CHAPTER-5 -Matrices

Exercise-5A

Q1

For any matrix of order $m \times n$ have m rows , n columns , and total $m \times n$ members. Thus

i. The number of rows in given matrix A is 3.

ii. The number of columns in given matrix A is 4.

iii. The order of the given matrix is 3×4.

iv. The number of all entries in A is $3 \times 4 = 12$.

v. The elements are:

$a_{23} = 8$, $a_{31} = \sqrt{2}$, $a_{14} = 1$, $a_{33} = 4$, $a_{22} = 0$.

Q2

i. The order of the matrix $A = 2 \times 4$.

ii. The order of the matrix $B = 3 \times 2$.

iii. The order of the matrix $C = 1 \times 4$.

iv. The order of the matrix $D = 1 \times 2$.

v. The order of the matrix $E = 3 \times 1$.

vi. The order of the matrix $A = 1 \times 1$.

Q3

In total, if the matrix have 18 elements then the possible orders of the matrix are

(i) 18×1 (ii) 9×2 (iii) 6×3 (iv) 3×6 (v) 2×9 (vi) 1×18.

Q4

In total, if the matrix have 18 elements then the possible orders of the matrix are

(i) 7×1 (ii) 1×7.

Q5

From the given equation we have

$a_{11} = 2(1) - 1 = 1$, $a_{12} = 2(1) - 2 = 0$;

$a_{21} = 2(2) - 1 = 3$, $a_{22} = 2(2) - 2 = 2$;

$a_{31} = 2(3) - 1 = 5$, $a_{32} = 2(3) - 2 = 4$.

Thus the required matrix of the order 3×2 with satisfying the given equation is given by

$$\begin{pmatrix} 1 & 0 \\ 3 & 2 \\ 5 & 4 \end{pmatrix}.$$

Q6

From the given equation we have

$a_{11} = \frac{1}{1} = 1$, $a_{12} = \frac{1}{2}$, $a_{13} = \frac{1}{3}$;

$a_{21} = \frac{2}{1} = 2$, $a_{22} = \frac{2}{2} = 1$, $a_{23} = \frac{2}{3}$;

$$a_{31} = \frac{3}{1} = 3 \, , a_{32} = \frac{3}{2} , \quad a_{33} = \frac{3}{3} = 1 \, ,$$

$$a_{41} = \frac{4}{1} = 4 \, , a_{42} = \frac{4}{2} = 2 \, , \quad a_{43} = \frac{4}{3}.$$

Thus the required matrix of the order 3×2 with satisfying the given equation is given by

$$\begin{pmatrix} 1 & \frac{1}{2} & \frac{1}{3} \\ 2 & 1 & \frac{2}{3} \\ 3 & \frac{3}{2} & 1 \\ 4 & 2 & \frac{4}{3} \end{pmatrix}.$$

Q7

From the given equation we have

$a_{11} = \frac{(1+2(1))^2}{2} = \frac{9}{2}$, $a_{12} = \frac{(1+2(2))^2}{2} = \frac{25}{2}$;

$a_{21} = \frac{(2+2(1))^2}{2} = 8$, $a_{22} = \frac{(2+2(2))^2}{2} = 18$.

Thus the required matrix of the order 3×2 with satisfying the given equation is given by

$\begin{pmatrix} \frac{9}{2} & \frac{25}{2} \\ 8 & 18 \end{pmatrix}$.

Q8

From the given equation we have

$a_{11} = \frac{(1-2(1))^2}{2} = \frac{1}{2}$, $a_{12} = \frac{(1-2(2))^2}{2} = \frac{9}{2}$, $a_{13} = \frac{(1-2(3))^2}{2} = \frac{25}{2}$;

$a_{21} = \frac{(2-2(1))^2}{2} = 0$, $a_{22} = \frac{(2-2(2))^2}{2} = \frac{4}{2} = 2$, $a_{23} = \frac{(2-2(3))^2}{2} =$ 8 ;

Thus the required matrix of the order 3×2 with satisfying the given equation is given by

$\begin{pmatrix} \frac{1}{2} & \frac{9}{2} & \frac{25}{2} \\ 0 & 2 & 8 \end{pmatrix}$.

Q9

From the given equation we have

$\boldsymbol{a_{11} = \frac{1}{2}|-3(1)+1| = 1}$, $\boldsymbol{a_{12} = \frac{1}{2}|-3(1)+2| = \frac{1}{2}}$, $\boldsymbol{a_{13} = \frac{1}{2}|-3(1)+3| = 0}$, $\boldsymbol{a_{14} = \frac{1}{2}|-3(1)+4| = \frac{1}{2}}$;

$a_{21} = \frac{1}{2}|-3(2)+1| = \frac{5}{2}, a_{22} = \frac{1}{2}|-3(2)+2| = 2$, $a_{23} = \frac{1}{2}|-3(2)+3| = \frac{3}{2}$, $a_{24} = \frac{1}{2}|-3(2)+4| = 1$;

$a_{31} = \frac{1}{2}|-3(3)+1| = 4, a_{32} = \frac{1}{2}|-3(3)+2| = \frac{7}{2}$, $a_{33} = \frac{1}{2}|-3(3)+3| = 3$, $a_{34} = \frac{1}{2}|-3(3)+4| = \frac{5}{2}$;

Thus the required matrix of the order 3×2 with satisfying the given equation is given by

$$\begin{pmatrix} 1 & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{5}{2} & 2 & \frac{3}{2} & 1 \\ 4 & \frac{7}{2} & 3 & \frac{5}{2} \end{pmatrix}.$$

Exercise:5B

Q1

We are given $A = \begin{pmatrix} 2 & -3 & 5 \\ -1 & 0 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & 2 & -2 \\ 4 & -3 & 1 \end{pmatrix}$. The addition both matrices is possible as order of both matrices are similar.

Now operating addition of two matrices using addition of corresponding elements we get

$$A + B = \begin{pmatrix} 2 & -3 & 5 \\ -1 & 0 & 3 \end{pmatrix} + \begin{pmatrix} 3 & 2 & -2 \\ 4 & -3 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 2+3 & -3+2 & 5-2 \\ -1+4 & 0-3 & 3+1 \end{pmatrix} = \begin{pmatrix} 5 & -1 & 3 \\ 3 & -3 & 4 \end{pmatrix}$$

$$B + A = \begin{pmatrix} 3 & 2 & -2 \\ 4 & -3 & 1 \end{pmatrix} + \begin{pmatrix} 2 & -3 & 5 \\ -1 & 0 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 3+2 & 2-3 & -2+5 \\ 4-1 & -3+0 & 1+3 \end{pmatrix} = \begin{pmatrix} 5 & -1 & 3 \\ 3 & -3 & 4 \end{pmatrix}$$

Here we can say that $A + B = B + A$.

Q2

We are given $A = \begin{pmatrix} 3 & 5 \\ -2 & 0 \\ 6 & -1 \end{pmatrix}$, $B = \begin{pmatrix} -1 & -3 \\ 4 & 2 \\ -2 & 3 \end{pmatrix}$ and $C = \begin{pmatrix} 0 & 2 \\ 3 & -4 \\ 1 & 6 \end{pmatrix}$. The addition all matrices is possible as order of all matrices are similar.

The addition of two matrices is done by the addition of corresponding elements.

Now

LHL= (A+B)+C

$$= \left(\begin{pmatrix} 3 & 5 \\ -2 & 0 \\ 6 & -1 \end{pmatrix} + \begin{pmatrix} -1 & -3 \\ 4 & 2 \\ -2 & 3 \end{pmatrix} \right) + \begin{pmatrix} 0 & 2 \\ 3 & -4 \\ 1 & 6 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 2 \\ 2 & 2 \\ 4 & 2 \end{pmatrix} + \begin{pmatrix} 0 & 2 \\ 3 & -4 \\ 1 & 6 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 4 \\ 5 & -2 \\ 5 & 8 \end{pmatrix}$$

RHL= A+(B+C)

$$= \begin{pmatrix} 3 & 5 \\ -2 & 0 \\ 6 & -1 \end{pmatrix} + \left(\begin{pmatrix} -1 & -3 \\ 4 & 2 \\ -2 & 3 \end{pmatrix} + \begin{pmatrix} 0 & 2 \\ 3 & -4 \\ 1 & 6 \end{pmatrix} \right)$$

$$= \begin{pmatrix} 3 & 5 \\ -2 & 0 \\ 6 & -1 \end{pmatrix} + \begin{pmatrix} -1 & -1 \\ 7 & -2 \\ -1 & 7 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 4 \\ 5 & -2 \\ 5 & 8 \end{pmatrix}$$

Thus LHL=RHL

Therefore $(\boldsymbol{A} + \boldsymbol{B}) + \boldsymbol{C} = \boldsymbol{A} + (\boldsymbol{B} + \boldsymbol{C})$.

Q3

We have $\boldsymbol{A} = \begin{pmatrix} 3 & 1 & 2 \\ 1 & 2 & -3 \end{pmatrix}$ and $\boldsymbol{B} = \begin{pmatrix} -2 & 0 & 4 \\ 5 & -3 & 2 \end{pmatrix}$.

Now

$$\boldsymbol{2A - B} = 2\begin{pmatrix} 3 & 1 & 2 \\ 1 & 2 & -3 \end{pmatrix} - \begin{pmatrix} -2 & 0 & 4 \\ 5 & -3 & 2 \end{pmatrix}$$

$=\begin{pmatrix}6 & 2 & 4\\ 2 & 4 & -6\end{pmatrix}-\begin{pmatrix}-2 & 0 & 4\\ 5 & -3 & 2\end{pmatrix}$

$=\begin{pmatrix}8 & 2 & 0\\ -3 & 7 & -8\end{pmatrix}$

Q4

We are given $\boldsymbol{A}=\begin{pmatrix}2 & 4\\ 3 & 2\end{pmatrix}$, B= $\begin{pmatrix}1 & 3\\ -2 & 5\end{pmatrix}$ and $\boldsymbol{C}=\begin{pmatrix}-2 & 5\\ 5 & 4\end{pmatrix}$.

Next using the addition of two matrices and scalar multiplication of two matrices we get

i. $\boldsymbol{A}+2\boldsymbol{B}=\begin{pmatrix}2 & 4\\ 3 & 2\end{pmatrix}+2\begin{pmatrix}1 & 3\\ -2 & 5\end{pmatrix}=\begin{pmatrix}2 & 4\\ 3 & 2\end{pmatrix}+\begin{pmatrix}2 & 6\\ -4 & 10\end{pmatrix}=$

$\begin{pmatrix}4 & 10\\ -1 & 12\end{pmatrix}$

ii. $\boldsymbol{B}-4\boldsymbol{C}=\begin{pmatrix}1 & 3\\ -2 & 5\end{pmatrix}-4\begin{pmatrix}-2 & 5\\ 5 & 4\end{pmatrix}=\begin{pmatrix}1 & 3\\ -2 & 5\end{pmatrix}-\begin{pmatrix}-8 & 20\\ 12 & 16\end{pmatrix}=$

$\begin{pmatrix}9 & -17\\ -14 & -11\end{pmatrix}$

iii. $\boldsymbol{A}-2\boldsymbol{B}+3\boldsymbol{C}=\begin{pmatrix}2 & 4\\ 3 & 2\end{pmatrix}-2\begin{pmatrix}1 & 3\\ -2 & 5\end{pmatrix}+3\begin{pmatrix}-2 & 5\\ 5 & 4\end{pmatrix}=$

$\begin{pmatrix}2 & 4\\ 3 & 2\end{pmatrix}-\begin{pmatrix}2 & 6\\ -4 & 10\end{pmatrix}+\begin{pmatrix}-6 & 15\\ 9 & 12\end{pmatrix}=\begin{pmatrix}-6 & 13\\ 16 & 4\end{pmatrix}$

Q5

We are given $= \begin{pmatrix} 0 & 1 & -2 \\ 5 & -1 & -4 \end{pmatrix}$, $\boldsymbol{B} = \begin{pmatrix} 1 & -3 & -1 \\ 0 & -2 & 5 \end{pmatrix}$ and $\boldsymbol{C} = \begin{pmatrix} 2 & -5 & 1 \\ -4 & 0 & 6 \end{pmatrix}$.

Now using the addition of matrices we get

$$5\boldsymbol{A} - 3\boldsymbol{B} + 4\boldsymbol{C} = 5\begin{pmatrix} 0 & 1 & -2 \\ 5 & -1 & -4 \end{pmatrix} - 3\begin{pmatrix} 1 & -3 & -1 \\ 0 & -2 & 5 \end{pmatrix} + 4\begin{pmatrix} 2 & -5 & 1 \\ -4 & 0 & 6 \end{pmatrix}.$$

$$= \begin{pmatrix} 0 & 5 & -10 \\ 25 & -5 & -20 \end{pmatrix} - \begin{pmatrix} 3 & -9 & -3 \\ 0 & -6 & 15 \end{pmatrix} + \begin{pmatrix} 8 & -20 & 4 \\ -16 & 0 & 24 \end{pmatrix}.$$

$$= \begin{pmatrix} 5 & -6 & -3 \\ 9 & 1 & -11 \end{pmatrix}$$

Q6

We have given $5A = \begin{pmatrix} 5 & 10 & -15 \\ 2 & 3 & 4 \\ 1 & 0 & -5 \end{pmatrix}$.

Now to find A, let us divide it by 5 we get

$A = \frac{1}{5}(5\boldsymbol{A})$

$$= \frac{1}{5}\begin{pmatrix} 5 & 10 & -15 \\ 2 & 3 & 4 \\ 1 & 0 & -5 \end{pmatrix}$$

Dividing each elements with the help of 5 we get

$$= \begin{pmatrix} 1 & 2 & -3 \\ \frac{2}{5} & \frac{3}{5} & \frac{4}{5} \\ \frac{1}{5} & 0 & -1 \end{pmatrix}$$

Q7

We have given $\boldsymbol{A+B} = \begin{pmatrix} 1 & 0 & 2 \\ 5 & 4 & -6 \\ 7 & 3 & 8 \end{pmatrix}$ and $\boldsymbol{A-B} =$

$\begin{pmatrix} -5 & -4 & 8 \\ 11 & 2 & 0 \\ -1 & 7 & 4 \end{pmatrix}$.

Now by addition of both matrices we get

$\boldsymbol{2A} = (\boldsymbol{A+B}) + (\boldsymbol{A-B})$

$$= \begin{pmatrix} 1 & 0 & 2 \\ 5 & 4 & -6 \\ 7 & 3 & 8 \end{pmatrix} + \begin{pmatrix} -5 & -4 & 8 \\ 11 & 2 & 0 \\ -1 & 7 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} -4 & -4 & 10 \\ 16 & 6 & -6 \\ 6 & 10 & 12 \end{pmatrix}$$

Dividing above matrix by 2 we get

$$\boldsymbol{A} = \begin{pmatrix} -2 & -2 & 5 \\ 8 & 3 & -3 \\ 3 & 5 & 6 \end{pmatrix}$$

Now by subtraction of both matrices we get

$\boldsymbol{2B} = (\boldsymbol{A+B}) - (\boldsymbol{A-B})$

$$= \begin{pmatrix} 1 & 0 & 2 \\ 5 & 4 & -6 \\ 7 & 3 & 8 \end{pmatrix} - \begin{pmatrix} -5 & -4 & 8 \\ 11 & 2 & 0 \\ -1 & 7 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} 6 & 4 & -6 \\ -6 & 2 & -6 \\ 8 & -4 & 4 \end{pmatrix}$$

Dividing above matrix by 2 we get

$$\boldsymbol{B} = \begin{pmatrix} 3 & 4 & -3 \\ -3 & 1 & -3 \\ 4 & -2 & 2 \end{pmatrix}$$

Q8

We have given $\boldsymbol{2A - B} = \begin{pmatrix} 6 & -6 & 0 \\ -4 & 2 & 1 \end{pmatrix}$ and $\boldsymbol{2B + A} =$

$\begin{pmatrix} 3 & 2 & 5 \\ -2 & 1 & -7 \end{pmatrix}$.

Now by addition of double of $\boldsymbol{2A - B}$ and $2\boldsymbol{B + A}$ we get

$$\boldsymbol{5A = 2(2A - B) + (2B + A)}$$

$$= \begin{pmatrix} 6 & -6 & 0 \\ -4 & 2 & 1 \end{pmatrix} + \begin{pmatrix} 3 & 2 & 5 \\ -2 & 1 & -7 \end{pmatrix}$$

$$= \begin{pmatrix} 15 & -10 & 5 \\ -10 & 5 & -5 \end{pmatrix}$$

Dividing above matrix with 5 we get

$$\boldsymbol{A} = \frac{1}{5}(5\boldsymbol{A}) = \frac{1}{5}\begin{pmatrix} 15 & -10 & 5 \\ -10 & 5 & -5 \end{pmatrix} = \begin{pmatrix} 3 & -2 & 1 \\ -2 & 1 & -1 \end{pmatrix}.$$

Now rewriting the equation $\boldsymbol{2A - B} = \begin{pmatrix} 6 & -6 & 0 \\ -4 & 2 & 1 \end{pmatrix}$ we get

$$B = 2A - \begin{pmatrix} 6 & -6 & 0 \\ -4 & 2 & 1 \end{pmatrix}$$

$$= 2\begin{pmatrix} 3 & -2 & 1 \\ -2 & 1 & -1 \end{pmatrix} - \begin{pmatrix} 6 & -6 & 0 \\ -4 & 2 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 6 & -4 & 2 \\ -4 & 2 & -2 \end{pmatrix} - \begin{pmatrix} 6 & -6 & 0 \\ -4 & 2 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 2 & 2 \\ 0 & 0 & -3 \end{pmatrix}$$

Q9

We are given $\begin{pmatrix} 3 & 5 & -9 \\ -1 & 4 & -7 \end{pmatrix} + X = \begin{pmatrix} 6 & 2 & 3 \\ 4 & 8 & 6 \end{pmatrix}$.

Now to find X , subtract $\begin{pmatrix} 3 & 5 & -9 \\ -1 & 4 & -7 \end{pmatrix}$ from left side we get

$$X = \begin{pmatrix} 6 & 2 & 3 \\ 4 & 8 & 6 \end{pmatrix} - \begin{pmatrix} 3 & 5 & -9 \\ -1 & 4 & -7 \end{pmatrix} = \begin{pmatrix} 3 & -3 & 12 \\ 5 & 4 & 13 \end{pmatrix}.$$

Q10

We are given $A = \begin{pmatrix} -2 & 3 \\ 4 & 5 \\ 1 & -6 \end{pmatrix}$, $B = \begin{pmatrix} 5 & 2 \\ -7 & 3 \\ 6 & 4 \end{pmatrix}$. and $A + B - C =$

O.

Now We can have the equation as $A + B - O = C$.

Therefore

$C = A + B - C$

$$= \begin{pmatrix} -2 & 3 \\ 4 & 5 \\ 1 & -6 \end{pmatrix} + \begin{pmatrix} 5 & 2 \\ -7 & 3 \\ 6 & 4 \end{pmatrix} - \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 3 & 2 \\ -3 & 3 \\ 7 & 4 \end{pmatrix}$$

Q11

We are given $\boldsymbol{A} = \begin{pmatrix} 3 & 1 \\ 0 & 2 \end{pmatrix}$, $\boldsymbol{B} = \begin{pmatrix} -2 & 1 \\ 0 & 3 \end{pmatrix}$. and $\boldsymbol{2A - B + X = O}$.

Now We can have the equation as $\boldsymbol{2A - B = 0 - X = X}$.

Thus $\boldsymbol{X = 2A - B}$

$$= 2\begin{pmatrix} 3 & 1 \\ 0 & 2 \end{pmatrix} - \begin{pmatrix} -2 & 1 \\ 0 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} -8 & -1 \\ 0 & -1 \end{pmatrix}$$

Q. 12

We have given $\boldsymbol{A} = \begin{pmatrix} 1 & -3 & 2 \\ 2 & 0 & 2 \end{pmatrix}$, $\boldsymbol{B} = \begin{pmatrix} 2 & -1 & -1 \\ 1 & 0 & -1 \end{pmatrix}$ **and** $\boldsymbol{A + B + C = O}$.

Now by addition of $\boldsymbol{A}$ and $\boldsymbol{B}$ we get

$$\boldsymbol{A + B} = \begin{pmatrix} 1 & -3 & 2 \\ 2 & 0 & 2 \end{pmatrix} + \begin{pmatrix} 2 & -1 & -1 \\ 1 & 0 & -1 \end{pmatrix} = \begin{pmatrix} 3 & -4 & 1 \\ 3 & 0 & 1 \end{pmatrix}$$

Substracing above matrix frommatrix $\boldsymbol{O}$ we get

$\boldsymbol{C = O - (A + B)} = \frac{1}{5} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} - \begin{pmatrix} 3 & -4 & 1 \\ 3 & 0 & 1 \end{pmatrix} =$

$\begin{pmatrix} -3 & 4 & -1 \\ -3 & 0 & -1 \end{pmatrix}$.

Q13

We have given $\boldsymbol{A = diag[2, -5, 9]} = \begin{pmatrix} 2 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & 9 \end{pmatrix}$ and $\boldsymbol{B} =$

$\boldsymbol{diag[-3, 7, 14]} = \begin{pmatrix} -3 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 14 \end{pmatrix}$.

