac-ab \\ 1 & c & ab \end{vmatrix} = \begin{vmatrix} 0 & a-b & a^2-b^2 \\ 0 & b-c & b^2-c^2 \\ 1 & c & c^2 \end{vmatrix}$$

$$\begin{vmatrix} 0 & a-b & -c(a-b) \\ 0 & b-c & -a(b-c) \\ 1 & c & ab \end{vmatrix} = \begin{vmatrix} 0 & a-b & (a-b)(a+b) \\ 0 & b-c & (b-c)(b+c) \\ 1 & c & c^2 \end{vmatrix}$$

Taking (a-b) and (b-c) from R_1, R_2

$$(a-b)(b-c)\begin{vmatrix} 0 & 1 & -c \\ 0 & 1 & -a \\ 1 & c & ab \end{vmatrix} = (a-b)(b-c)\begin{vmatrix} 0 & 1 & (a+b) \\ 0 & 1 & (b+c) \\ 1 & c & c^2 \end{vmatrix}$$

For the two determinants to be equal, their difference must be 0.

$$= \begin{vmatrix} 0 & 1 & -c \\ 0 & 1 & -a \\ 1 & c & ab \end{vmatrix} - \begin{vmatrix} 0 & 1 & a+b \\ 0 & 1 & b+c \\ 1 & c & c^2 \end{vmatrix} = \begin{vmatrix} 0-0 & 1-1 & -(a+b+c) \\ 0-0 & 1-1 & -(a+b+c) \\ 1-1 & c-c & ab-c^2 \end{vmatrix}$$

$$= \begin{vmatrix} 0 & 0 & -(a+b+c) \\ 0 & 0 & -(a+b+c) \\ 0 & 0 & ab-c^2 \end{vmatrix}$$

=0 as we have two columns is zero columns.

Q37

We want to show that

$$\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = \begin{vmatrix} 1 & bc & b+c \\ 1 & ca & c+a \\ 1 & ab & a+b \end{vmatrix}$$

Operating $R1 \to R1 - R_3$, $R_2 \to R2 - R3$

$$\begin{vmatrix} 0 & a-c & a^2-c^2 \\ 0 & b-c & b^2-c^2 \\ 1 & c & c^2 \end{vmatrix} = \begin{vmatrix} 0 & bc-ab & b+c-a-b \\ 0 & ac-ab & c+a-a-b \\ 1 & ab & a+b \end{vmatrix}$$

$$\begin{vmatrix} 0 & a-c & (a-c)(a+c) \\ 0 & b-c & (b-c)(b+c) \\ 1 & c & c^2 \end{vmatrix} = \begin{vmatrix} 0 & -b(a-c) & -(a-c) \\ 0 & -a(b-c) & -(b-c) \\ 1 & ab & a+b \end{vmatrix}$$

Taking (a-c) and (b-c) common from R_1, R_2

$$(a-c)(b-c)\begin{vmatrix} 0 & 1 & a+c \\ 0 & 1 & b+c \\ 1 & c & c^2 \end{vmatrix} = (a-c)(b-c)\begin{vmatrix} 0 & -b & -1 \\ 0 & -a & -1 \\ 1 & ab & a+b \end{vmatrix}$$

If the determinants are equal, their difference must also be equal.

(a-c) and (b-c) get cancelled.

$$= \begin{vmatrix} 0 & 1 & a+c \\ 0 & 1 & b+c \\ 1 & c & c^2 \end{vmatrix} - \begin{vmatrix} 0 & -b & -1 \\ 0 & -a & -1 \\ 1 & ab & a+b \end{vmatrix}$$

$$= \begin{vmatrix} 0-0 & 1+b & a+c+1 \\ 0-0 & 1+a & b+c+1 \\ 1-1 & c-ab & c^2+a+b \end{vmatrix}$$

$$= \begin{vmatrix} 0 & 1+b & a+c+1 \\ 0 & 1+a & b+c+1 \\ 0 & c-ab & c^2+a+b \end{vmatrix}$$

=0 as column 1 is zero.

Q38

We have the equation as

$$\begin{vmatrix} x & -6 & -1 \\ 2 & -3x & x-3 \\ -3 & 2x & 2+x \end{vmatrix} = 0.$$

Operating $R_1 \rightarrow R_1 - R_2$

$$0 = \begin{vmatrix} x-2 & -6+3x & -1-x+3 \\ 2 & -3x & x-3 \\ -3 & 2x & 2+x \end{vmatrix}$$

$$0 = \begin{vmatrix} x-2 & 3(x-2) & -(x-2) \\ 2 & -3x & x-3 \\ -3 & 2x & 2+x \end{vmatrix}$$

Taking (x-2) common from R_1

$$0 = (x-2)\begin{vmatrix} 1 & 1 & -1 \\ 2 & -3x & x-3 \\ -3 & 2x & 2+x \end{vmatrix}$$

Here, we can see that x-2 is a factor of the determinant.

We can say that when x-2 is put in the equation, we get 0.

x-2=0

Thus

x=2

Q39

We have given equation as

$$\begin{vmatrix} 1 & x & x^3 \\ 1 & b & b^3 \\ 1 & c & x^3 \end{vmatrix} = 0$$

Operating $R1 \rightarrow R1 - R_2$, $R_2 \rightarrow R2 - R_3$

$$\begin{vmatrix} 0 & x-b & x^3-b^3 \\ 0 & b-c & b^3-c^3 \\ 1 & c & c^3 \end{vmatrix} = 0$$

$$0 = \begin{vmatrix} 0 & x-c & (x-b)^3+3xb(x-b) \\ 0 & b-c & (b-c)^3+3bc(b-c) \\ 1 & c & c^3 \end{vmatrix}$$

$$0 = (x-c)(b-c)\begin{vmatrix} 0 & 1 & (x-b)^2+3xb \\ 0 & 1 & (b-c)^2+3bc \\ 1 & c & c^3 \end{vmatrix}$$

Expanding with C_1

$0=(x-c)(b-c)(b^2-2bc+c^2+3bc-x^2+2xb-b^2-3xb)$

$0= (x-c)(b-c)(bc+c^2-x^2-xb)$

$0=(x-c)(b-c)(-b(-c+x)-(c-x)(-c-x))$

$0=(x-c)^2(b-c)(-b-c-x)$

Either x-c=0 or b-c=0 or (-b-c-x)=0

$\therefore$x=c or b=c or x=-(b+c)

If b=c, x=b

$\therefore$ x=c or x=b or x=-(b+c)

Q40

We have given equation as

$$\begin{vmatrix} x+a & b & c \\ a & x+b & c \\ b & b & x+c \end{vmatrix} = 0$$

Operating $C1 \rightarrow C1+C_2+C_3$

$$\begin{vmatrix} x+a+b+c & b & c \\ x+a+b+c & x+b & c \\ x+a+b+c & b & x+c \end{vmatrix} = 0$$

Taking (x+a+b+c) common from C_1

$$(x+a+b+c)\begin{vmatrix} 1 & b & c \\ 1 & x+b & c \\ 1 & b & x+c \end{vmatrix} = 0$$

Operating $R_1 \rightarrow R_1-R_3$, $R_2 \rightarrow R_2-R_3$

$$(x+a+b+c)\begin{vmatrix} 0 & 0 & -x \\ 0 & x & -x \\ 1 & b & x+c \end{vmatrix} = 0$$

Expanding with C_1

$0=(x+a+b+c)(0+x^2)$

$0=x^2(x+a+b+c)$

Either $x^2=0$ or $(x+a+b+c)=0$

$\therefore$ x=0 or x=-(a+b+c)

Q41

We have given equation as :

$$\begin{vmatrix} 3x-8 & 3 & 3 \\ 3 & 3x-8 & 3 \\ 3 & 3 & 3x-8 \end{vmatrix} = 0$$

Operating $C_1 \rightarrow C_1 + C_2 + C_3$

$$0 = \begin{vmatrix} 3x-8+3+3 & 3 & 3 \\ 3+3x-8+3 & 3x-8 & 3 \\ 3+3+3x-8 & 3 & 3x-8 \end{vmatrix}$$

$$0 = \begin{vmatrix} 3x-2 & 3 & 3 \\ 3x-2 & 3x-8 & 3 \\ 3x-2 & 3 & 3x-8 \end{vmatrix}$$

Taking (3x-2) common from C_1

$$0 = (3x-2)\begin{vmatrix} 1 & 3 & 3 \\ 1 & 3x-8 & 3 \\ 1 & 3 & 3x-8 \end{vmatrix}$$