(i) Now by addition of $\boldsymbol{A}$ +2B we get

$\boldsymbol{A + 2B}$

$= \begin{pmatrix} 2 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & 9 \end{pmatrix} + 2\begin{pmatrix} -3 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 14 \end{pmatrix}$

$= \begin{pmatrix} -4 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 37 \end{pmatrix} = \boldsymbol{diag[-4, 9, 37]}$

(ii) Now to find $\boldsymbol{B + C - A}$

$\boldsymbol{B + C - A} = \begin{pmatrix} -3 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 14 \end{pmatrix} + \begin{pmatrix} 4 & 0 & 0 \\ 0 & -6 & 0 \\ 0 & 0 & 3 \end{pmatrix} - \begin{pmatrix} 2 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & 9 \end{pmatrix}$

$= \begin{pmatrix} 2 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & 9 \end{pmatrix}$

Q14

(i) We are given that $\begin{pmatrix} x+y \\ x-y \end{pmatrix} = \begin{pmatrix} 8 \\ 4 \end{pmatrix}$.

By equality of matrices we get

$$x + y = 8 \; ; x - y = 4$$

Thus $x = 6$ **and** $y = 2$.

(ii) We have given $\begin{pmatrix} 2x+5 & 7 \\ 0 & 3y-7 \end{pmatrix} = \begin{pmatrix} x-3 & 7 \\ 0 & -5 \end{pmatrix}$.

Thus from the equality of matrices we get

$$2x + 5 = x - 3 \; ; 3y - 7 = -5$$

By solving linear equations we get

$$x = -8 \; ; y = \frac{2}{3}$$

(iii) We are given

$$2\begin{pmatrix} x & 5 \\ 7y-3 \end{pmatrix} + \begin{pmatrix} 3 & -4 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 7 & 6 \\ 15 & 14 \end{pmatrix}.$$

This we can write as

$$\begin{pmatrix} 2x+3 & 6 \\ 15 & 2y-4 \end{pmatrix} = \begin{pmatrix} 7 & 6 \\ 15 & 14 \end{pmatrix}$$

Now by using equality of matrices we get

$$2x + 3 = 7 \; ; 2y - 4 = 14$$

By solving given linear equations we get

$$x = 2 \; ; y = 9$$

Q15

We are given equation as $\mathbf{2}\begin{pmatrix}\mathbf{1}&\mathbf{3}\\\mathbf{0}&\boldsymbol{x}\end{pmatrix}+\begin{pmatrix}\boldsymbol{y}&\mathbf{0}\\\mathbf{1}&\mathbf{2}\end{pmatrix}=\begin{pmatrix}\mathbf{5}&\mathbf{6}\\\mathbf{1}&\mathbf{8}\end{pmatrix}$.

Now by using addition rule of matrices we get

$$\begin{pmatrix}\mathbf{2}+\boldsymbol{y}&\mathbf{6}\\\mathbf{1}&\mathbf{2}\boldsymbol{x}+\mathbf{2}\end{pmatrix}=\begin{pmatrix}\mathbf{5}&\mathbf{6}\\\mathbf{1}&\mathbf{8}\end{pmatrix}$$

By using equality of two matrices we get

$$\mathbf{2}+\boldsymbol{y}=\mathbf{5}\ ;\mathbf{2}\boldsymbol{x}+\mathbf{2}=\mathbf{8}$$

Thus by solving above equations we get

$$\boldsymbol{x}=\mathbf{3}\ ;\boldsymbol{y}=\mathbf{3}$$

Therefore we have $\boldsymbol{x}+\boldsymbol{y}=\mathbf{3}+\mathbf{3}=\mathbf{6}$.

Q16

We are given that $\begin{pmatrix}\boldsymbol{x}-\boldsymbol{y}&\mathbf{2}\boldsymbol{y}\\\mathbf{2}\boldsymbol{y}+\boldsymbol{z}&\boldsymbol{x}+\boldsymbol{y}\end{pmatrix}=\begin{pmatrix}\mathbf{1}&\mathbf{4}\\\mathbf{9}&\mathbf{5}\end{pmatrix}$.

By equality of matrices we get

$$\boldsymbol{x}-\boldsymbol{y}=\mathbf{1}\,;\mathbf{2}\boldsymbol{y}+\boldsymbol{z}=\mathbf{9}\ ;\ \boldsymbol{x}+\boldsymbol{y}=\mathbf{5}\,;\mathbf{2}\boldsymbol{y}=\mathbf{4}$$

By from above equations we get

$\boldsymbol{x}+\boldsymbol{y}=\mathbf{5}$.

Exercise:5C

Q1A

We have $\boldsymbol{A} = \begin{pmatrix} \mathbf{2} & \mathbf{-1} \\ \mathbf{3} & \mathbf{0} \\ \mathbf{-1} & \mathbf{4} \end{pmatrix}$ and $\boldsymbol{B} = \begin{pmatrix} \mathbf{-2} & \mathbf{3} \\ \mathbf{0} & \mathbf{4} \end{pmatrix}$.

Now multiplication of two matrices is possible if number of columns in left matrix is equals to the number of rows in right matrix.

Next let us discuss the order of the matrices which are given.

The order of matrix $\boldsymbol{A}$ is $\mathbf{3 \times 2}$ and matrix $\boldsymbol{B}$ is $\mathbf{2 \times 2}$.

Thus the multiplications we can proceed as

(i) The multiplication $\boldsymbol{AB}$ is possible.

Thus $\boldsymbol{AB} = \begin{pmatrix} \mathbf{2} & \mathbf{-1} \\ \mathbf{3} & \mathbf{0} \\ \mathbf{-1} & \mathbf{4} \end{pmatrix} \begin{pmatrix} \mathbf{-2} & \mathbf{3} \\ \mathbf{0} & \mathbf{4} \end{pmatrix} =$

$$\begin{pmatrix} \mathbf{(2)(-2)+(-1)(0)} & \mathbf{(2)3+(-1)4} \\ \mathbf{3(-1)+(0)(0)} & \mathbf{(3)3+(0)4} \\ \mathbf{(-1)(-2)+(4)(0)} & \mathbf{(-1)3+(4)4} \end{pmatrix} = \begin{pmatrix} \mathbf{-4} & \mathbf{2} \\ \mathbf{-6} & \mathbf{9} \\ \mathbf{2} & \mathbf{13} \end{pmatrix}$$

(ii) The multiplication of $\boldsymbol{BA}$ is not possible as if number of columns in left matrix is 2 which is not equals to the number of rows in right matrix i.e, 3.

Q1B

We have $\boldsymbol{A} = \begin{pmatrix} \mathbf{-1} & \mathbf{1} \\ \mathbf{-2} & \mathbf{2} \\ \mathbf{-3} & \mathbf{3} \end{pmatrix}$ and $\boldsymbol{B} = \begin{pmatrix} \mathbf{3} & \mathbf{-2} & \mathbf{1} \\ \mathbf{0} & \mathbf{1} & \mathbf{2} \\ \mathbf{-3} & \mathbf{4} & \mathbf{-5} \end{pmatrix}$.

Now multiplication of two matrices is possible if number of columns in left matrix is equals to the number of rows in right matrix.

Next let us discuss the order of the matrices which are given.

The order of matrix $\boldsymbol{A}$ is $\mathbf{3 \times 2}$ and matrix $\boldsymbol{B}$ is $\mathbf{3 \times 3}$.

Thus the multiplications we can proceed as

(i) The multiplication $\boldsymbol{AB}$ is not possible as if number of columns in left matrix is 2 which is not equals to the number of rows in right matrix i.e, 3.

(ii) The multiplication $\boldsymbol{BA}$ is possible.

Thus $\boldsymbol{BA} = \begin{pmatrix} \mathbf{3} & \mathbf{-2} & \mathbf{1} \\ \mathbf{0} & \mathbf{1} & \mathbf{2} \\ \mathbf{-3} & \mathbf{4} & \mathbf{-5} \end{pmatrix} \begin{pmatrix} \mathbf{-1} & \mathbf{1} \\ \mathbf{-2} & \mathbf{2} \\ \mathbf{-3} & \mathbf{3} \end{pmatrix}$

$$= \begin{pmatrix} \mathbf{3(-1) - 2(-2) + 1(-3)} & \mathbf{3(1) - 2(2) + 1(3)} \\ \mathbf{0(-1) + 1(-2) + 2(-3)} & \mathbf{0(1) + 1(2) + 2(3)} \\ \mathbf{-3(-1) + 4(-2) - 5(-3)} & \mathbf{-3(1) + 4(2) - 5(3)} \end{pmatrix}$$

$$= \begin{pmatrix} \mathbf{-2} & \mathbf{2} \\ \mathbf{-8} & \mathbf{8} \\ \mathbf{10} & \mathbf{-10} \end{pmatrix}$$

Q1C

We have $\boldsymbol{A} = \begin{pmatrix} \mathbf{0} & \mathbf{1} & \mathbf{-5} \\ \mathbf{2} & \mathbf{4} & \mathbf{0} \end{pmatrix}$ and $\boldsymbol{B} = \begin{pmatrix} \mathbf{1} & \mathbf{3} \\ \mathbf{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{5} \end{pmatrix}$.

Now multiplication of two matrices is possible if number of columns in left matrix is equals to the number of rows in right matrix.

Next let us discuss the order of the matrices which are given.

The order of matrix $\boldsymbol{A}$ is $\mathbf{2 \times 3}$ and matrix$\boldsymbol{B}$ is $\mathbf{3 \times 2}$.

Thus the multiplications we can proceed as

(i) The multiplication $\boldsymbol{BA}$ is possible.

$$\boldsymbol{BA} = \begin{pmatrix} \mathbf{1} & \mathbf{3} \\ \mathbf{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{5} \end{pmatrix} \begin{pmatrix} \mathbf{0} & \mathbf{1} & \mathbf{-5} \\ \mathbf{2} & \mathbf{4} & \mathbf{0} \end{pmatrix}$$

$$= \begin{pmatrix} \mathbf{0(1) + 1(-1) - 5(0)} & \mathbf{0(3) + 1(0) - 5(5)} \\ \mathbf{2(1) + 4(-1) + 0(0)} & \mathbf{2(3) + 4(0) + 0(5)} \end{pmatrix}$$

$$= \begin{pmatrix} \mathbf{-1} & \mathbf{-25} \\ \mathbf{-2} & \mathbf{6} \end{pmatrix}$$

(ii) The multiplication $\boldsymbol{AB}$ is possible.

$$\boldsymbol{AB} = \begin{pmatrix} \mathbf{0} & \mathbf{1} & \mathbf{-5} \\ \mathbf{2} & \mathbf{4} & \mathbf{0} \end{pmatrix} \begin{pmatrix} \mathbf{1} & \mathbf{3} \\ \mathbf{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{5} \end{pmatrix}$$

$$= \begin{pmatrix} \mathbf{1(0) + 3(2)} & \mathbf{1(1) + 3(4)} & \mathbf{1(-5) + 3(0)} \\ \mathbf{-1(0) + 0(2)} & \mathbf{-1(1) + 0(4)} & \mathbf{-1(-5) + 0(0)} \\ \mathbf{0(0) + 5(2)} & \mathbf{0(1) + 5(4)} & \mathbf{0(-5) + 5(0)} \end{pmatrix}$$

$$= \begin{pmatrix} 6 & 13 & -5 \\ 0 & -1 & 5 \\ 10 & 20 & 0 \end{pmatrix}$$

Q1D

We have $\boldsymbol{A} = (1\ \ 2\ \ 3\ \ 4)$ and $\boldsymbol{B} = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}$.

Now multiplication of two matrices is possible if number of columns in left matrix is equals to the number of rows in right matrix.

Next let us discuss the order of the matrices which are given.

The order of matrix $\boldsymbol{A}$ is $\boldsymbol{1 \times 4}$ and matrix $\boldsymbol{B}$ is $\boldsymbol{4 \times 1}$.

Thus the multiplications we can proceed as

(i) The multiplication $\boldsymbol{AB}$ is possible.

$$\boldsymbol{AB} = (1\ \ 2\ \ 3\ \ 4)\begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix} = \big(\ \ 1(1) + 2(2) + 3(3) + 4(4)\big) = (30)$$

(ii) The multiplication $\boldsymbol{BA}$ is possible.

$$BA = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix} (1 \;\; 2 \;\; 3 \;\; 4) = \begin{pmatrix} 1(1) & 1(2) & 1(3) & 1(4) \\ 2(1) & 2(2) & 2(3) & 2(4) \\ 3(1) & 3(2) & 3(3) & 3(4) \\ 4(1) & 4(2) & 4(3) & 4(4) \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ 3 & 6 & 9 & 12 \\ 4 & 8 & 12 & 16 \end{pmatrix}$$

Q1E

We have $B = \begin{pmatrix} 1 & 0 & 1 \\ -1 & 2 & 1 \end{pmatrix}$ and $A = \begin{pmatrix} 2 & 1 \\ 3 & 2 \\ -1 & 1 \end{pmatrix}$.

Now multiplication of two matrices is possible if number of columns in left matrix is equals to the number of rows in right matrix.

Next let us discuss the order of the matrices which are given.

The order of matrix B is 2×3 and matrix A is 3×2.

Thus the multiplications we can proceed as

(i) The multiplication AB is possible.

$$AB = \begin{pmatrix} 2 & 1 \\ 3 & 2 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 1 \\ -1 & 2 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 2(1)+1(-1) & 2(0)+1(2) & 2(1)+1(1) \\ 3(1)+2(-1) & 3(0)+2(2) & 3(1)+2(1) \\ -1(1)+1(-1) & -1(0)+1(2) & -1(1)+1(1) \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 2 & 3 \\ 1 & 4 & 5 \\ -2 & 2 & 0 \end{pmatrix}$$

(ii) The multiplication BA is possible.

$$BA = \begin{pmatrix} 1 & 0 & 1 \\ -1 & 2 & 1 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 3 & 2 \\ -1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1(2)+0(3)+1(-1) & 1(1)+0(2)+1(1) \\ -1(2)+2(3)+1(-1) & -1(1)+2(2)+1(1) \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

Q2A

We have $A = \begin{pmatrix} 5 & -1 \\ 6 & 7 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix}$.

Now multiplication of two matrices is possible if number of columns in left matrix is equals to the number of rows in right matrix.

Next let us discuss the order of the matrices which are given.

The order of matrix $\boldsymbol{B}$ is $\mathbf{2 \times 2}$ and matrix $\boldsymbol{A}$ is $\mathbf{2 \times 2}$.

Thus the multiplications we can proceed as

(i) The multiplication $\boldsymbol{AB}$ is possible.

$$\boldsymbol{AB} = \begin{pmatrix} \mathbf{5} & \mathbf{-1} \\ \mathbf{6} & \mathbf{7} \end{pmatrix}\begin{pmatrix} \mathbf{2} & \mathbf{1} \\ \mathbf{3} & \mathbf{4} \end{pmatrix} = \begin{pmatrix} \mathbf{5(2)-1(3)} & \mathbf{5(1)-1(4)} \\ \mathbf{6(2)+7(3)} & \mathbf{6(1)+7(4)} \end{pmatrix}$$

$$= \begin{pmatrix} \mathbf{7} & \mathbf{1} \\ \mathbf{33} & \mathbf{34} \end{pmatrix}$$

(ii) The multiplication $\boldsymbol{BA}$ is possible.

$$\boldsymbol{BA} = \begin{pmatrix} \mathbf{2} & \mathbf{1} \\ \mathbf{3} & \mathbf{4} \end{pmatrix}\begin{pmatrix} \mathbf{5} & \mathbf{-1} \\ \mathbf{6} & \mathbf{7} \end{pmatrix}$$

$$= \begin{pmatrix} \mathbf{2(5)+1(6)} & \mathbf{2(-1)+1(7)} \\ \mathbf{3(5)+4(6)} & \mathbf{3(-1)+4(7)} \end{pmatrix}$$

$$= \begin{pmatrix} \mathbf{16} & \mathbf{5} \\ \mathbf{39} & \mathbf{25} \end{pmatrix}$$

From (i) and (ii) we can observe that both the matrices $\boldsymbol{AB}$ and $\boldsymbol{BA}$ are of same order $\mathbf{2 \times 2}$ but the corresponding elements are different.

Thus by equality of two matrices we can say here that our matrices are not equal.

Thus$\boldsymbol{AB \neq BA}$.

Q2B

We have $\boldsymbol{A} = \begin{pmatrix} \mathbf{1} & \mathbf{2} & \mathbf{3} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{1} & \mathbf{1} & \mathbf{0} \end{pmatrix}$ and $\boldsymbol{B} = \begin{pmatrix} \mathbf{-1} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{-1} & \mathbf{1} \\ \mathbf{2} & \mathbf{2} & \mathbf{4} \end{pmatrix}$.

Now multiplication of two matrices is possible if number of columns in left matrix is equals to the number of rows in right matrix.

Next let us discuss the order of the matrices which are given.

The order of matrix $\boldsymbol{B}$ is $\mathbf{3 \times 3}$ and matrix $\boldsymbol{A}$ is $\mathbf{3 \times 3}$.

Thus the multiplications we can proceed as

(i) The multiplication $\boldsymbol{AB}$ is possible.

$$\boldsymbol{AB} = \begin{pmatrix} \mathbf{1} & \mathbf{2} & \mathbf{3} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{1} & \mathbf{1} & \mathbf{0} \end{pmatrix} \begin{pmatrix} \mathbf{5} & \mathbf{10} & \mathbf{-15} \\ \mathbf{2} & \mathbf{3} & \mathbf{4} \\ \mathbf{1} & \mathbf{0} & \mathbf{-5} \end{pmatrix}$$

$$= \begin{pmatrix} \mathbf{1(-1)+2(0)+3(2)} & \mathbf{1(1)+2(-1)+3(3)} & \mathbf{1(0)+2(1)+3(4)} \\ \mathbf{0(-1)+1(0)+0(2)} & \mathbf{0(1)+1(-1)+0(3)} & \mathbf{0(0)+1(1)+0(4)} \\ \mathbf{1(-1)+1(0)+0(2)} & \mathbf{1(1)+1(-1)+0(3)} & \mathbf{1(0)+1(1)+0(4)} \end{pmatrix}$$

$$= \begin{pmatrix} \mathbf{5} & \mathbf{8} & \mathbf{14} \\ \mathbf{0} & \mathbf{-1} & \mathbf{1} \\ \mathbf{-1} & \mathbf{0} & \mathbf{1} \end{pmatrix}$$

(ii) The multiplication $\boldsymbol{BA}$ is possible.

$$\boldsymbol{BA} = \begin{pmatrix} \mathbf{-1} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{-1} & \mathbf{1} \\ \mathbf{2} & \mathbf{2} & \mathbf{4} \end{pmatrix} \begin{pmatrix} \mathbf{1} & \mathbf{2} & \mathbf{3} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{1} & \mathbf{1} & \mathbf{0} \end{pmatrix}$$

$$= \begin{pmatrix} -1(1)+1(0)+0(1) & -1(2)+1(1)+0(1) & -1(3)+1(0)+0(0) \\ 0(1)-1(0)+1(1) & 0(2)-1(1)+1(1) & 0(3)-1(0)+1(0) \\ 2(1)+3(0)+4(1) & 2(2)+3(1)+4(1) & 2(3)+3(0)+4(0) \end{pmatrix}$$

$$= \begin{pmatrix} -1 & -1 & -3 \\ 0 & 0 & 0 \\ 2 & 11 & 6 \end{pmatrix}$$

From (i) and (ii) we can observe that both the matrices $\boldsymbol{AB}$ and $\boldsymbol{BA}$ are of same order $\mathbf{3 \times 3}$ but the corresponding elements are different.

Thus by equality of two matrices we can say here that our matrices are not equal.

Thus $\boldsymbol{AB \neq BA}$.

Q3A

We have $\boldsymbol{A} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$ and $\boldsymbol{B} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix}$.

Now multiplication of two matrices is possible if number of columns in left matrix is equals to the number of rows in right matrix.

Next let us discuss the order of the matrices which are given.

The order of matrix $\boldsymbol{B}$ is $\mathbf{2 \times 2}$ and matrix $\boldsymbol{A}$ is $\mathbf{2 \times 2}$.

Thus the multiplications we can proceed as

(i) The multiplication $\boldsymbol{AB}$ is possible.

$$AB = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix}$$

$$= \begin{pmatrix} \cos\theta\cos\phi - \sin\theta\sin\phi & -\cos\theta\sin\theta - \sin\theta\sin\phi \\ \sin\theta\cos\phi + \cos\theta\cos\phi & -\sin\theta\sin\phi + \cos\theta\cos\phi \end{pmatrix}$$

(ii) The multiplication BA is possible.

$$BA = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix}\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

$$= \begin{pmatrix} \cos\theta\cos\phi - \sin\theta\sin\phi & -\cos\theta\sin\theta - \sin\theta\sin\phi \\ \sin\theta\cos\phi + \cos\theta\cos\phi & -\sin\theta\sin\phi + \cos\theta\cos\phi \end{pmatrix}$$

From (i) and (ii) we can observe that both the matrices AB and BA are of same order $2 \times 2$and the corresponding elements are same. Thus by equality of two matrices we can say here that our matrices are equal.