Operating $R_1 \rightarrow R_1 - R_3$, $R_2 \rightarrow R_2 - R_3$

$$0 = (3x-2)\begin{vmatrix} 0 & 0 & -(3x-11) \\ 0 & 3x-11 & -3x+11 \\ 1 & 3 & 3x-8 \end{vmatrix}$$

Expanding with C_1

$0 = (3x-2)(0+(3x-11)^2)$

$0 = (3x-2)(3x-11)^2$

Either $3x-2=0$ or $3x-11=0$

$\therefore x = 2/3$ or $x = 11/3$

Q42

We have given equation as

$$\begin{vmatrix} x+1 & 3 & 5 \\ 2 & x+2 & 5 \\ 2 & 3 & x+4 \end{vmatrix} = 0$$

Operating $C_1 \rightarrow C_1+C_2+C_3$

$$0 = \begin{vmatrix} x+9 & 3 & 5 \\ x+9 & x+2 & 5 \\ x+9 & 3 & x+4 \end{vmatrix}$$

Taking (x+9) common from C_1

$$0 = (x+9)\begin{vmatrix} 1 & 3 & 5 \\ 1 & x+2 & 5 \\ 1 & 3 & x+4 \end{vmatrix}$$

Operating $R_1 \rightarrow R_1-R_3$, $R_2 \rightarrow R_2-R_3$

$$0 = (x+9)\begin{vmatrix} 0 & 0 & 1-x \\ 0 & x-1 & 1-x \\ 1 & 3 & x+4 \end{vmatrix}$$

$0= (x+9)(0-x+x^2+1-x)$

$0=(x+9)(x^2-2x+1)$

$0=(x+9)(x-1)^2$

∴ Either x+9=0 or x-1=0

x=-9, x=1

Q43

We have given equation as

$$\begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix} = 0$$

Operating $R1 \rightarrow R1 + R_2 + R_3$

$$0 = \begin{vmatrix} x+9 & x+9 & x+9 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix}$$

Taking (x+9) common from R_1

$$0 = (x+9)\begin{vmatrix} 1 & 1 & 1 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix}$$

Operating $C_1 \rightarrow C_1 - C_3$, $C_2 \rightarrow C_2 - C_3$

$$0 = (x+9)\begin{vmatrix} 0 & 0 & 1 \\ 0 & x-2 & 2 \\ 7-x & 6-x & x \end{vmatrix}$$

Expanding with R_1

$0=(x+9)(0-(x-2)(7-x))$

$0=(x+9)(7-x)(2-x)$

Either $x+9=0$ or $7-x=0$ or $2-x=0$

$\therefore$ $x=-9$ or $x=7$ or $x=2$

Q44

We have given equation:

$$\begin{vmatrix} x & -6 & -1 \\ 2 & -3x & x-3 \\ -3 & 2x & x+2 \end{vmatrix} = 0$$

Expanding with R1

$0= x(-3x^2-6x-2x^2+6x)+6(2x+4+3x-9)-1(4x-9x)$

$0=x(-5x^2)+6(5x-5)-1(-5x)$

$0= -5x^3+30x-30+5x$

$0=-5x^3+35x-30$

$x^3-7x+6=0$

$x^3-x-6x+6=0$

$x(x^2-1)-6(x-1)=0$

$x(x-1)(x+1)-6(x-1)=0$

$(x-1)(x^2+x-6)=0$

$(x-1)(x^2+3x-2x-6)=0$

$(x-1)(x(x+3)-2(x+3)(=0$

$(x-1)(x+3)(x-2)=0$

Either $x-1=0$ or $x+3=0$ or $x-2=0$

$\therefore$ $x=1$ or $x=-3$ or $x=2$

Q45

To show

$$\begin{vmatrix} a & b-c & c+b \\ a+c & b & c-a \\ a-b & b+a & c \end{vmatrix} = (a+b+c)(a^2+b^2+c^2)$$

Let us Operating $C_1 \rightarrow aC_1$

$$= \frac{1}{a}\begin{vmatrix} a^2 & b-c & c+b \\ a^2+ac & b & c-a \\ a^2-ab & b+a & c \end{vmatrix}$$