Thus$AB = BA$.

Q3B

We have $A = \begin{pmatrix} 1 & 2 & 1 \\ 3 & 4 & 2 \\ 1 & 3 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 10 & -4 & -1 \\ -11 & 5 & 0 \\ 9 & -5 & 1 \end{pmatrix}$.

Now multiplication of two matrices is possible if number of columns in left matrix is equals to the number of rows in right matrix.

Next let us discuss the order of the matrices which are given.

The order of matrix $\boldsymbol{B}$ is $\mathbf{3 \times 3}$ and matrix $\boldsymbol{A}$ is $\mathbf{3 \times 3}$.

Thus the multiplications we can proceed as

(i) The multiplication $\boldsymbol{AB}$ is possible.

$$\boldsymbol{AB} = \begin{pmatrix} \mathbf{1} & \mathbf{2} & \mathbf{1} \\ \mathbf{3} & \mathbf{4} & \mathbf{2} \\ \mathbf{1} & \mathbf{3} & \mathbf{2} \end{pmatrix} \begin{pmatrix} \mathbf{10} & \mathbf{-4} & \mathbf{-1} \\ \mathbf{-11} & \mathbf{5} & \mathbf{0} \\ \mathbf{9} & \mathbf{-5} & \mathbf{1} \end{pmatrix} = \begin{pmatrix} \mathbf{-3} & \mathbf{1} & \mathbf{0} \\ \mathbf{-32} & \mathbf{-2} & \mathbf{-1} \\ \mathbf{-41} & \mathbf{1} & \mathbf{1} \end{pmatrix}$$

(ii) The multiplication $\boldsymbol{BA}$ is possible.

$$\boldsymbol{BA} = \begin{pmatrix} \mathbf{10} & \mathbf{-4} & \mathbf{-1} \\ \mathbf{-11} & \mathbf{5} & \mathbf{0} \\ \mathbf{9} & \mathbf{-5} & \mathbf{1} \end{pmatrix} \begin{pmatrix} \mathbf{1} & \mathbf{2} & \mathbf{1} \\ \mathbf{3} & \mathbf{4} & \mathbf{2} \\ \mathbf{1} & \mathbf{3} & \mathbf{2} \end{pmatrix} = \begin{pmatrix} \mathbf{-3} & \mathbf{1} & \mathbf{0} \\ \mathbf{-4} & \mathbf{-2} & \mathbf{-1} \\ \mathbf{-5} & \mathbf{1} & \mathbf{1} \end{pmatrix}$$

From (i) and (ii) we can observe that both the matrices $\boldsymbol{AB}$ and $\boldsymbol{BA}$ are of same order $\mathbf{3 \times 3}$but the corresponding elements are different.

Thus by equality of two matrices we can say here that our matrices are not equal.

Thus$\boldsymbol{AB} \neq \boldsymbol{BA}$.

Q3C

We have $\boldsymbol{A} = \begin{pmatrix} \mathbf{1} & \mathbf{3} & \mathbf{-1} \\ \mathbf{2} & \mathbf{2} & \mathbf{-1} \\ \mathbf{3} & \mathbf{0} & \mathbf{-1} \end{pmatrix}$ and $\boldsymbol{B} = \begin{pmatrix} \mathbf{-2} & \mathbf{3} & \mathbf{-1} \\ \mathbf{-1} & \mathbf{2} & \mathbf{-1} \\ \mathbf{-6} & \mathbf{9} & \mathbf{-4} \end{pmatrix}$.

Now multiplication of two matrices is possible if number of columns in left matrix is equals to the number of rows in right matrix.

Next let us discuss the order of the matrices which are given.

The order of matrix $\boldsymbol{B}$ is $\mathbf{3 \times 3}$ and matrix $\boldsymbol{A}$ is $\mathbf{3 \times 3}$.

Thus the multiplications we can proceed as

(i) The multiplication $\boldsymbol{AB}$ is possible.

$$\boldsymbol{AB} = \begin{pmatrix} \mathbf{1} & \mathbf{3} & \mathbf{-1} \\ \mathbf{2} & \mathbf{2} & \mathbf{-1} \\ \mathbf{3} & \mathbf{0} & \mathbf{-1} \end{pmatrix} \begin{pmatrix} \mathbf{-2} & \mathbf{3} & \mathbf{-1} \\ \mathbf{-1} & \mathbf{2} & \mathbf{-1} \\ \mathbf{-6} & \mathbf{9} & \mathbf{-4} \end{pmatrix} = \begin{pmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} \end{pmatrix}$$

(ii) The multiplication $\boldsymbol{BA}$ is possible.

$$\boldsymbol{BA} = \begin{pmatrix} \mathbf{-2} & \mathbf{3} & \mathbf{-1} \\ \mathbf{-1} & \mathbf{2} & \mathbf{-1} \\ \mathbf{-6} & \mathbf{9} & \mathbf{-4} \end{pmatrix} \begin{pmatrix} \mathbf{1} & \mathbf{3} & \mathbf{-1} \\ \mathbf{2} & \mathbf{2} & \mathbf{-1} \\ \mathbf{3} & \mathbf{0} & \mathbf{-1} \end{pmatrix} = \begin{pmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} \end{pmatrix}$$

From (i) and (ii) we can observe that both the matrices $\boldsymbol{AB}$ and $\boldsymbol{BA}$ are of same order $\mathbf{3 \times 3}$ and the corresponding elements are same.

Thus by equality of two matrices we can say here that our matrices are equal.

Thus $\boldsymbol{AB} = \boldsymbol{BA}$.

Q4

We have $\boldsymbol{A} = \begin{pmatrix} \mathbf{2} & \mathbf{-3} & \mathbf{-5} \\ \mathbf{-1} & \mathbf{4} & \mathbf{5} \\ \mathbf{1} & \mathbf{-3} & \mathbf{-4} \end{pmatrix}$ and $\boldsymbol{B} = \begin{pmatrix} \mathbf{2} & \mathbf{-2} & \mathbf{-4} \\ \mathbf{-1} & \mathbf{3} & \mathbf{4} \\ 1 & \mathbf{-2} & \mathbf{-3} \end{pmatrix}$.

Now multiplication of two matrices is possible if number of columns in left matrix is equals to the number of rows in right matrix.

Next let us discuss the order of the matrices which are given.

The order of matrix $\boldsymbol{B}$ is $\mathbf{3 \times 3}$ and matrix $\boldsymbol{A}$ is $\mathbf{3 \times 3}$.

Thus the multiplications we can proceed as

(i) The multiplication $\boldsymbol{AB}$ is possible.

$$\boldsymbol{AB} = \begin{pmatrix} \mathbf{2} & \mathbf{-3} & \mathbf{-5} \\ \mathbf{-1} & \mathbf{4} & \mathbf{5} \\ \mathbf{1} & \mathbf{-3} & \mathbf{-4} \end{pmatrix} \begin{pmatrix} \mathbf{2} & \mathbf{-2} & \mathbf{-4} \\ \mathbf{-1} & \mathbf{3} & \mathbf{4} \\ \mathbf{1} & \mathbf{-2} & \mathbf{-3} \end{pmatrix} = \begin{pmatrix} \mathbf{2} & \mathbf{-3} & \mathbf{-5} \\ \mathbf{-1} & \mathbf{4} & \mathbf{5} \\ \mathbf{1} & \mathbf{-3} & \mathbf{-4} \end{pmatrix}$$

$$= \boldsymbol{A}$$

(ii) The multiplication $\boldsymbol{BA}$ is possible.

$$\boldsymbol{BA} = \begin{pmatrix} \mathbf{2} & \mathbf{-2} & \mathbf{-4} \\ \mathbf{-1} & \mathbf{3} & \mathbf{4} \\ \mathbf{1} & \mathbf{-2} & \mathbf{-3} \end{pmatrix} \begin{pmatrix} \mathbf{2} & \mathbf{-3} & \mathbf{-5} \\ \mathbf{-1} & \mathbf{4} & \mathbf{5} \\ \mathbf{1} & \mathbf{-3} & \mathbf{-4} \end{pmatrix} = \begin{pmatrix} \mathbf{2} & \mathbf{-2} & \mathbf{-4} \\ \mathbf{-1} & \mathbf{3} & \mathbf{4} \\ \mathbf{1} & \mathbf{-2} & \mathbf{-3} \end{pmatrix}$$

$$= \boldsymbol{B}$$

Q5

We have $\boldsymbol{A}=\begin{pmatrix}\boldsymbol{0} & \boldsymbol{c} & \boldsymbol{-b}\\ \boldsymbol{-c} & \boldsymbol{0} & \boldsymbol{a}\\ \boldsymbol{b} & \boldsymbol{-a} & \boldsymbol{0}\end{pmatrix}$ and $\boldsymbol{B}=\begin{pmatrix}\boldsymbol{a^2} & \boldsymbol{ab} & \boldsymbol{ac}\\ \boldsymbol{ab} & \boldsymbol{b^2} & \boldsymbol{bc}\\ \boldsymbol{ac} & \boldsymbol{bc} & \boldsymbol{c^2}\end{pmatrix}$.

Now multiplication of two matrices is possible if number of columns in left matrix is equals to the number of rows in right matrix.

Next let us discuss the order of the matrices which are given.

The order of matrix $\boldsymbol{B}$ is $\boldsymbol{3\times 3}$ and matrix $\boldsymbol{A}$ is $\boldsymbol{3\times 3}$.

Thus the multiplications we can proceed as the multiplication $\boldsymbol{AB}$ is possible.

$$\boldsymbol{AB}=\begin{pmatrix}\boldsymbol{0} & \boldsymbol{c} & \boldsymbol{-b}\\ \boldsymbol{-c} & \boldsymbol{0} & \boldsymbol{a}\\ \boldsymbol{b} & \boldsymbol{-a} & \boldsymbol{0}\end{pmatrix}\begin{pmatrix}\boldsymbol{a^2} & \boldsymbol{ab} & \boldsymbol{ac}\\ \boldsymbol{ab} & \boldsymbol{b^2} & \boldsymbol{bc}\\ \boldsymbol{ac} & \boldsymbol{bc} & \boldsymbol{c^2}\end{pmatrix}=\begin{pmatrix}\boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0}\end{pmatrix}=\boldsymbol{O}.$$

Q6A

We have $\boldsymbol{A}=\begin{pmatrix}\boldsymbol{1} & \boldsymbol{2} & \boldsymbol{5}\\ \boldsymbol{0} & \boldsymbol{1} & \boldsymbol{3}\end{pmatrix}$,$\boldsymbol{B}=\begin{pmatrix}\boldsymbol{2} & \boldsymbol{3} & \boldsymbol{0}\\ \boldsymbol{1} & \boldsymbol{0} & \boldsymbol{4}\\ \boldsymbol{1} & \boldsymbol{-1} & \boldsymbol{2}\end{pmatrix}$.and $\boldsymbol{C}=\begin{pmatrix}\boldsymbol{1}\\ \boldsymbol{4}\\ \boldsymbol{5}\end{pmatrix}$

Now multiplication of two matrices is possible if number of columns in left matrix is equals to the number of rows in right matrix.

Next let us discuss the order of the matrices which are given.

The order of matrix $\boldsymbol{B}$ is $\mathbf{3 \times 3}$, matrix $\boldsymbol{A}$ is $\mathbf{2 \times 3}$ and matrix $\boldsymbol{C}$ is $\mathbf{3 \times 1}$.

Thus the multiplications we can proceed as

(i) The multiplication $\boldsymbol{AB}$ is possible.

$$\boldsymbol{AB} = \begin{pmatrix} \mathbf{1} & \mathbf{2} & \mathbf{5} \\ \mathbf{0} & \mathbf{1} & \mathbf{3} \end{pmatrix} \begin{pmatrix} \mathbf{2} & \mathbf{3} & \mathbf{0} \\ \mathbf{1} & \mathbf{0} & \mathbf{4} \\ \mathbf{1} & \mathbf{-1} & \mathbf{2} \end{pmatrix} = \begin{pmatrix} \mathbf{9} & \mathbf{-2} & \mathbf{18} \\ \mathbf{4} & \mathbf{-3} & \mathbf{10} \end{pmatrix}$$

(ii) The multiplication $\boldsymbol{BC}$ is possible.

$$\boldsymbol{BC} = \begin{pmatrix} \mathbf{2} & \mathbf{3} & \mathbf{0} \\ \mathbf{1} & \mathbf{0} & \mathbf{4} \\ \mathbf{1} & \mathbf{-1} & \mathbf{2} \end{pmatrix} \begin{pmatrix} \mathbf{1} \\ \mathbf{4} \\ \mathbf{5} \end{pmatrix} = \begin{pmatrix} \mathbf{14} \\ \mathbf{21} \\ \mathbf{7} \end{pmatrix}$$

(iii) The multiplication $\boldsymbol{A(BC)}$ is possible.

$$\boldsymbol{A(BC)} = \begin{pmatrix} \mathbf{1} & \mathbf{2} & \mathbf{5} \\ \mathbf{0} & \mathbf{1} & \mathbf{3} \end{pmatrix} \begin{pmatrix} \mathbf{14} \\ \mathbf{21} \\ \mathbf{7} \end{pmatrix} = \begin{pmatrix} \mathbf{91} \\ \mathbf{42} \end{pmatrix}$$

(iv) The multiplication $\boldsymbol{(AB)C}$ is possible.

$$\boldsymbol{(AB)C} = \begin{pmatrix} \mathbf{9} & \mathbf{-2} & \mathbf{18} \\ \mathbf{4} & \mathbf{-3} & \mathbf{10} \end{pmatrix} \begin{pmatrix} \mathbf{1} \\ \mathbf{4} \\ \mathbf{5} \end{pmatrix} = \begin{pmatrix} \mathbf{91} \\ \mathbf{42} \end{pmatrix}$$

From (iii) and (iv) we can observe that both the matrices $\boldsymbol{A(BC)}$ and $\boldsymbol{(AB)C}$ are of same order $\mathbf{2 \times 1}$ and the corresponding elements are same.

Thus by equality of two matrices we can say here that our matrices are equal.

Thus $(\boldsymbol{ABC}) = \boldsymbol{A}(\boldsymbol{BC})$.

Q6B

We have $\boldsymbol{A} = \begin{pmatrix} \mathbf{2} & \mathbf{3} & \mathbf{-1} \\ \mathbf{3} & \mathbf{0} & \mathbf{2} \end{pmatrix}$, $\boldsymbol{B} = \begin{pmatrix} \mathbf{1} \\ \mathbf{1} \\ \mathbf{2} \end{pmatrix}$.and C=$(\mathbf{1} \quad \mathbf{-2})$.

Now multiplication of two matrices is possible if number of columns in left matrix is equals to the number of rows in right matrix.

Next let us discuss the order of the matrices which are given.

The order of matrix $\boldsymbol{B}$ is $\mathbf{2 \times 3}$, matrix $\boldsymbol{A}$ is $\mathbf{3 \times 1}$ and matrix $\boldsymbol{C}$ is $\mathbf{1 \times 2}$.

Thus the multiplications we can proceed as

(i) The multiplication $\boldsymbol{AB}$ is possible.

$$\boldsymbol{AB} = \begin{pmatrix} \mathbf{2} & \mathbf{3} & \mathbf{-1} \\ \mathbf{3} & \mathbf{0} & \mathbf{2} \end{pmatrix} \begin{pmatrix} \mathbf{1} \\ \mathbf{1} \\ \mathbf{2} \end{pmatrix} = \begin{pmatrix} \mathbf{3} \\ \mathbf{7} \end{pmatrix}$$

(ii) The multiplication $\boldsymbol{BC}$ is possible.

$$\boldsymbol{BC} = \begin{pmatrix} \mathbf{1} \\ \mathbf{1} \\ \mathbf{2} \end{pmatrix} (\mathbf{1} \quad \mathbf{-2}) = \begin{pmatrix} \mathbf{1} & \mathbf{-2} \\ \mathbf{1} & \mathbf{-2} \\ \mathbf{2} & \mathbf{-4} \end{pmatrix}$$

(iii) The multiplication $\boldsymbol{A}(\boldsymbol{BC})$ is possible.

$$A(BC) = \begin{pmatrix} 2 & 3 & -1 \\ 3 & 0 & 2 \end{pmatrix} \begin{pmatrix} 1 & -2 \\ 1 & -2 \\ 2 & -4 \end{pmatrix} = \begin{pmatrix} 3 & -6 \\ 7 & -14 \end{pmatrix}$$

(iv) The multiplication $(AB)C$ is possible.

$$(AB)C = \begin{pmatrix} 3 \\ 7 \end{pmatrix} (1 \quad -2) = \begin{pmatrix} 3 & -6 \\ 7 & -14 \end{pmatrix}$$

From (iii) and (iv) we can observe that both the matrices $A(BC)$ and $(AB)C$ are of same order 2×2 and the corresponding elements are same.

Thus by equality of two matrices we can say here that our matrices are equal.

Thus $(AB)C = A(BC)$.

Q7A

We have $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 0 \\ 1 & -3 \end{pmatrix}$.and C=$\begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$.

Now addition of two matrices is possible if order of both the matrices are same and multiplication of two matrices is possible if number of columns in left matrix is equals to the number of rows in right matrix

Next let us discuss the order of the matrices which are given.

The order of matrix B is 2×2, matrix A is 2×2 and matrix C is 2×2.

Thus the multiplications we can proceed as

(i) The multiplication $\boldsymbol{AB}$ is possible.

$$\boldsymbol{AB} = \begin{pmatrix} \mathbf{1} & \mathbf{2} \\ \mathbf{3} & \mathbf{4} \end{pmatrix}\begin{pmatrix} \mathbf{2} & \mathbf{0} \\ \mathbf{1} & \mathbf{-3} \end{pmatrix} = \begin{pmatrix} \mathbf{4} & \mathbf{-6} \\ \mathbf{10} & \mathbf{-12} \end{pmatrix}$$

(ii) The multiplication $\boldsymbol{AC}$ is possible.

$$\boldsymbol{AC} = \begin{pmatrix} \mathbf{1} & \mathbf{2} \\ \mathbf{3} & \mathbf{4} \end{pmatrix}\begin{pmatrix} \mathbf{2} & \mathbf{0} \\ \mathbf{1} & \mathbf{-3} \end{pmatrix} = \begin{pmatrix} \mathbf{1} & \mathbf{1} \\ \mathbf{3} & \mathbf{1} \end{pmatrix}$$

(iii) The addition$\boldsymbol{B + C}$ is possible.

$$\boldsymbol{B + C} = \begin{pmatrix} \mathbf{1} & \mathbf{2} \\ \mathbf{3} & \mathbf{4} \end{pmatrix} + \begin{pmatrix} \mathbf{2} & \mathbf{0} \\ \mathbf{1} & \mathbf{-3} \end{pmatrix} = \begin{pmatrix} \mathbf{3} & \mathbf{-1} \\ \mathbf{1} & \mathbf{-2} \end{pmatrix}$$

(iv) The multiplication $\boldsymbol{A(B + C)}$ is possible.

$$\boldsymbol{A(B + C)} = \begin{pmatrix} \mathbf{1} & \mathbf{2} \\ \mathbf{3} & \mathbf{4} \end{pmatrix}\begin{pmatrix} \mathbf{3} & \mathbf{-1} \\ \mathbf{1} & \mathbf{-2} \end{pmatrix} = \begin{pmatrix} \mathbf{5} & \mathbf{-5} \\ \mathbf{13} & \mathbf{-11} \end{pmatrix}$$

(v) The addition$\boldsymbol{AB + AC}$ is possible.

$$\boldsymbol{AB + AC} = \begin{pmatrix} \mathbf{4} & \mathbf{-6} \\ \mathbf{10} & \mathbf{-12} \end{pmatrix} + \begin{pmatrix} \mathbf{1} & \mathbf{1} \\ \mathbf{3} & \mathbf{1} \end{pmatrix} = \begin{pmatrix} \mathbf{5} & \mathbf{-5} \\ \mathbf{13} & \mathbf{-11} \end{pmatrix}$$

From (iv) and (v) we can observe that both the matrices $\boldsymbol{A(B + C)}$ and $\boldsymbol{AB + AC}$are of same order $\mathbf{2 \times 2}$ and the corresponding elements are same.

Thus by equality of two matrices we can say here that our matrices are equal.

Thus $\boldsymbol{A(B + C) = AB + AC}$.

Q7B

We have $\boldsymbol{A} = \begin{pmatrix} \mathbf{2} & \mathbf{3} \\ \mathbf{-1} & \mathbf{4} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}$, $\boldsymbol{B} = \begin{pmatrix} \mathbf{5} & \mathbf{-3} \\ \mathbf{2} & \mathbf{1} \end{pmatrix}$.and C=$\begin{pmatrix} \mathbf{-1} & \mathbf{2} \\ \mathbf{3} & \mathbf{4} \end{pmatrix}$.

Now addition of two matrices is possible if order of both the matrices are same and multiplication of two matrices is possible if number of columns in left matrix is equals to the number of rows in right matrix

Next let us discuss the order of the matrices which are given.

The order of matrix $\boldsymbol{B}$ is $\mathbf{3 \times 2}$, matrix $\boldsymbol{A}$ is $\mathbf{2 \times 2}$ and matrix $\boldsymbol{C}$ is $\mathbf{2 \times 2}$.

Thus the multiplications we can proceed as

(i) The multiplication $\boldsymbol{AB}$ is possible.

$$\boldsymbol{AB} = \begin{pmatrix} \mathbf{2} & \mathbf{3} \\ \mathbf{-1} & \mathbf{4} \\ \mathbf{0} & \mathbf{1} \end{pmatrix} \begin{pmatrix} \mathbf{5} & \mathbf{-3} \\ \mathbf{2} & \mathbf{1} \end{pmatrix} = \begin{pmatrix} \mathbf{16} & \mathbf{-3} \\ \mathbf{3} & \mathbf{7} \\ \mathbf{2} & \mathbf{1} \end{pmatrix}$$

(ii) The multiplication $\boldsymbol{AC}$ is possible.

$$\boldsymbol{AC} = \begin{pmatrix} \mathbf{2} & \mathbf{3} \\ \mathbf{-1} & \mathbf{4} \\ \mathbf{0} & \mathbf{1} \end{pmatrix} \begin{pmatrix} \mathbf{-1} & \mathbf{2} \\ \mathbf{3} & \mathbf{4} \end{pmatrix} = \begin{pmatrix} \mathbf{7} & \mathbf{16} \\ \mathbf{13} & \mathbf{14} \\ \mathbf{3} & \mathbf{4} \end{pmatrix}$$

(iii) The addition$\boldsymbol{B + C}$ is possible.

$$\boldsymbol{B + C} = \begin{pmatrix} \mathbf{5} & \mathbf{-3} \\ \mathbf{2} & \mathbf{1} \end{pmatrix} + \begin{pmatrix} \mathbf{-1} & \mathbf{2} \\ \mathbf{3} & \mathbf{4} \end{pmatrix} = \begin{pmatrix} \mathbf{4} & \mathbf{-1} \\ \mathbf{5} & \mathbf{5} \end{pmatrix}$$

(iv) The multiplication $\boldsymbol{A(B + C)}$ is possible.

$$\boldsymbol{A(B+C)=\begin{pmatrix}2 & 3\\ -1 & 4\\ 0 & 1\end{pmatrix}\begin{pmatrix}3 & -1\\ 1 & -2\end{pmatrix}=\begin{pmatrix}23 & 13\\ 16 & 21\\ 5 & 5\end{pmatrix}}$$

(v) The addition$\boldsymbol{AB+AC}$ is possible.

$$\boldsymbol{AB+AC=\begin{pmatrix}16 & -3\\ 3 & 7\\ 2 & 1\end{pmatrix}+\begin{pmatrix}7 & 16\\ 13 & 14\\ 3 & 4\end{pmatrix}=\begin{pmatrix}23 & 13\\ 16 & 21\\ 5 & 5\end{pmatrix}}$$

From (iv) and (v) we can observe that both the matrices $\boldsymbol{A(B+C)}$ and $\boldsymbol{AB+AC}$ are of same order $\boldsymbol{3\times 2}$ and the corresponding elements are same.

Thus by equality of two matrices we can say here that our matrices are equal.

Thus $\boldsymbol{A(B+C)=AB+AC}$.

Q8

We have $\boldsymbol{A=\begin{pmatrix}1 & 0 & 2\\ 3 & -1 & 0\\ -2 & 1 & 1\end{pmatrix}, B=\begin{pmatrix}0 & 5 & -4\\ -2 & 1 & 3\\ 1 & 0 & 2\end{pmatrix}}$.and

C=$\boldsymbol{\begin{pmatrix}1 & 5 & 2\\ -1 & 1 & 0\\ 0 & -1 & 1\end{pmatrix}}$.

Now sustraction of two matrices is possible if order of both the matrices are same and multiplication of two matrices is possible if

number of columns in left matrix is equals to the number of rows in right matrix

Next let us discuss the order of the matrices which are given.

The order of matrix $\boldsymbol{B}$ is $\mathbf{3 \times 3}$, matrix $\boldsymbol{A}$ is $\mathbf{3 \times 3}$ and matrix $\boldsymbol{C}$ is $\mathbf{3 \times 3}$.

Thus the multiplications we can proceed as

(i) The multiplication $\boldsymbol{AB}$ is possible.

$$\boldsymbol{AB} = \begin{pmatrix} \mathbf{1} & \mathbf{0} & \mathbf{2} \\ \mathbf{3} & \mathbf{-1} & \mathbf{0} \\ \mathbf{-2} & \mathbf{1} & \mathbf{1} \end{pmatrix} \begin{pmatrix} \mathbf{0} & \mathbf{5} & \mathbf{-4} \\ \mathbf{-2} & \mathbf{1} & \mathbf{3} \\ \mathbf{1} & \mathbf{0} & \mathbf{2} \end{pmatrix} = \begin{pmatrix} \mathbf{-2} & \mathbf{5} & \mathbf{-8} \\ \mathbf{2} & \mathbf{14} & \mathbf{-15} \\ \mathbf{-1} & \mathbf{-9} & \mathbf{13} \end{pmatrix}$$

(ii) The multiplication $\boldsymbol{AC}$ is possible.

$$\boldsymbol{AC} = \begin{pmatrix} \mathbf{1} & \mathbf{0} & \mathbf{2} \\ \mathbf{3} & \mathbf{-1} & \mathbf{0} \\ \mathbf{-2} & \mathbf{1} & \mathbf{1} \end{pmatrix} \begin{pmatrix} \mathbf{1} & \mathbf{5} & \mathbf{2} \\ \mathbf{-1} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{-1} & \mathbf{1} \end{pmatrix} = \begin{pmatrix} \mathbf{1} & \mathbf{7} & \mathbf{0} \\ \mathbf{4} & \mathbf{16} & \mathbf{6} \\ \mathbf{-3} & \mathbf{-10} & \mathbf{-3} \end{pmatrix}$$

(iii) The subtraction $\boldsymbol{B - C}$ is possible.

$$\boldsymbol{B - C} = \begin{pmatrix} \mathbf{0} & \mathbf{5} & \mathbf{-4} \\ \mathbf{-2} & \mathbf{1} & \mathbf{3} \\ \mathbf{1} & \mathbf{0} & \mathbf{2} \end{pmatrix} - \begin{pmatrix} \mathbf{1} & \mathbf{5} & \mathbf{2} \\ \mathbf{-1} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{-1} & \mathbf{1} \end{pmatrix} = \begin{pmatrix} \mathbf{-1} & \mathbf{0} & \mathbf{-6} \\ \mathbf{-1} & \mathbf{0} & \mathbf{3} \\ \mathbf{1} & \mathbf{1} & \mathbf{1} \end{pmatrix}$$

(iv) The multiplication $\boldsymbol{A(B - C)}$ is possible.

$$\boldsymbol{A(B - C)} = \begin{pmatrix} \mathbf{1} & \mathbf{0} & \mathbf{2} \\ \mathbf{3} & \mathbf{-1} & \mathbf{0} \\ \mathbf{-2} & \mathbf{1} & \mathbf{1} \end{pmatrix} \begin{pmatrix} \mathbf{-1} & \mathbf{0} & \mathbf{-6} \\ \mathbf{-1} & \mathbf{0} & \mathbf{3} \\ \mathbf{1} & \mathbf{1} & \mathbf{1} \end{pmatrix}$$

$$= \begin{pmatrix} \mathbf{-3} & \mathbf{-2} & \mathbf{-8} \\ \mathbf{-2} & \mathbf{0} & \mathbf{-21} \\ \mathbf{2} & \mathbf{1} & 16 \end{pmatrix}$$

(v) The subtraction $\boldsymbol{AB - AC}$ is possible.

$$\boldsymbol{AB - AC} = \begin{pmatrix} \mathbf{-2} & \mathbf{5} & \mathbf{-8} \\ \mathbf{2} & \mathbf{14} & \mathbf{-15} \\ \mathbf{-1} & \mathbf{-9} & \mathbf{13} \end{pmatrix} - \begin{pmatrix} \mathbf{1} & \mathbf{7} & \mathbf{0} \\ \mathbf{4} & \mathbf{16} & \mathbf{6} \\ \mathbf{-3} & \mathbf{-10} & \mathbf{-3} \end{pmatrix}$$

$$= \begin{pmatrix} \mathbf{-3} & \mathbf{-2} & \mathbf{-8} \\ \mathbf{-2} & \mathbf{0} & \mathbf{-21} \\ \mathbf{2} & \mathbf{1} & \mathbf{16} \end{pmatrix}$$

From (iv) and (v) we can observe that both the matrices $\boldsymbol{A(B + C)}$ and $\boldsymbol{AB + AC}$ are of same order $\mathbf{3 \times 3}$ and the corresponding elements are same.

Thus by equality of two matrices we can say here that our matrices are equal.

Thus $\boldsymbol{A(B + C) = AB + AC}$.

Q9

We have $\boldsymbol{A} = \begin{pmatrix} \boldsymbol{ab} & \boldsymbol{b^2} \\ \boldsymbol{-a^2} & \boldsymbol{-ab} \end{pmatrix}$.

To show: $\boldsymbol{A^2 = O}$.

Now multiplication of two matrices is possible if number of columns in left matrix is equals to the number of rows in right matrix.

Next let us discuss the order of the matrices which are given.

The order of matrix $\boldsymbol{A}$ is $\boldsymbol{2 \times 2}$.

Thus the multiplications we can proceed as the multiplication $\boldsymbol{A^2}$is possible.

$$\boldsymbol{A^2 = AA = \begin{pmatrix} ab & b^2 \\ -a^2 & -ab \end{pmatrix}\begin{pmatrix} ab & b^2 \\ -a^2 & -ab \end{pmatrix}}$$

$$\boldsymbol{= \begin{pmatrix} a^2b^2 - a^2b^2 & ab^3 - ab^3 \\ -a^3b + a^3b & -a^2b^2 + a^2b^2 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}}$$

Q10

We have $\boldsymbol{A = \begin{pmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{pmatrix}}$.

To show: $\boldsymbol{A^2 = A}$.

Now multiplication of two matrices is possible if number of columns in left matrix is equals to the number of rows in right matrix.

Next let us discuss the order of the matrices which are given.

The order of matrix $\boldsymbol{A}$ is $\boldsymbol{3 \times 3}$.

Thus the multiplications we can proceed as the multiplication $\boldsymbol{A^2}$is possible.

$\boldsymbol{A^2 = AA}$

$$= \begin{pmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{pmatrix}\begin{pmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{pmatrix}$$

$$= \begin{pmatrix} 4+2-4 & -4-6+8 & -8-8+12 \\ -2-3+4 & 2+9-8 & 4+12-12 \\ 2+2-3 & -2-6+6 & -4-8+9 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{pmatrix}$$

$$= A$$

Q11

We have $A = \begin{pmatrix} 4 & -1 & -4 \\ 3 & 0 & -4 \\ 3 & -1 & -3 \end{pmatrix}$.

To show: $A^2 = A$.

Now multiplication of two matrices is possible if number of columns in left matrix is equals to the number of rows in right matrix.

Next let us discuss the order of the matrices which are given.

The order of matrix A is 3×3.

Thus the multiplications we can proceed as the multiplication A^2 is possible.

$A^2 = AA$

$$= \begin{pmatrix} 4 & -1 & -4 \\ 3 & 0 & -4 \\ 3 & -1 & -3 \end{pmatrix}\begin{pmatrix} 4 & -1 & -4 \\ 3 & 0 & -4 \\ 3 & -1 & -3 \end{pmatrix}$$

$$= \begin{pmatrix} 16-3-12 & -4+0+4 & -16+4+12 \\ 12+0-12 & -3+0+4 & -12+0+12 \\ 12-3-9 & -3+0+3 & -12+4+9 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \mathbf{I}$$

Q12

We have $\boldsymbol{A} = \begin{pmatrix} 2 & -1 \\ 3 & 2 \end{pmatrix}$ and $\boldsymbol{B} = \begin{pmatrix} 0 & 4 \\ -1 & 7 \end{pmatrix}$.

Now subtraction/addition of two matrices is possible if order of both the matrices are same and multiplication of two matrices is possible if number of columns in left matrix is equals to the number of rows in right matrix

Next let us discuss the order of the matrices which are given.

The order of matrix $\boldsymbol{B}$ is $\mathbf{2 \times 2}$ and matrix $\boldsymbol{A}$ is $\mathbf{2 \times 2}$.

Thus the multiplications we can proceed as

(i) The multiplication $\boldsymbol{A^2}$ is possible.

$$\boldsymbol{A^2 = AA} = \begin{pmatrix} 2 & -1 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} 2 & -1 \\ 3 & 2 \end{pmatrix} = \begin{pmatrix} 1 & -4 \\ 12 & 1 \end{pmatrix}$$

(ii) The subtraction/addition $\boldsymbol{3A^2 - 2B + I}$ is possible.

$$3A^2 - 2B + I = 3\begin{pmatrix} 1 & -4 \\ 12 & 1 \end{pmatrix} - 2\begin{pmatrix} 0 & 4 \\ -1 & 7 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 3 - 0 + 1 & -12 - 8 + 0 \\ 36 + 2 + 0 & 3 - 14 + 1 \end{pmatrix}$$

$$= \begin{pmatrix} 4 & -20 \\ 38 & -10 \end{pmatrix}.$$

Q13

We have $A = \begin{pmatrix} 2 & -2 \\ -3 & 4 \end{pmatrix}$.

Now addition of two matrices is possible if order of both the matrices are same and multiplication of two matrices is possible if number of columns in left matrix is equals to the number of rows in right matrix

Next let us discuss the order of the matrices which are given.

The order of matrix A is 2×2.

Thus the multiplications we can proceed as

(i) The multiplication A^2 is possible.

$$A^2 = AA = \begin{pmatrix} 2 & -2 \\ -3 & 4 \end{pmatrix}\begin{pmatrix} 2 & -2 \\ -3 & 4 \end{pmatrix} = \begin{pmatrix} 10 & -12 \\ -18 & 22 \end{pmatrix}$$

(ii) The addition $-A^2 + 6A$ is possible.

$$-A^2 + 6A = -1\begin{pmatrix} 10 & -12 \\ -18 & 22 \end{pmatrix} + 6\begin{pmatrix} 2 & -2 \\ -3 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} -10 + 12 & 12 - 12 \\ 18 - 18 & -22 + 24 \end{pmatrix}$$

$= \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$.

Q14

We have $\boldsymbol{A} = \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$.

Now addition of two matrices is possible if order of both the matrices are same and multiplication of two matrices is possible if number of columns in left matrix is equals to the number of rows in right matrix

Next let us discuss the order of the matrices which are given.

The order of matrix $\boldsymbol{A}$ is $\boldsymbol{2 \times 2}$.

Thus the multiplications we can proceed as

(i) The multiplication $\boldsymbol{A^2}$ is possible.

$$\boldsymbol{A^2 = AA = \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} 8 & 5 \\ -5 & 3 \end{pmatrix}}$$

(ii) The addition $\boldsymbol{A^2 - 5A + 7I}$ is possible.

$$\boldsymbol{A^2 - 5A + 7I = \begin{pmatrix} 8 & 5 \\ -5 & 3 \end{pmatrix} - 5\begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix} + 7\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}}$$

$$= \begin{pmatrix} 8 - 15 + 7 & 5 - 5 + 0 \\ -5 + 5 + 0 & 3 - 10 + 7 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$= \boldsymbol{O}.$$

Q15

We have $\boldsymbol{A} = \begin{pmatrix} \mathbf{2} & \mathbf{3} \\ \mathbf{1} & \mathbf{2} \end{pmatrix}$.

Now addition of two matrices is possible if order of both the matrices are same and multiplication of two matrices is possible if number of columns in left matrix is equals to the number of rows in right matrix

Next let us discuss the order of the matrices which are given.

The order of matrix $\boldsymbol{A}$ is $\mathbf{2} \times \mathbf{2}$.

Thus the multiplications we can proceed as

(i) The multiplication $\boldsymbol{A^2}$ is possible.

$$\boldsymbol{A^2 = AA = \begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 7 & 12 \\ 4 & 7 \end{pmatrix}}$$

(ii) The multiplication $\boldsymbol{A^3}$ is possible.

$$\boldsymbol{A^3 = A^2A = \begin{pmatrix} 7 & 12 \\ 4 & 7 \end{pmatrix}\begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 26 & 45 \\ 15 & 28 \end{pmatrix}}$$

(iii) The addition $\boldsymbol{A^3 - 4A^2 + A}$ is possible.

$$\boldsymbol{A^3 - 4A^2 + A = \begin{pmatrix} 26 & 45 \\ 15 & 28 \end{pmatrix} - 4\begin{pmatrix} 7 & 12 \\ 4 & 7 \end{pmatrix} + \begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix}}$$

$$= \begin{pmatrix} \mathbf{26 - 28 + 2} & \mathbf{45 - 48 + 3} \\ \mathbf{15 - 16 + 1} & \mathbf{26 - 28 + 2} \end{pmatrix}$$

$$= \begin{pmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{pmatrix}$$

$$= \boldsymbol{O}.$$

Q16

We have $\boldsymbol{A} = \begin{pmatrix} \mathbf{3} & \mathbf{-2} \\ \mathbf{4} & \mathbf{-2} \end{pmatrix}$.

Now addition/subtraction of two matrices is possible if order of both the matrices are same and multiplication of two matrices is possible if number of columns in left matrix is equals to the number of rows in right matrix

Next let us discuss the order of the matrices which are given.

The order of matrix $\boldsymbol{A}$ is $\mathbf{2 \times 2}$.

Thus the multiplications we can proceed as

(i) The multiplication $\boldsymbol{A^2}$ is possible.

$$\boldsymbol{A^2 = AA = \begin{pmatrix} 3 & -2 \\ 4 & -2 \end{pmatrix}\begin{pmatrix} 3 & -2 \\ 4 & -2 \end{pmatrix} = \begin{pmatrix} 1 & -2 \\ 4 & -4 \end{pmatrix}}$$

(ii) We have the equation $\boldsymbol{A^2 = kA - 2I}$.

$$\boldsymbol{A^2 = kA - 2I}$$

$$\boldsymbol{\Rightarrow \begin{pmatrix} 1 & -2 \\ 4 & -4 \end{pmatrix} = k\begin{pmatrix} 3 & -2 \\ 4 & -2 \end{pmatrix} - 2\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}}$$

$$\boldsymbol{\Rightarrow \begin{pmatrix} 1 & -2 \\ 4 & -4 \end{pmatrix} = \begin{pmatrix} 3k-2 & -2k \\ 4k & -2k-2 \end{pmatrix}}$$

Thus we have

$$\boldsymbol{3k - 2 = 1 \Rightarrow k = 1}$$

Q17

We have $\boldsymbol{A} = \begin{pmatrix} \mathbf{-1} & \mathbf{2} \\ \mathbf{3} & \mathbf{1} \end{pmatrix}$.

Now addition/subtraction of two matrices is possible if order of both the matrices are same and multiplication of two matrices is possible if number of columns in left matrix is equals to the number of rows in right matrix

Next let us discuss the order of the matrices which are given the order of matrix $\boldsymbol{A}$ is $\mathbf{2 \times 2}$.

Thus the multiplications we can proceed as

(i) The multiplication $\boldsymbol{A^2}$ is possible.

$$\boldsymbol{A^2 = AA = \begin{pmatrix} -1 & 2 \\ 3 & 1 \end{pmatrix}\begin{pmatrix} -1 & 2 \\ 3 & 1 \end{pmatrix} = \begin{pmatrix} 7 & 0 \\ 0 & 7 \end{pmatrix}}$$

(ii) We have the equation $\boldsymbol{f(x) = x^2 - 2x + 3}$.

$$\boldsymbol{f(A) = A^2 - 2A + 3I_2}$$

$$= \begin{pmatrix} 7 & 0 \\ 0 & 7 \end{pmatrix} - 2\begin{pmatrix} -1 & 2 \\ 3 & 1 \end{pmatrix} + 3\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 7 + 2 + 3 & -4 + 0 \\ 0 - 6 + 0 & 7 - 2 + 3 \end{pmatrix}$$

$$= \begin{pmatrix} 12 & -4 \\ -6 & 8 \end{pmatrix}$$

Q18

We have $\boldsymbol{A} = \begin{pmatrix} \mathbf{1} & \mathbf{2} \\ \mathbf{4} & \mathbf{-3} \end{pmatrix}$.

Now addition/subtraction of two matrices is possible if order of both the matrices are same and multiplication of two matrices is possible if number of columns in left matrix is equals to the number of rows in right matrix

Next let us discuss the order of the matrices which are given the order of matrix $\boldsymbol{A}$ is $\mathbf{2 \times 2}$.