Operating $C_1 \rightarrow C_1+bC_2+cC_3$

$$= \frac{1}{a}\begin{vmatrix} a^2+b^2+c^2 & b-c & c+b \\ a^2+b^2+c^2 & b & c-a \\ a^2+b^2+c^2 & b+a & c \end{vmatrix}$$

Taking $(a^2+b^2+c^2)$

$$= \frac{a^2+b^2+c^2}{a}\begin{vmatrix} 1 & b-c & c+b \\ 1 & b & c-a \\ 1 & b+a & c \end{vmatrix}$$

Operating $C_2 \rightarrow C_2-bC_1$, $C_3 \rightarrow C_3-cC_3$

$$= \frac{a^2+b^2+c^2}{a}\begin{vmatrix} 1 & -c & b \\ 1 & 0 & -a \\ 1 & a & 0 \end{vmatrix}$$

Expanding with R_3

$$= \frac{a^2+b^2+c^2}{a}(ac-0+a^2+ab)$$

$$=(a^2+b^2+c^2)\frac{a}{a}(a+b+c)$$

$$= (a^2+b^2+c^2)(a+b+c)$$

Exercise:6C

Q1A

We have A(3, 8), B(-4, 2) and C(5, -1)

Area of a triangle =

$$\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} 3 & 8 & 1 \\ -4 & 2 & 1 \\ 5 & -1 & 1 \end{vmatrix}$$

Expanding with C_3

$$= \frac{1}{2}[(4-10)-(-3-40)+(6+32)]$$

$$= \frac{1}{2}[-6+43+38]$$

$$= \frac{75}{2}$$

= 37.5 sq. units

Q1B

We have given A(-2, 4), B(2, -6) and C(5, 4)

Area of a triangle = $\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

$$= \frac{1}{2}\begin{vmatrix} -2 & 4 & 1 \\ 2 & -6 & 1 \\ 5 & 4 & 1 \end{vmatrix}$$

Expanding with C_3

$$= \frac{1}{2}[(8+30)-(-8-20)+(12-8)]$$

$$= \frac{1}{2}[38+28+4] = \frac{68}{2}$$

= 34 sq. units

Q1C

We have A(-8, -2), B(-4, -6) and C(-1, 5)

Area of a triangle $= \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

$$= \frac{1}{2}\begin{vmatrix} -8 & -2 & 1 \\ -4 & -6 & 1 \\ -1 & 5 & 1 \end{vmatrix}$$

Expanding with R_3

$$= \frac{1}{2}[(-20-6)-(-40-2)+(48-8)]$$

$$= \frac{1}{2}[-26+42+40]$$

$$= \frac{56}{2}$$

=28 sq. units

Q1D

We have given P(0, 0), Q(6, 0) and R(4, 3)

Area of a triangle $= \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

$= \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ 6 & 0 & 1 \\ 4 & 3 & 1 \end{vmatrix}$

Expanding with R_1

$= \frac{1}{2}[18]$

$= 9$ sq. units

Q1E

We have given P(1, 1), Q(2, 7) and R(10, 8)

Area of a triangle $= \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

$= \frac{1}{2}\begin{vmatrix} 1 & 1 & 1 \\ 2 & 7 & 1 \\ 10 & 8 & 1 \end{vmatrix}$

Operating $R_1 \rightarrow R_1 - R_3$, $R_2 \rightarrow R_2 - R_3$

$= \frac{1}{2}\begin{vmatrix} -9 & -7 & 0 \\ -8 & -1 & 0 \\ 10 & 8 & 1 \end{vmatrix}$

Expanding with C_3

$= \frac{1}{2}[9 - 56]$

$= \frac{1}{2}[-47]$

$= \frac{-47}{2}$

= -23.5 sq. units = 23.5 sq units

Q2A

We have given A(2, 3), B(-1, -2) and C(5, 8)

Area of a triangle $= \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

$= \frac{1}{2}\begin{vmatrix} 2 & 3 & 1 \\ -1 & -2 & 1 \\ 5 & 8 & 1 \end{vmatrix}$

Expanding with C_3

$= \frac{1}{2}[(-8+10)-(16-15)+(-4+3)]$

$= \frac{1}{2}[2-1-1]$

=0

Since the area between the 3 points is 0, the three points lie in a straight line, i.e. they are collinear.