Thus the multiplications we can proceed as

(i) The multiplication $\boldsymbol{A^2}$ is possible.

$$\boldsymbol{A^2 = AA = \begin{pmatrix} 1 & 2 \\ 4 & -3 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 4 & -3 \end{pmatrix} = \begin{pmatrix} 9 & -4 \\ -8 & 17 \end{pmatrix}}$$

(ii) The multiplication $\boldsymbol{A^3}$ is possible.

$$\boldsymbol{A^3 = A^2A = \begin{pmatrix} 9 & -4 \\ -8 & 17 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 4 & -3 \end{pmatrix} = \begin{pmatrix} -7 & 30 \\ 60 & -67 \end{pmatrix}}$$

(iii) We have the equation $\boldsymbol{f(x) = 2x^3 + 4x + 5}$.

$$\boldsymbol{f(A) = A^3 + 4A + 5I_2}$$

$$= \begin{pmatrix} -7 & 30 \\ 60 & -67 \end{pmatrix} + 4\begin{pmatrix} 1 & 2 \\ 4 & -3 \end{pmatrix} + 5\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} -14 + 4 + 5 & 60 + 8 + 0 \\ 120 + 16 + 0 & -134 - 12 + 5 \end{pmatrix}$$

$= \begin{pmatrix} -5 & 68 \\ 136 & -141 \end{pmatrix}$

Q19

We have $\boldsymbol{A} = \begin{pmatrix} 2 & -3 \\ 1 & 1 \end{pmatrix}, \boldsymbol{B} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ $\boldsymbol{and}$ $\boldsymbol{X} = \begin{pmatrix} x \\ y \end{pmatrix}$.

To find $\boldsymbol{X}$.

Now multiplication of two matrices is possible if number of columns in left matrix is equals to the number of rows in right matrix

Next let us discuss the order of the matrices which are given the order of matrix $\boldsymbol{A}$ is $\boldsymbol{2 \times 2}$, $\boldsymbol{order\ of\ the\ matrix\ B\ is\ 2 \times 1}$ $\boldsymbol{and\ order\ of\ the\ matrix\ C\ is\ 2 \times 1}$.

Thus the multiplications we can proceed as

(i) The multiplication $\boldsymbol{AX}$ is possible.

$$\boldsymbol{AX} = \begin{pmatrix} 2 & -3 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2x - 3y \\ x + y \end{pmatrix}$$

(ii)We have$\boldsymbol{AX = B}$.

$$\begin{pmatrix} 2x - 3y \\ x + y \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

Therefore we can write

$\boldsymbol{2x - 3y = 1\ ;\ x + y = 3}$

By solving the equations we get

$x = 2, y = 1$

Q20

We have $A = \begin{pmatrix} 3 & -4 \\ 1 & 2 \end{pmatrix}, B = \begin{pmatrix} 3 \\ 11 \end{pmatrix}$ *and* $X = \begin{pmatrix} x \\ y \end{pmatrix}$.

To find X.

Now multiplication of two matrices is possible if number of columns in left matrix is equals to the number of rows in right matrix

Next let us discuss the order of the matrices which are given the order of matrix A is 2×2, *order of the matrix* B *is* 2×1 *and order of the matrix* C *is* 2×1.

Thus the multiplications we can proceed as

(i) The multiplication AX is possible.

$$AX = \begin{pmatrix} 3 & -4 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3x - 4y \\ x + 2y \end{pmatrix}$$

(ii)We have $AX = B$.

$$\begin{pmatrix} 3x - 4y \\ x + 2y \end{pmatrix} = \begin{pmatrix} 3 \\ 11 \end{pmatrix}$$

Therefore we can write

$3x - 4y = 3 ; x + 2y = 11$

By solving the equations we get

$x = 5, y = 3$

Q21

We have $A = \begin{pmatrix} 3 & 1 \\ 7 & 5 \end{pmatrix}$.

To find x, y such that $A^2 + xI = yA$.

Now multiplication of two matrices is possible if number of columns in left matrix is equals to the number of rows in right matrix

Next let us discuss the order of the matrices which are given the order of matrix A is 2×2.

Thus the multiplications we can proceed as

(i) The multiplication A^2 is possible.

$$A^2 = AA = \begin{pmatrix} 3 & 1 \\ 7 & 5 \end{pmatrix}\begin{pmatrix} 3 & 1 \\ 7 & 5 \end{pmatrix} = \begin{pmatrix} 16 & 8 \\ 56 & 32 \end{pmatrix}$$

(ii)We have $A^2 + xI = yA$.

$$\begin{pmatrix} 16 & 8 \\ 56 & 32 \end{pmatrix} + \begin{pmatrix} x & 0 \\ 0 & x \end{pmatrix} = \begin{pmatrix} 3y & 1y \\ 7y & 5y \end{pmatrix}$$

Therefore we can write

$16 + x = 3y\,; 8 = y$

By solving the equations we get

$$x = 8, y = 8$$

Q22

We have $\boldsymbol{A} = \begin{pmatrix} \mathbf{3} & \mathbf{2} \\ \mathbf{1} & \mathbf{1} \end{pmatrix}$.

To find $\boldsymbol{a}, \boldsymbol{b}$ such that $\boldsymbol{A^2 + aA + bI = O}$.

Now multiplication of two matrices is possible if number of columns in left matrix is equals to the number of rows in right matrix

Next let us discuss the order of the matrices which are given the order of matrix $\boldsymbol{A}$ is $\mathbf{2 \times 2}$.

Thus the multiplications we can proceed as

(i) The multiplication $\boldsymbol{A^2}$ is possible.

$$\boldsymbol{A^2 = AA} = \begin{pmatrix} \mathbf{3} & \mathbf{2} \\ \mathbf{1} & \mathbf{1} \end{pmatrix}\begin{pmatrix} \mathbf{3} & \mathbf{2} \\ \mathbf{1} & \mathbf{1} \end{pmatrix} = \begin{pmatrix} \mathbf{11} & \mathbf{8} \\ \mathbf{4} & \mathbf{3} \end{pmatrix}$$

(ii)We have $\boldsymbol{A^2 + aA + bI = O}$.

$$\begin{pmatrix} \mathbf{11} & \mathbf{8} \\ \mathbf{4} & \mathbf{3} \end{pmatrix} + \boldsymbol{a}\begin{pmatrix} \mathbf{3} & \mathbf{2} \\ \mathbf{1} & \mathbf{1} \end{pmatrix} + \boldsymbol{b}\begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix} = \begin{pmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{pmatrix}$$

Therefore we can write

$$\boldsymbol{4 + a = 0\ ;\ 3 + a + b = 0}$$

By solving the equations we get

$$\boldsymbol{a = -4\ ,\ b = 1}$$

Q23

We have $\boldsymbol{B} = \begin{pmatrix} \mathbf{5} & \mathbf{-7} \\ \mathbf{-2} & \mathbf{3} \end{pmatrix}$ and $\boldsymbol{C} = \begin{pmatrix} \mathbf{-16} & \mathbf{-6} \\ \mathbf{7} & \mathbf{2} \end{pmatrix}$.

To find matrix $\boldsymbol{A}$ such that $\boldsymbol{BA} = \boldsymbol{C}$.

We will use the inverse application here by modification of the equation we have as $\boldsymbol{A} = \boldsymbol{B^{-1}C}$

(i) The inverse of $\boldsymbol{B}$ is computed as follow:

We have $\boldsymbol{B} = \begin{pmatrix} \mathbf{5} & \mathbf{-7} \\ \mathbf{-2} & \mathbf{3} \end{pmatrix}$. Then $\mathbf{det}(\boldsymbol{B}) = \begin{vmatrix} \mathbf{5} & \mathbf{-7} \\ \mathbf{-2} & \mathbf{3} \end{vmatrix} = \mathbf{15} - \mathbf{14} = \mathbf{1} \neq \mathbf{0}$.

Thus inverse of $\boldsymbol{B}$ exist.

Then $\boldsymbol{C_B} = \begin{pmatrix} \mathbf{3} & \mathbf{2} \\ \mathbf{7} & \mathbf{5} \end{pmatrix}$

Now $\boldsymbol{adj(B)} = \boldsymbol{C_B^T} = \begin{pmatrix} \mathbf{3} & \mathbf{7} \\ \mathbf{2} & \mathbf{5} \end{pmatrix}$.

Thus the inverse of the given matrix is $\boldsymbol{B^{-1}} = \frac{\mathbf{1}}{\mathbf{det}(\boldsymbol{B})}\boldsymbol{adj(B)} = \begin{pmatrix} \mathbf{3} & \mathbf{7} \\ \mathbf{2} & \mathbf{5} \end{pmatrix}$

(ii)Finally we have $\boldsymbol{A} = \boldsymbol{B^{-1}C}$

$= \begin{pmatrix} \mathbf{3} & \mathbf{7} \\ \mathbf{2} & \mathbf{5} \end{pmatrix}\begin{pmatrix} \mathbf{-16} & \mathbf{-6} \\ \mathbf{7} & \mathbf{2} \end{pmatrix}$

$= \begin{pmatrix} \mathbf{1} & \mathbf{-4} \\ \mathbf{3} & \mathbf{-6} \end{pmatrix}$

Q24

We have $\boldsymbol{B} = \begin{pmatrix} \mathbf{2} & \mathbf{3} \\ \mathbf{4} & \mathbf{5} \end{pmatrix}$ and $\boldsymbol{C} = \begin{pmatrix} \mathbf{0} & \mathbf{-4} \\ \mathbf{10} & \mathbf{3} \end{pmatrix}$.

To find matrix $\boldsymbol{A}$ such that $\boldsymbol{AB} = \boldsymbol{C}$.

We will use the inverse application here by modification of the equation we have as $\boldsymbol{A} = \boldsymbol{CB^{-1}}$

(i) The inverse of $\boldsymbol{B}$ is computed as follow:

We have $\boldsymbol{B} = \begin{pmatrix} \mathbf{2} & \mathbf{3} \\ \mathbf{4} & \mathbf{5} \end{pmatrix}$. Then $|\boldsymbol{B}| = \begin{vmatrix} \mathbf{2} & \mathbf{3} \\ \mathbf{4} & \mathbf{5} \end{vmatrix} = \mathbf{10 - 12 = -2 \neq 0}$.

Thus inverse of $\boldsymbol{B}$ exist.

Then $\boldsymbol{C_B} = \begin{pmatrix} \mathbf{5} & \mathbf{-4} \\ \mathbf{-3} & \mathbf{2} \end{pmatrix}$

Now $\boldsymbol{adj(B)} = \boldsymbol{C_B^T} = \begin{pmatrix} \mathbf{5} & \mathbf{-3} \\ \mathbf{-4} & \mathbf{2} \end{pmatrix}$.

Thus the inverse of the given matrix is $\boldsymbol{B^{-1}} = \frac{\mathbf{1}}{|\boldsymbol{B}|}\boldsymbol{adj(B)} =$

$\frac{\mathbf{1}}{\mathbf{-2}}\begin{pmatrix} \mathbf{5} & \mathbf{-3} \\ \mathbf{-4} & \mathbf{2} \end{pmatrix}$

(ii)Finally we have $\boldsymbol{A} = \boldsymbol{B^{-1}C}$

$$\boldsymbol{A} = \frac{\mathbf{1}}{\mathbf{-2}}\begin{pmatrix} \mathbf{5} & \mathbf{-3} \\ \mathbf{-4} & \mathbf{2} \end{pmatrix}\begin{pmatrix} \mathbf{0} & \mathbf{-4} \\ \mathbf{10} & \mathbf{3} \end{pmatrix}$$

$$= \begin{pmatrix} \mathbf{-8} & \mathbf{4} \\ \mathbf{-19} & \mathbf{12} \end{pmatrix}$$

Q25

We have $\boldsymbol{A} = \begin{pmatrix} \mathbf{1} & \mathbf{-1} \\ \mathbf{2} & \mathbf{-1} \end{pmatrix}, \boldsymbol{B} = \begin{pmatrix} \boldsymbol{a} & \mathbf{-1} \\ \boldsymbol{b} & \mathbf{-1} \end{pmatrix}$ $\boldsymbol{and}$ $(\boldsymbol{A}+\boldsymbol{B})^{\mathbf{2}} = (\boldsymbol{A}^{\mathbf{2}} + \boldsymbol{B}^{\mathbf{2}})$.

To find matrix $\boldsymbol{a}, \boldsymbol{b}$.

We will use the inverse application here by modification of the equation we have as $\boldsymbol{A} = \boldsymbol{CB}^{-\mathbf{1}}$

(i) The of $\boldsymbol{A}+\boldsymbol{B}$ is computed as follow:

We have $\boldsymbol{A}+\boldsymbol{B} = \begin{pmatrix} \mathbf{1} & -1 \\ \mathbf{2} & \mathbf{-1} \end{pmatrix} + \begin{pmatrix} \boldsymbol{a} & \mathbf{-1} \\ \boldsymbol{b} & \mathbf{-1} \end{pmatrix} = \begin{pmatrix} \mathbf{1}+\boldsymbol{a} & \mathbf{-1-1} \\ \mathbf{2}+\boldsymbol{b} & \mathbf{-1-1} \end{pmatrix} = \begin{pmatrix} \boldsymbol{a}+\mathbf{1} & \mathbf{-2} \\ \boldsymbol{b}+\mathbf{2} & \mathbf{-2} \end{pmatrix}$.

(ii)We can have $\boldsymbol{A}^{\mathbf{2}}$ as

$$\boldsymbol{A}^{\mathbf{2}} = \boldsymbol{AA} = \begin{pmatrix} \mathbf{1} & \mathbf{-1} \\ \mathbf{2} & \mathbf{-1} \end{pmatrix}\begin{pmatrix} \mathbf{1} & \mathbf{-1} \\ \mathbf{2} & \mathbf{-1} \end{pmatrix} = \begin{pmatrix} \mathbf{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{pmatrix}$$

(iii)We can have $\boldsymbol{B}^{\mathbf{2}}$ as

$$\boldsymbol{B}^{\mathbf{2}} = \boldsymbol{BB} = \begin{pmatrix} \boldsymbol{a} & \mathbf{-1} \\ \boldsymbol{b} & \mathbf{-1} \end{pmatrix}\begin{pmatrix} \boldsymbol{a} & \mathbf{-1} \\ \boldsymbol{b} & \mathbf{-1} \end{pmatrix} = \begin{pmatrix} \boldsymbol{a}^{\mathbf{2}} - \boldsymbol{b} & -\boldsymbol{a}+\mathbf{1} \\ \boldsymbol{ab}-\boldsymbol{b} & -\boldsymbol{b}+\mathbf{1} \end{pmatrix}$$

(iv)We can have $\boldsymbol{A}^{\mathbf{2}} + \boldsymbol{B}^{\mathbf{2}}$ as

$$\boldsymbol{A}^{\mathbf{2}} + \boldsymbol{B}^{\mathbf{2}} = \begin{pmatrix} \mathbf{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{pmatrix} + \begin{pmatrix} \boldsymbol{a}^{\mathbf{2}} - \boldsymbol{b} & -\boldsymbol{a}+\mathbf{1} \\ \boldsymbol{ab}-\boldsymbol{b} & -\boldsymbol{b}+\mathbf{1} \end{pmatrix}$$

$$= \begin{pmatrix} \mathbf{-1}+\boldsymbol{a}^{\mathbf{2}} - \boldsymbol{b} & -\boldsymbol{a}+\mathbf{1} \\ \boldsymbol{ab}-\boldsymbol{b} & -\boldsymbol{b}+\mathbf{1} \end{pmatrix}$$

(v)We can have $\boldsymbol{A}+\boldsymbol{B})^{\mathbf{2}}$ as

$(A+B)^2 = (A+B)(A+B)$

$= \begin{pmatrix} a+1 & -2 \\ b+2 & -2 \end{pmatrix}\begin{pmatrix} a+1 & -2 \\ b+2 & -2 \end{pmatrix}$

$= \begin{pmatrix} a^2+2a-2b-3 & 2-2a \\ 2a-b+ab-2 & -2b \end{pmatrix}$

(vi) We are given equation as $(A+B)^2 = (A^2+B^2)$.

$\begin{pmatrix} a^2+2a-2b-3 & 2-2a \\ 2a-b+ab-2 & -2b \end{pmatrix} = \begin{pmatrix} -1+a^2-b & -a+1 \\ ab-b & -b+1 \end{pmatrix}$

Therefore we have the relations as follow:

$-2b = -b+1 \; ; \; 2-2a = -a+1$

Thus by solving the equations we get

$a = 1 \; ; b = -1$

Q26

We have $F(X) = \begin{pmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{pmatrix}$ and to show $F(X)F(Y) = F(X+Y)$.

Let us get the multiplication $F(X)F(Y)$ as follow:

$F(X)F(Y) = \begin{pmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} \cos y & -\sin y & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$= \begin{pmatrix} \cos x\cos y - \sin x \sin y & -\cos x \sin x - \sin x \cos y & 0 \\ \sin x \cos y + \cos x \sin y & -\sin x \sin y + \cos x \cos y & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Using the trigonometric rules we get

$$=\begin{pmatrix}\cos(x+y) & -\sin(x+y) & 0\\ \sin(x+y) & \cos(x+y) & 0\\ 0 & 0 & 1\end{pmatrix}$$

$$= F(X+Y)$$

Q27

We have $A = \begin{pmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{pmatrix}$ and to show $A^2 = \begin{pmatrix}\cos 2\alpha & \sin 2\alpha\\ -\sin 2\alpha & \cos 2\alpha\end{pmatrix}$.

Let us get the multiplication A^2 as follow:

$$A^2 = AA = \begin{pmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{pmatrix}\begin{pmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{pmatrix}$$

$$= \begin{pmatrix}\cos^2\alpha - \sin^2\alpha & -2\sin\alpha\cos\alpha\\ -2\sin\alpha\cos\alpha & -\sin^2\alpha + \cos^2\alpha\end{pmatrix}$$

Using the trigonometric rules we get

$$=\begin{pmatrix}\cos 2\alpha & \sin 2\alpha\\ -\sin 2\alpha & \cos 2\alpha\end{pmatrix}$$

Q28

We have $A = \begin{pmatrix}1 & 2 & 3\\ 4 & 5 & 6\\ 3 & 2 & 5\end{pmatrix}$, $B = \begin{pmatrix}1 & x & 1\end{pmatrix}$ and $C = \begin{pmatrix}1\\ -2\\ 3\end{pmatrix}$.

We also have $CAB = O$.

Therefore

$$(1 \quad x \quad 1)\begin{pmatrix}1 & 2 & 3\\ 4 & 5 & 6\\ 3 & 2 & 5\end{pmatrix}\begin{pmatrix}1\\ -2\\ 3\end{pmatrix}$$

$$= (1+4x+3 \quad 2+5x+2 \quad 6x+8)\begin{pmatrix}1\\ -2\\ 3\end{pmatrix}$$

$$= (4x+4 \quad 5x+4 \quad 6x+8)\begin{pmatrix}1\\ -2\\ 3\end{pmatrix}$$

$$=\big(1(4x+4)-2(5x+4)+3(6x+8)\big)$$

$$= (12x+20)$$

Thus finally we have

$$12x+20=0$$

Therefore $x=-\frac{5}{3}$.

Q29

We have $A=\begin{pmatrix}2 & 1 & 2\\ 1 & 0 & 2\\ 0 & 2 & -4\end{pmatrix}$, $B=(x \quad 4 \quad 1)$ and $C=\begin{pmatrix}x\\ 4\\ -1\end{pmatrix}$.

We also have $BAC=O$.

Therefore

$$(x \quad 4 \quad 1)\begin{pmatrix}2 & 1 & 2\\ 1 & 0 & 2\\ 0 & 2 & -4\end{pmatrix}\begin{pmatrix}x\\ 4\\ -1\end{pmatrix}$$

$= (2x + 4 \quad x + 2 \quad 2x + 4)\begin{pmatrix} x \\ 4 \\ -1 \end{pmatrix}$

$=\left(x(2x + 4) + 4(x + 2) - 1(2x + 4)\right)$

$= \left(2x^2 + 6x + 4\right)$

Thus finally we have

$2x^2 + 6x + 4 = (x + 1)(x + 2) = 0$

Therefore $x = -1 \; or \; x = -2$.

Q30

We have $A = \begin{pmatrix} a & b \\ -a & 2b \end{pmatrix}$, $B = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$ and $C = \begin{pmatrix} 5 \\ 4 \end{pmatrix}$.

We also have $AB = C$.

Therefore

$\begin{pmatrix} a & b \\ -a & 2b \end{pmatrix}\begin{pmatrix} 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 5 \\ 4 \end{pmatrix}$

By multiplication of left side matrices we get

$\begin{pmatrix} 2a - b \\ -2a - 2b \end{pmatrix} = \begin{pmatrix} 5 \\ 4 \end{pmatrix}$

By comparison of matrices we get

$2a - b = 5 \; ; \; -2a - 2b = 4$

Thus finally we have

$$a = 1 \; or \; b = -3.$$

Q31

We have $\boldsymbol{A} = \begin{pmatrix} \mathbf{3} & \mathbf{4} \\ \mathbf{-4} & \mathbf{-3} \end{pmatrix}$ and equation $\boldsymbol{f(x) = x^2 - 5x + 7}$.

(i) Let us compute first $\boldsymbol{A^2}$

$$\boldsymbol{A^2 = AA = \begin{pmatrix} 3 & 4 \\ -4 & -3 \end{pmatrix}\begin{pmatrix} 3 & 4 \\ -4 & -3 \end{pmatrix} = \begin{pmatrix} -7 & 0 \\ 0 & -7 \end{pmatrix}}$$

(ii) To compute $\boldsymbol{f(A)}$ use the equation $\boldsymbol{f(x) = x^2 - 5x + 7}$ by replacing the $\boldsymbol{A}$ in place of $\boldsymbol{x}$ we get

$$\boldsymbol{f(A) = A^2 - 5A + 7I}$$

$$= \begin{pmatrix} -7 & 0 \\ 0 & -7 \end{pmatrix} - 5\begin{pmatrix} 3 & 4 \\ -4 & -3 \end{pmatrix} + 7\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} -15 & -20 \\ 20 & 15 \end{pmatrix}$$

Q32

We have $\boldsymbol{A} = \begin{pmatrix} \mathbf{1} & \mathbf{1} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}$.

To show: $\boldsymbol{A^n} = \begin{pmatrix} \mathbf{1} & \boldsymbol{n} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}; \forall \boldsymbol{n} \in \boldsymbol{N}$.

(i) Let us compute first $\boldsymbol{A^2}$

$$\boldsymbol{A^2 = AA = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}}$$

Thus the result is true for $\boldsymbol{n = 2}$.

(ii) Now suppose the result is true for $\boldsymbol{n = k - 1}$.

Thus we have $\boldsymbol{A^{k-1}} = \begin{pmatrix} \mathbf{1} & \boldsymbol{k-1} \\ 0 & \mathbf{1} \end{pmatrix}$.

(iii)Let us show that result is true for $\boldsymbol{n = k}$ also.

$$\boldsymbol{A^k = A^{k-1}A = \begin{pmatrix} 1 & k-1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & k \\ 0 & 1 \end{pmatrix}}$$

Thus from the principle of mathematical induction we can state that the above result is true for all $\boldsymbol{k \in N}$.

Therefore we have $\boldsymbol{A^n = \begin{pmatrix} 1 & n \\ 0 & 1 \end{pmatrix}; \forall n \in N}$

Q33

We have $\boldsymbol{A \neq O\ , B \neq O\ , AB = O\ and\ BA \neq O}$.

To find: $\boldsymbol{A, B}$.

Let us take $\boldsymbol{A = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \neq O}$ and $\boldsymbol{B = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \neq O}$.

We have here $\boldsymbol{AB = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} = O}$

And $\boldsymbol{BA = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \neq O}$.

Therefore our required matrices are:

$\boldsymbol{A = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}}$ and $\boldsymbol{B = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}}$.

Q34

We have $\boldsymbol{AB} = \boldsymbol{AC}$ but $\boldsymbol{B} \neq \boldsymbol{C}$.

To find: $\boldsymbol{A}, \boldsymbol{B}$.

Let us take $A = \begin{pmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{pmatrix} = \boldsymbol{O}$, $\boldsymbol{C} = \begin{pmatrix} \mathbf{2} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 1 \\ 3 & 2 \end{pmatrix}$.

We have here $AB = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} = O$

And $AC = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} = O$.

Thus we have $AB = AC$ but $B \neq C$.

Therefore our required matrices are:

$A = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} = O$, $C = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 1 \\ 3 & 2 \end{pmatrix}$.

Q35

We have $A = \begin{pmatrix} 1 & 0 \\ -1 & 7 \end{pmatrix}$, $B = \begin{pmatrix} 0 & 4 \\ -1 & 7 \end{pmatrix}$.

(i) Let us compute first A^2

$A^2 = AA = \begin{pmatrix} 1 & 0 \\ -1 & 7 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -1 & 7 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -8 & 49 \end{pmatrix}$

(ii) To compute $3A^2 - 2B + I$

$3A^2 - 2B + I$

$= 3\begin{pmatrix} 1 & 0 \\ -8 & 49 \end{pmatrix} - 2\begin{pmatrix} 0 & 4 \\ -1 & 7 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$=\begin{pmatrix} 4 & -8 \\ -22 & 134 \end{pmatrix}$

Q36

We have $A = \begin{pmatrix} 2 & 3 \\ 5 & 7 \end{pmatrix}$, $B = \begin{pmatrix} 1 & -3 \\ -2 & 4 \end{pmatrix}$ and $C = \begin{pmatrix} -4 & 6 \\ -9 & x \end{pmatrix}$.

We also have $AB = C$.

Therefore $\begin{pmatrix} 2 & 3 \\ 5 & 7 \end{pmatrix}\begin{pmatrix} 1 & -3 \\ -2 & 4 \end{pmatrix} = \begin{pmatrix} -4 & 6 \\ -9 & x \end{pmatrix}$

$\begin{pmatrix} -4 & 6 \\ -9 & 13 \end{pmatrix} = \begin{pmatrix} -4 & 6 \\ -9 & x \end{pmatrix}$

Therefore $x = 13$

Exercise:5D

Q1

We have $A = \begin{pmatrix} 2 & -3 & 5 \\ 0 & 7 & -4 \end{pmatrix}$.

The transpose of the matrix is an operation of making interchange of elements by the rule on positioned element a_{ij} shifted to new position a_{ji}.

(i) The transposeof A is given by A^T which can be written as

$$A^T = \begin{pmatrix} 2 & -3 & 5 \\ 0 & 7 & -4 \end{pmatrix}^T = \begin{pmatrix} 2 & 0 \\ -3 & 7 \\ 5 & -4 \end{pmatrix}.$$

(ii) Next the transpose of A^T can be processed as

$$(A^T)^T = \begin{pmatrix} 2 & 0 \\ -3 & 7 \\ 5 & -4 \end{pmatrix}^T = \begin{pmatrix} 2 & -3 & 5 \\ 0 & 7 & -4 \end{pmatrix} = A$$

Thus we have $(A^T)^T = A$.

Q2

We have $A = \begin{pmatrix} 3 & 5 \\ -2 & 0 \\ 4 & -6 \end{pmatrix}$. Thus $2A = \begin{pmatrix} 6 & 10 \\ -4 & 0 \\ 8 & -12 \end{pmatrix}$

The transpose of the matrix is an operation of making interchange of elements by the rule on positioned element a_{ij} shifted to new position a_{ji}.

The transpose of A is given by A^T which can be written as

$$(2A)^T = \begin{pmatrix} 6 & 10 \\ -4 & 0 \\ 8 & -12 \end{pmatrix}^T = \begin{pmatrix} 6 & -4 & 8 \\ 10 & 0 & -12 \end{pmatrix} = 2\begin{pmatrix} 3 & -2 & 4 \\ 5 & 0 & -6 \end{pmatrix} =$$

$2A^T$.

Thus we have $(2A)^T = 2A^T$.

Q3

We have $A = \begin{pmatrix} 3 & 2 & -1 \\ -5 & 0 & -6 \end{pmatrix}$ and $B = \begin{pmatrix} -4 & -5 & -2 \\ 3 & 1 & 8 \end{pmatrix}$.

The transpose of the matrix is an operation of making interchange of elements by the rule on positioned element a_{ij} shifted to new position a_{ji}.

(i) The transpose of A and B are given by A^T and B^T which can be written as

$$A^T = \begin{pmatrix} 3 & 2 & -1 \\ -5 & 0 & -6 \end{pmatrix}^T = \begin{pmatrix} 3 & -5 \\ 2 & 0 \\ -1 & -6 \end{pmatrix}.$$

$$B^T = \begin{pmatrix} -4 & -5 & -2 \\ 3 & 1 & 8 \end{pmatrix}^T = \begin{pmatrix} -4 & 3 \\ -5 & 1 \\ -2 & 8 \end{pmatrix}.$$

(ii) Next the addition of A and B can be processed as

$$A + B = \begin{pmatrix} 3 & 2 & -1 \\ -5 & 0 & -6 \end{pmatrix} + \begin{pmatrix} -4 & -5 & -2 \\ 3 & 1 & 8 \end{pmatrix} = \begin{pmatrix} -1 & -3 & -3 \\ -2 & 1 & 2 \end{pmatrix}$$

(iii) Therefore $A^T + B^T = \begin{pmatrix} 3 & -5 \\ 2 & 0 \\ -1 & -6 \end{pmatrix} + \begin{pmatrix} -4 & 3 \\ -5 & 1 \\ -2 & 8 \end{pmatrix} = \begin{pmatrix} -1 & -2 \\ -3 & 1 \\ -3 & 2 \end{pmatrix}$

(iv) The transpose of $A + B$ is given by

$$(A + B)^T = \begin{pmatrix} -1 & -3 & -3 \\ -2 & 1 & 2 \end{pmatrix}^T = \begin{pmatrix} -1 & -2 \\ -3 & 1 \\ -3 & 2 \end{pmatrix}$$

From (iii) and (iv) we can say that $(A+B)^T = A^T + B^T$.

Q4

We have $P = \begin{pmatrix} 3 & 4 \\ 2 & -1 \\ 0 & 5 \end{pmatrix}$ and $Q = \begin{pmatrix} 7 & -5 \\ -4 & 0 \\ 2 & 6 \end{pmatrix}$.

The transpose of the matrix is an operation of making interchange of elements by the rule on positioned element a_{ij} shifted to new position a_{ji}.

(i) The transpose of P and Q are given by P^T and Q^T which can be written as

$$P^T = \begin{pmatrix} 3 & 4 \\ 2 & -1 \\ 0 & 5 \end{pmatrix}^T = \begin{pmatrix} 2 & 2 & 0 \\ 4 & -1 & 5 \end{pmatrix}.$$

$$Q^T = \begin{pmatrix} 7 & -5 \\ -4 & 0 \\ 2 & 6 \end{pmatrix}^T = \begin{pmatrix} 7 & -4 & 2 \\ -5 & 0 & 6 \end{pmatrix}.$$

(ii) Next the addition of P and Q can be processed as

$$P + Q = \begin{pmatrix} 3 & 4 \\ 2 & -1 \\ 0 & 5 \end{pmatrix} + \begin{pmatrix} 7 & -5 \\ -4 & 0 \\ 2 & 6 \end{pmatrix} = \begin{pmatrix} 10 & -1 \\ -2 & -1 \\ 2 & 11 \end{pmatrix}$$

(iii) Therefore $P^T + Q^T = \begin{pmatrix} 2 & 2 & 0 \\ 4 & -1 & 5 \end{pmatrix} + \begin{pmatrix} 7 & -4 & 2 \\ -5 & 0 & 6 \end{pmatrix} =$

$\begin{pmatrix} 10 & -2 & 2 \\ -1 & -1 & 11 \end{pmatrix}$.

(iv) The transpose of $P + Q$ is given by

$$(P + Q)^T = \begin{pmatrix} 10 & -1 \\ -2 & -1 \\ 2 & 11 \end{pmatrix}^T = \begin{pmatrix} 10 & -2 & 2 \\ -1 & -1 & 11 \end{pmatrix}$$

From (iii) and (iv) we can say that $(P + Q)^T = P^T + Q^T$.

Q5

We have $A = \begin{pmatrix} 4 & 1 \\ 5 & 8 \end{pmatrix}$.

The transpose of the matrix is an operation of making interchange of elements by the rule on positioned element a_{ij} shifted to new position a_{ji}.

The symmetric matrix is defined as similarity of transpose of matrix with it self. i.e, $A^T = A$.

(i) The transpose of $A$are given by A^Twhich can be written as

$A^T = \begin{pmatrix} 4 & 1 \\ 5 & 8 \end{pmatrix}^T = \begin{pmatrix} 4 & 5 \\ 1 & 8 \end{pmatrix}$.

(ii) Next the addition of A and A^T can be processed as

$$A + A^T = \begin{pmatrix} 4 & 1 \\ 5 & 8 \end{pmatrix} + \begin{pmatrix} 4 & 5 \\ 1 & 8 \end{pmatrix} = \begin{pmatrix} 8 & 6 \\ 6 & 16 \end{pmatrix}$$

(iii) To check symmetric nature of $A + A^T$ as follow:

$$(A + A^T)^T = \begin{pmatrix} 8 & 6 \\ 6 & 16 \end{pmatrix}^T = \begin{pmatrix} 8 & 6 \\ 6 & 16 \end{pmatrix} = A + A^T.$$

Thus we can say $A + A^T$ is symmetric.

Q6

We have $A = \begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix}$.

The transpose of the matrix is an operation of making interchange of elements by the rule on positioned element a_{ij} shifted to new position a_{ji}.

The skew-symmetric matrix is defined as similarity of transpose of matrix with it self. i.e, $A^T = -A$.

(i) The transpose of A are given by A^T which can be written as

$$A^T = \begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix}^T = \begin{pmatrix} 3 & 1 \\ -4 & -1 \end{pmatrix}.$$

(ii) Next the subtraction of A and A^T can be processed as

$$A - A^T = \begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix} - \begin{pmatrix} 3 & 1 \\ -4 & -1 \end{pmatrix} = \begin{pmatrix} 0 & -5 \\ 5 & 0 \end{pmatrix}$$

(iii) To check symmetric nature of $A + A^T$ as follow:

$$(A - A^T)^T = \begin{pmatrix} 0 & -5 \\ 5 & 0 \end{pmatrix}^T = \begin{pmatrix} 0 & 5 \\ -5 & 0 \end{pmatrix} = -\begin{pmatrix} 0 & -5 \\ 5 & 0 \end{pmatrix} = -(A - A^T).$$

Thus we can say $A - A^T$ is skew-symmetric.

Q7

We have $A = \begin{pmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{pmatrix}$.

The transpose of the matrix is an operation of making interchange of elements by the rule on positioned element a_{ij} shifted to new position a_{ji}.

The skew-symmetric matrix is defined as similarity of transpose of matrix with it self. i.e, $A^T = -A$.

The transpose of A are given by A^T which can be written as

$$A^T = \begin{pmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{pmatrix}^T = \begin{pmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{pmatrix} = -A.$$

Thus we can say A is skew-symmetric.

Q8

We have $A = \begin{pmatrix} 2 & 3 \\ -1 & 4 \end{pmatrix}$.

The transpose of the matrix is an operation of making interchange of elements by the rule on positioned element a_{ij} shifted to new position a_{ji}.

The symmetric matrix is defined as similarity of transpose of matrix with it self. i.e, $A^T = A$.

The skew-symmetric matrix is defined as similarity of transpose of matrix with it self. i.e, $A^T = -A$.

(i) The transpose of A are given by A^T which can be written as

$$A^T = \begin{pmatrix} 2 & 3 \\ -1 & 4 \end{pmatrix}^T = \begin{pmatrix} 2 & -1 \\ 3 & 4 \end{pmatrix}$$

(ii)Next the addition of A and A^T can be processed as

$$A + A^T = \begin{pmatrix} 2 & 3 \\ -1 & 4 \end{pmatrix} + \begin{pmatrix} 2 & -1 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 4 & 2 \\ 2 & 8 \end{pmatrix} \; ; \; P = \frac{1}{2}(A + A^T) = \frac{1}{2}\begin{pmatrix} 4 & 2 \\ 2 & 8 \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 1 & 4 \end{pmatrix}$$

We also have $P^T = \begin{pmatrix} 2 & 1 \\ 1 & 4 \end{pmatrix}^T = \begin{pmatrix} 2 & 1 \\ 1 & 4 \end{pmatrix} = P$

(iii)Next the subtraction of A and A^T can be processed as

$$A - A^T = \begin{pmatrix} 2 & 3 \\ -1 & 4 \end{pmatrix} - \begin{pmatrix} 2 & -1 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 0 & 4 \\ -4 & 0 \end{pmatrix} \; ; \; Q = \frac{1}{2}\begin{pmatrix} 0 & 4 \\ -4 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 2 \\ -2 & 0 \end{pmatrix}$$

We also have $Q^T = \begin{pmatrix} 0 & 2 \\ -2 & 0 \end{pmatrix}^T = \begin{pmatrix} 0 & -2 \\ 2 & 0 \end{pmatrix} = -\begin{pmatrix} 0 & 2 \\ -2 & 0 \end{pmatrix} = -Q$

(iii) Next the addition of P and Q can be processed as

$$A = P + Q = \begin{pmatrix} 2 & 1 \\ 1 & 4 \end{pmatrix} + \begin{pmatrix} 0 & 2 \\ -2 & 0 \end{pmatrix} = \begin{pmatrix} 2 & 3 \\ -1 & 4 \end{pmatrix}$$

Thus we can say P is symmetric and Q is skew-symmetric.

Q9

We have $A = \begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix}$.

The transpose of the matrix is an operation of making interchange of elements by the rule on positioned element a_{ij} shifted to new position a_{ji}.

The symmetric matrix is defined as similarity of transpose of matrix with it self. i.e, $A^T = A$.

The skew-symmetric matrix is defined as similarity of transpose of matrix with it self. i.e, $A^T = -A$.

(i) The transpose of A are given by A^T which can be written as

$$A^T = \begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix}^T = \begin{pmatrix} 3 & 1 \\ -4 & -1 \end{pmatrix}$$

(ii)Next the addition of A and A^T can be processed as

$$A + A^T = \begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix} + \begin{pmatrix} 3 & 1 \\ -4 & -1 \end{pmatrix} = \begin{pmatrix} 6 & -3 \\ -3 & -2 \end{pmatrix} \; ; \; P =$$

$$\frac{1}{2}(A + A^T) = \frac{1}{2}\begin{pmatrix} 6 & -3 \\ -3 & -2 \end{pmatrix} = \begin{pmatrix} 3 & -\frac{3}{2} \\ -\frac{3}{2} & -1 \end{pmatrix}$$

We also have $P^T = \begin{pmatrix} 3 & -\frac{3}{2} \\ -\frac{3}{2} & -1 \end{pmatrix}^T = \begin{pmatrix} 3 & -\frac{3}{2} \\ -\frac{3}{2} & -1 \end{pmatrix} = P$

(iii)Next the subtraction of A and A^T can be processed as

$$A - A^T = \begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix} - \begin{pmatrix} 3 & 1 \\ -4 & -1 \end{pmatrix} = \begin{pmatrix} 0 & -5 \\ 5 & 0 \end{pmatrix} \; ; \; Q = \frac{1}{2}(A - A^T)$$

$$= \frac{1}{2}\begin{pmatrix} 0 & -5 \\ 5 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -\frac{5}{2} \\ \frac{5}{2} & 0 \end{pmatrix}$$

We also have $Q^T = \begin{pmatrix} 0 & -\frac{5}{2} \\ \frac{5}{2} & 0 \end{pmatrix}^T = \begin{pmatrix} 0 & \frac{5}{2} \\ -\frac{5}{2} & 0 \end{pmatrix} = -\begin{pmatrix} 0 & -\frac{5}{2} \\ \frac{5}{2} & 0 \end{pmatrix} =$

$-Q$

(iii) Next the addition of P and Q can be processed as

$$A = P + Q = \begin{pmatrix} 3 & -\frac{3}{2} \\ -\frac{3}{2} & -1 \end{pmatrix} + \begin{pmatrix} 0 & -\frac{5}{2} \\ \frac{5}{2} & 0 \end{pmatrix} = \begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix}$$

Thus we can say P is symmetric and Q is skew-symmetric.

Q10

We have $A = \begin{pmatrix} -1 & 5 & 1 \\ 2 & 3 & 4 \\ 7 & 0 & 9 \end{pmatrix}$.

The transpose of the matrix is an operation of making interchange of elements by the rule on positioned element a_{ij} shifted to new position a_{ji}.

The symmetric matrix is defined as similarity of transpose of matrix with it self. i.e, $A^T = A$.

The skew-symmetric matrix is defined as similarity of transpose of matrix with it self. i.e, $A^T = -A$.