Q2B

We are given A(3, 8), B(-4, 2) and C(10, 14)

Area of a triangle $= \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

$= \frac{1}{2}\begin{vmatrix} 3 & 8 & 1 \\ -4 & 2 & 1 \\ 10 & 14 & 1 \end{vmatrix}$

Expanding with C_3

$= \frac{1}{2}\left[(-56-20)-(42-80)+(6+32)\right]$

$= \frac{1}{2}\left[-76+38+38\right]$

$=0$

Since the area between the 3 points is 0, the three points lie in a straight line, i.e. they are collinear.

Q2C

We are given P(-2, 5), Q(-6, -7) and R(-5, -4)

Area of a triangle $= \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

$= \frac{1}{2}\begin{vmatrix} -2 & 5 & 1 \\ -6 & -7 & 1 \\ -5 & -4 & 1 \end{vmatrix}$

Expanding with C_3

$= \frac{1}{2}\left[(24-35)-(8+25)+(14+30)\right]$

$= \frac{1}{2}\left[-11-33+44\right]$

$=0$

Since the area between the 3 points is 0, the three points lie in a straight line, i.e. they are collinear.

Q3

We have points A(3, -2), B(k, 2) and C(8, 8) are collinear.

Now

$$\textbf{Area of a triangle} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Since they are collinear, the area will be 0

$$0 = \frac{1}{2}\begin{vmatrix} 3 & -2 & 1 \\ k & 2 & 1 \\ 8 & 8 & 1 \end{vmatrix}$$

Expanding with C_3

$$0 = \frac{1}{2}\left[(8k-16)-(24+16)+(6+2k)\right]$$

$$0 = \frac{1}{2}\left[10k-50\right]$$

$10k - 50=0$

$10k=50$

$\therefore k=5$

Q4

We have the points P(5, 5), Q(k, 1) and R(11, 7) are collinear.

Now

Area of a triangle $= \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

Since they are collinear, the area will be 0

$0 = \frac{1}{2}\begin{vmatrix} 5 & 5 & 1 \\ k & 1 & 1 \\ 11 & 7 & 1 \end{vmatrix}$

Expanding with C_3

0 = (7k-11)-(35-55)+(5-5k)

0= 2k-14

2k=14

∴ k=7

Q5

We have the points A(1, -1), B(2, k) and C(4, 5) are collinear.

Now

Area of a triangle $= \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

Since they are collinear, the area will be 0

$0 = \frac{1}{2}\begin{vmatrix} 1 & -1 & 1 \\ 2 & k & 1 \\ 4 & 5 & 1 \end{vmatrix}$

Expanding with C_3

0 = (10-4k)-(5+4)+(k+2)

0=-3k+3

3k=3

∴ k= 1

Q6

We have given vertices A(2, -6), B(5, 4) and C(k, 4).

Also the area of triangle is 35 sq units.

Now

$$\textbf{Area of a triangle} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$35 = \frac{1}{2}\begin{vmatrix} 2 & -6 & 1 \\ 5 & 4 & 1 \\ k & 4 & 1 \end{vmatrix}$$

Expanding with C_3

70 = (20-4k)-(8+6k)+(8+30)

70= -10k+50

20=-2k

k=-2

Q7

We have given A(-2, 0), B(0, 4) and C(0, k) be three points such that area of a ABC is 4 sq units.

Now

$$\textbf{Area of a triangle} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$4 = \frac{1}{2}\begin{vmatrix} -2 & 0 & 1 \\ 0 & 4 & 1 \\ 0 & k & 1 \end{vmatrix}$$

Expanding with C_1

$8=-2(4-k)$

$-4=4-k$

$k=8$

Q8

We have given the points A(a, 0), B(0, b) and C(1, 1) are collinear.

To show $\frac{1}{a}+\frac{1}{b}=1.$

Now

$$\textbf{Area of a triangle} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Since the points are collinear, the area they enclose is 0

$$0 = \frac{1}{2}\begin{vmatrix} a & 0 & 1 \\ 0 & b & 1 \\ 1 & 1 & 1 \end{vmatrix}$$

Expanding with C_1

$0 = a(b-1)+(-b)$

0= ab-a-b

a+b=ab

Thus

$$\frac{a+b}{ab} = 1$$

Finally we have

$$\frac{1}{a} + \frac{1}{b} = 1$$