(i) The transpose of A are given by A^T which can be written as

$$A^T = \begin{pmatrix} -1 & 5 & 1 \\ 2 & 3 & 4 \\ 7 & 0 & 9 \end{pmatrix}^T = \begin{pmatrix} -1 & 2 & 7 \\ 5 & 3 & 0 \\ 1 & 4 & 9 \end{pmatrix}$$

(ii)Next the addition of A and A^T can be processed as

$$A + A^T = \begin{pmatrix} -1 & 5 & 1 \\ 2 & 3 & 4 \\ 7 & 0 & 9 \end{pmatrix} + \begin{pmatrix} -1 & 2 & 7 \\ 5 & 3 & 0 \\ 1 & 4 & 9 \end{pmatrix} = \begin{pmatrix} -2 & 7 & 8 \\ 7 & 6 & 4 \\ 8 & 4 & 16 \end{pmatrix} ;$$

$$P = \frac{1}{2}(A + A^T) = \frac{1}{2}\begin{pmatrix} -2 & 7 & 8 \\ 7 & 6 & 4 \\ 8 & 4 & 16 \end{pmatrix} = \begin{pmatrix} -1 & \frac{7}{2} & 4 \\ \frac{7}{2} & 3 & 2 \\ 4 & 2 & 8 \end{pmatrix}$$

We also have $P^T = \begin{pmatrix} -1 & \frac{7}{2} & 4 \\ \frac{7}{2} & 3 & 2 \\ 4 & 2 & 8 \end{pmatrix}^T = \begin{pmatrix} -1 & \frac{7}{2} & 4 \\ \frac{7}{2} & 3 & 2 \\ 4 & 2 & 8 \end{pmatrix} = P$

(iii)Next the subtraction of A and A^T can be processed as

$$A - A^T = \begin{pmatrix} -1 & 5 & 1 \\ 2 & 3 & 4 \\ 7 & 0 & 9 \end{pmatrix} - \begin{pmatrix} -1 & 2 & 7 \\ 5 & 3 & 0 \\ 1 & 4 & 9 \end{pmatrix} = \begin{pmatrix} 0 & 3 & -6 \\ -3 & 0 & 4 \\ 6 & -4 & 0 \end{pmatrix} ;$$

$$Q = \frac{1}{2}(A - A^T) = \frac{1}{2}\begin{pmatrix} 0 & 3 & -6 \\ -3 & 0 & 4 \\ 6 & -4 & 0 \end{pmatrix} = \begin{pmatrix} 0 & \frac{3}{2} & -3 \\ -\frac{3}{2} & 0 & 2 \\ 3 & -2 & 0 \end{pmatrix}$$

We also have $Q^T = \begin{pmatrix} 0 & \frac{3}{2} & -3 \\ -\frac{3}{2} & 0 & 2 \\ 3 & -2 & 0 \end{pmatrix}^T = \begin{pmatrix} 0 & \frac{-3}{2} & 3 \\ \frac{3}{2} & 0 & -2 \\ -3 & 2 & 0 \end{pmatrix} =$

$$-\begin{pmatrix} 0 & \frac{3}{2} & -3 \\ -\frac{3}{2} & 0 & 2 \\ 3 & -2 & 0 \end{pmatrix} = -Q$$

(iii) Next the addition of P and Q can be processed as

$$A = P + Q = \begin{pmatrix} -1 & \frac{7}{2} & 4 \\ \frac{7}{2} & 3 & 2 \\ 4 & 2 & 8 \end{pmatrix} + \begin{pmatrix} 0 & \frac{3}{2} & -3 \\ -\frac{3}{2} & 0 & 2 \\ 3 & -2 & 0 \end{pmatrix} = \begin{pmatrix} -1 & 5 & 1 \\ 2 & 3 & 4 \\ 7 & 0 & 9 \end{pmatrix}$$

Thus we can say P is symmetric and Q is skew-symmetric.

Q11

We have $A = \begin{pmatrix} 3 & -1 & 0 \\ 2 & 0 & 3 \\ 1 & -1 & 2 \end{pmatrix}$.

The transpose of the matrix is an operation of making interchange of elements by the rule on positioned element a_{ij} shifted to new position a_{ji}.

The symmetric matrix is defined as similarity of transpose of matrix with it self. i.e, $A^T = A$.

The skew-symmetric matrix is defined as similarity of transpose of matrix with it self. i.e, $A^T = -A$.

(i) The transpose of A are given by A^T which can be written as

$$A^T = \begin{pmatrix} 3 & -1 & 0 \\ 2 & 0 & 3 \\ 1 & -1 & 2 \end{pmatrix}^T = \begin{pmatrix} 3 & 2 & 1 \\ -1 & 0 & -1 \\ 0 & 3 & 2 \end{pmatrix}$$

(ii)Next the addition of A and A^T can be processed as

$$A + A^T = \begin{pmatrix} 3 & -1 & 0 \\ 2 & 0 & 3 \\ 1 & -1 & 2 \end{pmatrix} + \begin{pmatrix} 3 & 2 & 1 \\ -1 & 0 & -1 \\ 0 & 3 & 2 \end{pmatrix} = \begin{pmatrix} 6 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 4 \end{pmatrix} ;$$

$$P = \frac{1}{2}(A + A^T) = \frac{1}{2}\begin{pmatrix} 6 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 4 \end{pmatrix} = \begin{pmatrix} 3 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & 1 \\ \frac{1}{2} & 1 & 2 \end{pmatrix}$$

We also have $P^T = \begin{pmatrix} 3 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & 1 \\ \frac{1}{2} & 1 & 2 \end{pmatrix}^T = \begin{pmatrix} 3 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & 1 \\ \frac{1}{2} & 1 & 2 \end{pmatrix} = P$

(iii)Next the subtraction of A and A^T can be processed as

$$A - A^T = \begin{pmatrix} 3 & -1 & 0 \\ 2 & 0 & 3 \\ 1 & -1 & 2 \end{pmatrix} - \begin{pmatrix} 3 & 2 & 1 \\ -1 & 0 & -1 \\ 0 & 3 & 2 \end{pmatrix} = \begin{pmatrix} 0 & -3 & -1 \\ 3 & 0 & 4 \\ 1 & -4 & 0 \end{pmatrix} \quad ;$$

$$Q = \frac{1}{2}(A - A^T) = \frac{1}{2}\begin{pmatrix} 0 & -3 & -1 \\ 3 & 0 & 4 \\ 1 & -4 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -\frac{3}{2} & -\frac{1}{2} \\ \frac{3}{2} & 0 & 2 \\ \frac{1}{2} & -2 & 0 \end{pmatrix}$$

We also have $Q^T = \begin{pmatrix} 0 & -\frac{3}{2} & -\frac{1}{2} \\ \frac{3}{2} & 0 & 2 \\ \frac{1}{2} & -2 & 0 \end{pmatrix}^T = \begin{pmatrix} 0 & \frac{3}{2} & \frac{1}{2} \\ -\frac{3}{2} & 0 & -2 \\ -\frac{1}{2} & 2 & 0 \end{pmatrix} =$

$$-\begin{pmatrix} 0 & -\frac{3}{2} & -\frac{1}{2} \\ \frac{3}{2} & 0 & 2 \\ \frac{1}{2} & -2 & 0 \end{pmatrix} = -Q$$

(iii) Next the addition of P and Q can be processed as

$$A = P + Q = \begin{pmatrix} 3 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & 1 \\ \frac{1}{2} & 1 & 2 \end{pmatrix} + \begin{pmatrix} 0 & -\frac{3}{2} & -\frac{1}{2} \\ \frac{3}{2} & 0 & 2 \\ \frac{1}{2} & -2 & 0 \end{pmatrix} = \begin{pmatrix} 3 & -1 & 0 \\ 2 & 0 & 3 \\ 1 & -1 & 2 \end{pmatrix}$$

Thus we can say P is symmetric and Q is skew-symmetric.

Q12

We have $A = \begin{pmatrix} 3 & 2 & 5 \\ 4 & 1 & 3 \\ 0 & 6 & 7 \end{pmatrix}$.

The transpose of the matrix is an operation of making interchange of elements by the rule on positioned element a_{ij} shifted to new position a_{ji}.

The symmetric matrix is defined as similarity of transpose of matrix with it self. i.e, $A^T = A$.

The skew-symmetric matrix is defined as similarity of transpose of matrix with it self. i.e, $A^T = -A$.

(i) The transpose of A are given by A^T which can be written as

$$A^T = \begin{pmatrix} 3 & 2 & 5 \\ 4 & 1 & 3 \\ 0 & 6 & 7 \end{pmatrix}^T = \begin{pmatrix} 3 & 4 & 0 \\ 2 & 1 & 6 \\ 5 & 3 & 7 \end{pmatrix}$$

(ii)Next the addition of A and A^T can be processed as

$$A + A^T = \begin{pmatrix} 3 & 2 & 5 \\ 4 & 1 & 3 \\ 0 & 6 & 7 \end{pmatrix} + \begin{pmatrix} 3 & 4 & 0 \\ 2 & 1 & 6 \\ 5 & 3 & 7 \end{pmatrix} = \begin{pmatrix} 6 & 6 & 5 \\ 5 & 2 & 9 \\ 5 & 9 & 14 \end{pmatrix} ;$$

$$P = \frac{1}{2}(A + A^T) = \frac{1}{2}\begin{pmatrix} 6 & 6 & 5 \\ 5 & 2 & 9 \\ 5 & 9 & 14 \end{pmatrix} = \begin{pmatrix} 3 & 3 & \frac{5}{2} \\ \frac{5}{2} & 1 & \frac{9}{2} \\ \frac{5}{2} & \frac{9}{2} & 7 \end{pmatrix}$$

We also have $P^T = \begin{pmatrix} 3 & 3 & \frac{5}{2} \\ \frac{5}{2} & 1 & \frac{9}{2} \\ \frac{5}{2} & \frac{9}{2} & 7 \end{pmatrix}^T = \begin{pmatrix} 3 & 3 & \frac{5}{2} \\ \frac{5}{2} & 1 & \frac{9}{2} \\ \frac{5}{2} & \frac{9}{2} & 7 \end{pmatrix} = P$

(iii)Next the subtraction of A and A^T can be processed as

$$A - A^T = \begin{pmatrix} 3 & 2 & 5 \\ 4 & 1 & 3 \\ 0 & 6 & 7 \end{pmatrix} - \begin{pmatrix} 3 & 4 & 2 \\ 2 & 1 & 6 \\ 5 & 3 & 7 \end{pmatrix} = \begin{pmatrix} 0 & -2 & 5 \\ 2 & 0 & -3 \\ -5 & 3 & 0 \end{pmatrix} \quad ;$$

$$Q = \frac{1}{2}(A - A^T) = \frac{1}{2}\begin{pmatrix} 0 & -2 & 5 \\ 2 & 0 & -3 \\ -5 & 3 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -1 & \frac{5}{2} \\ 1 & 0 & -\frac{3}{2} \\ -\frac{5}{2} & \frac{3}{2} & 0 \end{pmatrix}$$

We also have $Q^T = \begin{pmatrix} 0 & -1 & \frac{5}{2} \\ 1 & 0 & -\frac{3}{2} \\ -\frac{5}{2} & \frac{3}{2} & 0 \end{pmatrix}^T = \begin{pmatrix} 0 & 1 & -\frac{5}{2} \\ -1 & 0 & \frac{3}{2} \\ \frac{5}{2} & -\frac{3}{2} & 0 \end{pmatrix} =$

$$-\begin{pmatrix} 0 & -1 & \frac{5}{2} \\ 1 & 0 & -\frac{3}{2} \\ -\frac{5}{2} & \frac{3}{2} & 0 \end{pmatrix} = -Q$$

(iii) Next the addition of P and Q can be processed as

$$A = P + Q = \begin{pmatrix} 3 & 3 & \frac{5}{2} \\ \frac{5}{2} & 1 & \frac{9}{2} \\ \frac{5}{2} & \frac{9}{2} & 7 \end{pmatrix} + \begin{pmatrix} 0 & -1 & \frac{5}{2} \\ 1 & 0 & -\frac{3}{2} \\ -\frac{5}{2} & \frac{3}{2} & 0 \end{pmatrix} = \begin{pmatrix} 3 & 2 & 5 \\ 4 & 1 & 3 \\ 0 & 6 & 7 \end{pmatrix}$$

Thus we can say P is symmetric and Q is skew-symmetric.

Q13A

We have $A = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 4 \\ 2 & 5 \end{pmatrix}$.

The transpose of the matrix is an operation of making interchange of elements by the rule on positioned element a_{ij} shifted to new position a_{ji}.

(i) The transpose of A is given by

$$A^T = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}^T = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

(ii) The transpose of B is given by

$$B^T = \begin{pmatrix} 1 & 4 \\ 2 & 5 \end{pmatrix}^T = \begin{pmatrix} 1 & 2 \\ 4 & 5 \end{pmatrix}$$

(iii) The multiplication of matrices A and B is given by

$$AB = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}\begin{pmatrix} 1 & 4 \\ 2 & 5 \end{pmatrix} = \begin{pmatrix} 7 & 19 \\ 10 & 28 \end{pmatrix}$$

(iv) The transpose of AB is given by

$$(AB)^T = \begin{pmatrix} 7 & 19 \\ 10 & 28 \end{pmatrix}^T = \begin{pmatrix} 7 & 10 \\ 19 & 28 \end{pmatrix}$$

(v) The multiplication of A^T and B^T is given by

$$B^T A^T = \begin{pmatrix} 1 & 2 \\ 4 & 5 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 7 & 10 \\ 19 & 28 \end{pmatrix}$$

Thus from (iv) and (v) we have

$$(AB)^T = B^T A^T$$

Q13B

We have $A = \begin{pmatrix} 3 & -1 \\ 2 & -2 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & -3 \\ 2 & -1 \end{pmatrix}$.

The transpose of the matrix is an operation of making interchange of elements by the rule on positioned element a_{ij} shifted to new position a_{ji}.

(i) The transpose of A is given by

$$A^T = \begin{pmatrix} 3 & -1 \\ 2 & -2 \end{pmatrix}^T = \begin{pmatrix} 3 & 2 \\ -1 & -2 \end{pmatrix}$$

(ii) The transpose of B is given by

$$B^T = \begin{pmatrix} 1 & -3 \\ 2 & -1 \end{pmatrix}^T = \begin{pmatrix} 1 & 2 \\ -3 & -1 \end{pmatrix}$$

(iii) The multiplication of matrices A and B is given by

$$AB = \begin{pmatrix} 3 & -1 \\ 2 & -2 \end{pmatrix}\begin{pmatrix} 1 & -3 \\ 2 & -1 \end{pmatrix} = \begin{pmatrix} 1 & -8 \\ -2 & -4 \end{pmatrix}$$

(iv) The transpose of AB is given by

$$(AB)^T = \begin{pmatrix} 1 & -8 \\ -2 & -4 \end{pmatrix}^T = \begin{pmatrix} 1 & -2 \\ -8 & -4 \end{pmatrix}$$

(v) The multiplication of A^T and B^T is given by

$$B^T A^T = \begin{pmatrix} 1 & 2 \\ -3 & -1 \end{pmatrix}\begin{pmatrix} 3 & 2 \\ -1 & -2 \end{pmatrix} = \begin{pmatrix} 1 & -2 \\ -8 & -4 \end{pmatrix}$$

Thus from (iv) and (v) we have

$$(AB)^T = B^T A^T$$

Q13C

We have $A = \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix}$ and $B = (-2 \quad -1 \quad -4)$.

The transpose of the matrix is an operation of making interchange of elements by the rule on positioned element a_{ij} shifted to new position a_{ji}.

(i) The transpose of A is given by

$$A^T = \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix}^T = (-1 \quad 2 \quad 3)$$

(ii) The transpose of B is given by

$$B^T = (-2 \quad -1 \quad -4)^T = \begin{pmatrix} -2 \\ -1 \\ -4 \end{pmatrix}$$

(iii) The multiplication of matrices A and B is given by

$$AB = \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix}(-2 \quad -1 \quad -4) = \begin{pmatrix} 2 & 1 & 4 \\ -4 & -2 & -8 \\ -6 & -3 & -12 \end{pmatrix}$$

(iv) The transpose of AB is given by

$$(AB)^T = \begin{pmatrix} 2 & 1 & 4 \\ -4 & -2 & -8 \\ -6 & -3 & -12 \end{pmatrix}^T = \begin{pmatrix} 2 & -4 & -6 \\ 1 & -2 & -3 \\ 4 & -8 & -12 \end{pmatrix}$$

(v) The multiplication of A^T and B^T is given by

$$B^T A^T = \begin{pmatrix} -2 \\ -1 \\ -4 \end{pmatrix}(-1 \quad 2 \quad 3) = \begin{pmatrix} 2 & -4 & -6 \\ 1 & -2 & -3 \\ 4 & -8 & -12 \end{pmatrix}$$

Thus from (iv) and (v) we have

$$(AB)^T = B^T A^T$$

Q13D

We have $A = \begin{pmatrix} -1 & 2 & -3 \\ 4 & -5 & 6 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & -4 \\ 2 & 1 \\ -1 & 0 \end{pmatrix}$.

The transpose of the matrix is an operation of making interchange of elements by the rule on positioned element a_{ij} shifted to new position a_{ji}.

(i) The transpose of A is given by

$$A^T = \begin{pmatrix} -1 & 2 & -3 \\ 4 & -5 & 6 \end{pmatrix}^T = \begin{pmatrix} -1 & 4 \\ 2 & -5 \\ -3 & 6 \end{pmatrix}$$

(ii) The transpose of B is given by

$$B^T = \begin{pmatrix} 3 & -4 \\ 2 & 1 \\ -1 & 0 \end{pmatrix}^T = \begin{pmatrix} 3 & 2 & -1 \\ -4 & 1 & 0 \end{pmatrix}$$

(iii) The multiplication of matrices A and B is given by

$$AB = \begin{pmatrix} -1 & 2 & -3 \\ 4 & -5 & 6 \end{pmatrix}\begin{pmatrix} 3 & -4 \\ 2 & 1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} 4 & 6 \\ -4 & -21 \end{pmatrix}$$

(iv) The transpose of AB is given by

$$(AB)^T = \begin{pmatrix} 4 & 6 \\ -4 & -21 \end{pmatrix}^T = \begin{pmatrix} 4 & -4 \\ 6 & -21 \end{pmatrix}$$

(v) The multiplication of A^T and B^T is given by

$$B^T A^T = \begin{pmatrix} 3 & 2 & -1 \\ -4 & 1 & 0 \end{pmatrix}\begin{pmatrix} -1 & 4 \\ 2 & -5 \\ -3 & 6 \end{pmatrix} = \begin{pmatrix} 4 & -4 \\ 6 & -21 \end{pmatrix}$$

Thus from (iv) and (v) we have

$$(AB)^T = B^T A^T$$

Q14

We have $A = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}$.

The transpose of the matrix is an operation of making interchange of elements by the rule on positioned element a_{ij} shifted to new position a_{ji}.

(i) The transpose of A is given by

$$A^T = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}^T = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$$

(ii) The multiplication of matrices A and B is given by

$$A^T A = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}\begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$$

Q15

We have $A = (1 \quad 2 \quad 3)$.

The transpose of the matrix is an operation of making interchange of elements by the rule on positioned element a_{ij} shifted to new position a_{ji}.

(i) The transpose of A is given by

$$A^T = (1 \quad 2 \quad 3)^T = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

(ii) The multiplication of matrices A and B is given by

$$A^TA = \begin{pmatrix}1\\2\\3\end{pmatrix}(1 \quad 2 \quad 3)=(14)$$

Exercise:5E

Q1

We have $A = \begin{pmatrix}1 & 2\\3 & 7\end{pmatrix}$.

To get the inverse we will proceed by augmented matrix with elementary row transformation process is as follow:

$$(A|I) \rightarrow (I|A^{-1})$$

Let us proceed

$$\left(\begin{array}{cc|cc}1 & 2 & 1 & 0\\3 & 7 & 0 & 1\end{array}\right)$$

$$\xrightarrow{R_2 \rightarrow R_2 - 3R_1} \left(\begin{array}{cc|cc}1 & 2 & 1 & 0\\0 & 1 & -3 & 1\end{array}\right) \xrightarrow{R_1 \rightarrow R_1 - 2R_2} \left(\begin{array}{cc|cc}1 & 0 & 7 & -2\\0 & 1 & -3 & 1\end{array}\right)$$

Thus we got the inverse of the matrix as

$$A^{-1} = \begin{pmatrix}7 & -2\\-3 & 1\end{pmatrix}.$$

Q2

We have $A = \begin{pmatrix} 1 & 2 \\ 2 & -1 \end{pmatrix}$.

To get the inverse we will proceed by augmented matrix with elementary row transformation process is as follow:

$$(A|I) \to (I|A^{-1})$$

Let us proceed

$$\left(\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 2 & -1 & 0 & 1 \end{array}\right) \xrightarrow{R_2 \to R_2 - 2R_1} \left(\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & -5 & -2 & 1 \end{array}\right)$$

$$\xrightarrow{R_2 \to -\frac{1}{5}R_2} \left(\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & 1 & \frac{2}{5} & -\frac{1}{5} \end{array}\right) \xrightarrow{R_1 \to R_1 - 2R_2} \left(\begin{array}{cc|cc} 1 & 0 & \frac{1}{5} & \frac{2}{5} \\ 0 & 1 & \frac{2}{5} & -\frac{1}{5} \end{array}\right)$$

Thus we got the inverse of the matrix as

$$A^{-1} = \begin{pmatrix} \frac{1}{5} & \frac{2}{5} \\ \frac{2}{5} & -\frac{1}{5} \end{pmatrix}.$$

Q3

We have $A = \begin{pmatrix} 2 & 5 \\ -3 & 1 \end{pmatrix}$.

To get the inverse we will proceed by augmented matrix with elementary row transformation process is as follow:

$$(A|I) \to (I|A^{-1})$$

Let us proceed

$$\begin{pmatrix} 2 & 5 \\ -3 & 1 \end{vmatrix} \begin{matrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \xrightarrow{R_2 \to R_2 + \frac{3}{2}R_1} \begin{pmatrix} 2 & 5 \\ 0 & \frac{17}{2} \end{vmatrix} \begin{matrix} 1 & 0 \\ \frac{3}{2} & 1 \end{pmatrix} \xrightarrow{R_2 \to \frac{2}{17}R_2 \,; R_1 \to \frac{1}{2}R_1}$$

$$\begin{pmatrix} 1 & \frac{5}{2} \\ 0 & 1 \end{vmatrix} \begin{matrix} 1 & 0 \\ \frac{3}{17} & \frac{2}{17} \end{pmatrix} \xrightarrow{R_1 \to R_1 - \frac{5}{2}R_2} \left(\begin{array}{cc|cc} 1 & 0 & \frac{1}{17} & -\frac{5}{17} \\ 0 & 1 & \frac{3}{17} & \frac{2}{17} \end{array} \right)$$

Thus we got the inverse of the matrix as

$$A^{-1} = \begin{pmatrix} \frac{1}{17} & -\frac{5}{17} \\ \frac{3}{17} & \frac{2}{17} \end{pmatrix}.$$

Q4

We have $A = \begin{pmatrix} 2 & -3 \\ 4 & 5 \end{pmatrix}$.

To get the inverse we will proceed by augmented matrix with elementary row transformation process is as follow:

$$(A|I) \to (I|A^{-1})$$

Let us proceed

$$\begin{pmatrix} 2 & -3 \\ 4 & 5 \end{vmatrix} \begin{matrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \xrightarrow{R_2 \to R_2 - 2R_1} \begin{pmatrix} 2 & -3 \\ 0 & 11 \end{vmatrix} \begin{matrix} 1 & 0 \\ -2 & 1 \end{pmatrix} \xrightarrow{R_2 \to \frac{1}{11}R_2 \,; R_1 \to \frac{1}{2}R_1}$$

$$\begin{pmatrix} 1 & -\frac{3}{2} \\ 0 & 1 \end{vmatrix} \begin{matrix} 1 & 0 \\ \frac{-2}{11} & \frac{1}{11} \end{pmatrix} \xrightarrow{R_1 \to R_1 + \frac{3}{2}R_2} \left(\begin{array}{cc|cc} 1 & 0 & \frac{5}{22} & \frac{3}{22} \\ 0 & 1 & \frac{-2}{11} & \frac{1}{11} \end{array} \right)$$

Thus we got the inverse of the matrix as

$$A^{-1} = \begin{pmatrix} \frac{5}{22} & \frac{3}{22} \\ \frac{-2}{11} & \frac{1}{11} \end{pmatrix}$$

Q5

We have $A = \begin{pmatrix} 4 & 0 \\ 2 & 5 \end{pmatrix}$.

To get the inverse we will proceed by augmented matrix with elementary row transformation process is as follow:

$$(A|I) \to (I|A^{-1})$$

Let us proceed

$$\left(\begin{array}{cc|cc} 4 & 0 & 1 & 0 \\ 2 & 5 & 0 & 1 \end{array}\right)$$

$$\xrightarrow{R_2 \to R_2 - 2R_1} \left(\begin{array}{cc|cc} 4 & 0 & 1 & 0 \\ 0 & 5 & -2 & 1 \end{array}\right) \xrightarrow{R_2 \to \frac{1}{5}R_2 \,; R_1 \to \frac{1}{4}R_1} \left(\begin{array}{cc|cc} 1 & 0 & \frac{1}{4} & 0 \\ 0 & 1 & \frac{-2}{5} & \frac{1}{5} \end{array}\right)$$

Thus we got the inverse of the matrix as

$$A^{-1} = \begin{pmatrix} \frac{1}{4} & 0 \\ \frac{-2}{5} & \frac{1}{5} \end{pmatrix}$$

Q6

We have $A = \begin{pmatrix} 6 & 7 \\ 8 & 9 \end{pmatrix}$.

To get the inverse we will proceed by augmented matrix with elementary row transformation process is as follow:

$$(A|I) \rightarrow (I|A^{-1})$$

Let us proceed

$$\left(\begin{array}{cc|cc} 6 & 7 & 1 & 0 \\ 8 & 9 & 0 & 1 \end{array}\right) \xrightarrow{R_2 \rightarrow R_2 - \frac{8}{6}R_1} \left(\begin{array}{cc|cc} 6 & 7 & 1 & 0 \\ 0 & -\frac{1}{3} & -\frac{4}{3} & 1 \end{array}\right) \xrightarrow{R_2 \rightarrow -3R_2 \,; R_1 \rightarrow \frac{1}{6}R_1}$$

$$\left(\begin{array}{cc|cc} 1 & \frac{7}{6} & \frac{1}{6} & 0 \\ 0 & 1 & -4 & -3 \end{array}\right) \xrightarrow{R_1 \rightarrow R_1 - \frac{7}{6}R_2} \left(\begin{array}{cc|cc} 1 & 0 & \frac{-9}{2} & \frac{7}{2} \\ 0 & 1 & 4 & -3 \end{array}\right)$$

Thus we got the inverse of the matrix as

$$A^{-1} = \begin{pmatrix} \frac{-9}{2} & \frac{7}{2} \\ 4 & -3 \end{pmatrix}$$

Q7

We have $A = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{pmatrix}$.

To get the inverse we will proceed by augmented matrix with elementary row transformation process is as follow:

$$(A|I) \rightarrow (I|A^{-1})$$

Let us proceed

$$\left(\begin{array}{ccc|ccc} 0 & 1 & 2 & 1 & 0 & 0 \\ 1 & 2 & 3 & 0 & 1 & 0 \\ 3 & 1 & 1 & 0 & 0 & 1 \end{array}\right) \xrightarrow{R_2 \leftrightarrow R_1} \left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 0 & 1 & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 \\ 3 & 1 & 1 & 1 & 0 & 0 \end{array}\right)$$

$$\xrightarrow{R_3 \to R_3 - 3R_1} \left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 0 & 1 & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 \\ 0 & -5 & -8 & 0 & -3 & 1 \end{array}\right)$$

$$\xrightarrow{R_3 \to R_3 + 5R_2} \left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 0 & 1 & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 \\ 0 & 0 & 2 & 5 & -3 & 1 \end{array}\right) \xrightarrow{R_3 \to \frac{1}{2}R_3} \left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 0 & 1 & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 \\ 0 & 0 & 1 & \frac{5}{2} & -\frac{3}{2} & \frac{1}{2} \end{array}\right)$$

$$\xrightarrow{R_1 \to R_1 - 2R_2} \left(\begin{array}{ccc|ccc} 1 & 0 & -1 & -2 & 1 & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 \\ 0 & 0 & 1 & \frac{5}{2} & -\frac{3}{2} & \frac{1}{2} \end{array}\right)$$

$$\xrightarrow{R_1 \to R_1 + R_3; R_2 \to R_2 - 2R_3} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ 0 & 1 & 0 & -4 & 3 & -1 \\ 0 & 0 & 1 & \frac{5}{2} & -\frac{3}{2} & \frac{1}{2} \end{array}\right)$$

Thus we have the inverse of the matrix as

$$A^{-1} = \begin{pmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -4 & 3 & -1 \\ \frac{5}{2} & -\frac{3}{2} & \frac{1}{2} \end{pmatrix}$$

Q8

We have $A = \begin{pmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{pmatrix}$.

To get the inverse we will proceed by augmented matrix with elementary row transformation process is as follow:

$$(A|I) \rightarrow (I|A^{-1})$$

Let us proceed

$$\begin{pmatrix} 2 & -3 & 3 & 1 & 0 & 0 \\ 2 & 2 & 3 & 0 & 1 & 0 \\ 3 & -2 & 2 & 0 & 0 & 1 \end{pmatrix} \xrightarrow{R_2 \rightarrow R_2 - R_1} \begin{pmatrix} 2 & -3 & 3 & 1 & 0 & 0 \\ 0 & 5 & 0 & -1 & 1 & 0 \\ 3 & -2 & 2 & 0 & 0 & 1 \end{pmatrix}$$

$$\xrightarrow{R_3 \rightarrow R_3 - R_1} \begin{pmatrix} 2 & -3 & 3 & 1 & 0 & 0 \\ 0 & 5 & 0 & -1 & 1 & 0 \\ 1 & 1 & -1 & -1 & 0 & 1 \end{pmatrix}$$

$$\xrightarrow{R_1 \rightarrow R_1 - R_3} \begin{pmatrix} 1 & -4 & 4 & 2 & 0 & -1 \\ 0 & 5 & 0 & -1 & 1 & 0 \\ 1 & 1 & -1 & -1 & 0 & 1 \end{pmatrix}$$

$$\xrightarrow{R_3 \rightarrow R_3 - R_1} \begin{pmatrix} 1 & -4 & 4 & 2 & 0 & -1 \\ 0 & 5 & 0 & -1 & 1 & 0 \\ 0 & 5 & -5 & -3 & 0 & 2 \end{pmatrix}$$

$$\xrightarrow{R_3 \rightarrow R_3 - R_2} \begin{pmatrix} 1 & -4 & 4 & 2 & 0 & -1 \\ 0 & 5 & 0 & -1 & 1 & 0 \\ 0 & 0 & -5 & -2 & -1 & 2 \end{pmatrix}$$

$$\xrightarrow{R_1 \to R_1 + R_2} \left(\begin{array}{ccc|ccc} 1 & 1 & 4 & 1 & 1 & -1 \\ 0 & 5 & 0 & -1 & 1 & 0 \\ 0 & 0 & -5 & -2 & -1 & 2 \end{array}\right)$$

$$\xrightarrow{R_2 \to \frac{1}{5}R_2; R_3 \to -\frac{1}{5}R_3} \left(\begin{array}{ccc|ccc} 1 & 1 & 4 & 1 & 1 & -1 \\ 0 & 1 & 0 & -\frac{1}{5} & \frac{1}{5} & 0 \\ 0 & 0 & 1 & \frac{2}{5} & \frac{1}{5} & -\frac{2}{5} \end{array}\right)$$

$$\xrightarrow{R_1 \to R_1 - R_2} \left(\begin{array}{ccc|ccc} 1 & 0 & 4 & \frac{6}{5} & \frac{4}{5} & -1 \\ 0 & 1 & 0 & -\frac{1}{5} & \frac{1}{5} & 0 \\ 0 & 0 & 1 & \frac{2}{5} & \frac{1}{5} & -\frac{2}{5} \end{array}\right)$$

$$\xrightarrow{R_1 \to R_1 - 4R_3} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{-2}{5} & 0 & \frac{3}{5} \\ 0 & 1 & 0 & -\frac{1}{5} & \frac{1}{5} & 0 \\ 0 & 0 & 1 & \frac{2}{5} & \frac{1}{5} & -\frac{2}{5} \end{array}\right)$$

Thus we got the inverse of the matrix as

$$A^{-1} = \begin{pmatrix} -\frac{2}{5} & 0 & \frac{3}{5} \\ -\frac{1}{5} & \frac{1}{5} & 0 \\ \frac{2}{5} & \frac{1}{5} & -\frac{2}{5} \end{pmatrix}$$

Q9

We have $A = \begin{pmatrix} 3 & 0 & 2 \\ 1 & 5 & 9 \\ 6 & 4 & 7 \end{pmatrix}$.

To get the inverse we will proceed by augmented matrix with elementary row transformation process is as follow:

$$(A|I) \rightarrow (I|A^{-1})$$

Let us proceed

$$\left(\begin{array}{ccc|ccc} 3 & 0 & 2 & 1 & 0 & 0 \\ 1 & 5 & 9 & 0 & 1 & 0 \\ 6 & 4 & 7 & 0 & 0 & 1 \end{array}\right)$$

$$\xrightarrow{R_2 \leftrightarrow R_1} \left(\begin{array}{ccc|ccc} 1 & 5 & 9 & 0 & 1 & 0 \\ 3 & 0 & 2 & 1 & 0 & 0 \\ 6 & 4 & 7 & 0 & 0 & 1 \end{array}\right) \xrightarrow{R_3 \rightarrow R_3 - 2R_2} \left(\begin{array}{ccc|ccc} 1 & 5 & 9 & 0 & 1 & 0 \\ 3 & 0 & 2 & 1 & 0 & 0 \\ 0 & 4 & 3 & -2 & 0 & 1 \end{array}\right)$$

$$\xrightarrow{R_2 \rightarrow R_2 - 3R_1} \left(\begin{array}{ccc|ccc} 1 & 5 & 9 & 0 & 1 & 0 \\ 0 & -15 & -25 & 1 & -3 & 0 \\ 0 & 4 & 3 & -2 & 0 & 1 \end{array}\right)$$

$$\xrightarrow{R_2 \rightarrow R_2 + 4R_1} \left(\begin{array}{ccc|ccc} 1 & 5 & 9 & 0 & 1 & 0 \\ 0 & 1 & -13 & -7 & -3 & 4 \\ 0 & 4 & 3 & -2 & 0 & 1 \end{array}\right)$$

$$\xrightarrow{R_1 \rightarrow R_1 - R_2} \left(\begin{array}{ccc|ccc} 1 & 1 & 6 & 2 & 1 & -1 \\ 0 & 1 & -13 & -7 & -3 & 4 \\ 0 & 4 & 3 & -2 & 0 & 1 \end{array}\right)$$

$$\xrightarrow{R_3 \to R_3 - 4R_2} \left(\begin{array}{ccc|ccc} 1 & 1 & 6 & 2 & 1 & -1 \\ 0 & 1 & -13 & -7 & -3 & 4 \\ 0 & 0 & 55 & 26 & 12 & -15 \end{array}\right)$$

$$\xrightarrow{R_3 \to \frac{1}{55} R_3} \left(\begin{array}{ccc|ccc} 1 & 1 & 6 & 2 & 1 & -1 \\ 0 & 1 & -13 & -7 & -3 & 4 \\ 0 & 0 & 1 & \frac{26}{55} & \frac{12}{55} & -\frac{15}{55} \end{array}\right)$$

$$\xrightarrow{R_2 \to R_2 + 13R_3} \left(\begin{array}{ccc|ccc} 1 & 1 & 6 & 2 & 1 & -1 \\ 0 & 1 & 0 & -\frac{47}{55} & \frac{9}{55} & \frac{25}{55} \\ 0 & 0 & 1 & \frac{26}{55} & \frac{12}{55} & -\frac{15}{55} \end{array}\right)$$

$$\xrightarrow{R_1 \to R_1 - R_2} \left(\begin{array}{ccc|ccc} 1 & 0 & 6 & \frac{157}{55} & \frac{64}{55} & -\frac{80}{55} \\ 0 & 1 & 0 & -\frac{47}{55} & \frac{9}{55} & \frac{25}{55} \\ 0 & 0 & 1 & \frac{26}{55} & \frac{12}{55} & -\frac{15}{55} \end{array}\right)$$

$$\xrightarrow{R_1 \to R_1 - 6R_3} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{55} & \frac{-8}{55} & -\frac{10}{55} \\ 0 & 1 & 0 & -\frac{47}{55} & \frac{9}{55} & \frac{25}{55} \\ 0 & 0 & 1 & \frac{26}{55} & \frac{12}{55} & -\frac{15}{55} \end{array}\right)$$

Thus we got the inverse of the matrix as

$$A^{-1} = \begin{pmatrix} \frac{1}{55} & \frac{-8}{55} & -\frac{10}{55} \\ -\frac{47}{55} & \frac{9}{55} & \frac{25}{55} \\ \frac{26}{55} & \frac{12}{55} & -\frac{15}{55} \end{pmatrix}$$

Q10

We have $A = \begin{pmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{pmatrix}$.

To get the inverse we will proceed by augmented matrix with elementary row transformation process is as follow:

$$(A|I) \to (I|A^{-1})$$

Let us proceed

$$\left(\begin{array}{ccc|ccc} 1 & 2 & -3 & 1 & 0 & 0 \\ 2 & 3 & 2 & 0 & 1 & 0 \\ 3 & -3 & -4 & 0 & 0 & 1 \end{array}\right)$$

$$\xrightarrow{R_3 \to R_3 - R_2} \left(\begin{array}{ccc|ccc} 1 & 2 & -3 & 1 & 0 & 0 \\ 2 & 3 & 2 & 0 & 1 & 0 \\ 1 & -6 & -3 & 0 & -1 & 1 \end{array}\right)$$

$$\xrightarrow{R_2 \to R_2 - R_1} \left(\begin{array}{ccc|ccc} 1 & 2 & -3 & 1 & 0 & 0 \\ 1 & 1 & 5 & -1 & 1 & 0 \\ 1 & -6 & -3 & 0 & -1 & 1 \end{array}\right)$$

$$\xrightarrow{R_3 \to R_3 - R_2} \left(\begin{array}{ccc|ccc} 1 & 2 & -3 & 1 & 0 & 0 \\ 1 & 1 & 5 & -1 & 1 & 0 \\ 0 & -7 & -11 & 1 & -2 & 1 \end{array}\right)$$

$$\xrightarrow{R_2 \to R_2 - R_1} \left(\begin{array}{ccc|ccc} 1 & 2 & -3 & 1 & 0 & 0 \\ 0 & -1 & 8 & -2 & 1 & 0 \\ 0 & -7 & -11 & 1 & -2 & 1 \end{array}\right)$$

$$\xrightarrow{R_2 \to -R_2} \left(\begin{array}{ccc|ccc} 1 & 2 & -3 & 1 & 0 & 0 \\ 0 & 1 & -8 & 2 & -1 & 0 \\ 0 & -7 & -11 & 1 & -2 & 1 \end{array}\right)$$

$$\xrightarrow{R_3 \to R_3 + 7R_2} \left(\begin{array}{ccc|ccc} 1 & 2 & -3 & 1 & 0 & 0 \\ 0 & 1 & -8 & 2 & -1 & 0 \\ 0 & 0 & -67 & 15 & -9 & 1 \end{array}\right)$$

$$\xrightarrow{R_3 \to -\frac{1}{67}R_3} \left(\begin{array}{ccc|ccc} 1 & 2 & -3 & 1 & 0 & 0 \\ 0 & 1 & -8 & 2 & -1 & 0 \\ 0 & 0 & 1 & -\frac{15}{67} & \frac{9}{67} & -\frac{1}{67} \end{array}\right)$$

$$\xrightarrow{R_2 \to R_2 + 8R_3} \left(\begin{array}{ccc|ccc} 1 & 2 & -3 & 1 & 0 & 0 \\ 0 & 1 & 0 & \frac{14}{67} & \frac{5}{67} & -\frac{8}{67} \\ 0 & 0 & 1 & -\frac{15}{67} & \frac{9}{67} & -\frac{1}{67} \end{array}\right)$$

$$\xrightarrow{R_1 \to R_1 - 2R_2} \left(\begin{array}{ccc|ccc} 1 & 0 & -3 & \frac{39}{67} & -\frac{10}{67} & \frac{16}{67} \\ 0 & 1 & 0 & \frac{14}{67} & \frac{5}{67} & -\frac{8}{67} \\ 0 & 0 & 1 & -\frac{15}{67} & \frac{9}{67} & -\frac{1}{67} \end{array}\right)$$

$$\xrightarrow{R_1 \to R_1 + 3R_3} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{6}{67} & \frac{17}{67} & \frac{13}{67} \\ 0 & 1 & 0 & \frac{14}{67} & \frac{5}{67} & -\frac{8}{67} \\ 0 & 0 & 1 & -\frac{15}{67} & \frac{9}{67} & -\frac{1}{67} \end{array}\right)$$

Thus we got the inverse of the matrix as

$$A^{-1} = \begin{pmatrix} \frac{-6}{67} & \frac{17}{67} & \frac{13}{67} \\ \frac{14}{67} & \frac{5}{67} & -\frac{8}{67} \\ -\frac{15}{67} & \frac{9}{67} & -\frac{1}{67} \end{pmatrix}$$

Q11

We have $A = \begin{pmatrix} 3 & -1 & -2 \\ 2 & 0 & -1 \\ 3 & -5 & 0 \end{pmatrix}$.

To get the inverse we will proceed by augmented matrix with elementary row transformation process is as follow:

$$(A|I) \to (I|A^{-1})$$

Let us proceed

$$\left(\begin{array}{ccc|ccc} 3 & -1 & -2 & 1 & 0 & 0 \\ 2 & 0 & -1 & 0 & 1 & 0 \\ 3 & -5 & 0 & 0 & 0 & 1 \end{array}\right)$$

$$\xrightarrow{R_3 \to R_3 - R_1} \left(\begin{array}{ccc|ccc} 3 & -1 & -2 & 1 & 0 & 0 \\ 2 & 0 & -1 & 0 & 1 & 0 \\ 0 & -4 & 2 & -1 & 0 & 1 \end{array}\right)$$

$$\xrightarrow{R_1 \to R_1 - R_2} \left(\begin{array}{ccc|ccc} 1 & -1 & -1 & 1 & -1 & 0 \\ 1 & 1 & 5 & -1 & 1 & 0 \\ 1 & -6 & -3 & 0 & -1 & 1 \end{array}\right)$$

$$\xrightarrow{R_2 \to R_2 - 2R_1} \left(\begin{array}{ccc|ccc} 1 & -1 & -1 & 1 & -1 & 0 \\ 0 & 2 & 1 & -2 & 3 & 0 \\ 1 & -6 & -3 & 0 & -1 & 1 \end{array}\right)$$

$$\xrightarrow{R_3 \to R_3 + 2R_2} \left(\begin{array}{ccc|ccc} 1 & -1 & -1 & 1 & -1 & 0 \\ 0 & 2 & 1 & -2 & 3 & 0 \\ 0 & 0 & 4 & -5 & 6 & 1 \end{array}\right)$$

$$\xrightarrow{R_3 \to \frac{1}{4}R_4} \left(\begin{array}{ccc|ccc} 1 & -1 & -1 & 1 & -1 & 0 \\ 0 & 2 & 1 & -2 & 3 & 0 \\ 0 & 0 & 1 & -\frac{5}{4} & \frac{6}{4} & \frac{1}{4} \end{array}\right)$$

$$\xrightarrow{R_2 \to R_2 - R_3} \left(\begin{array}{ccc|ccc} 1 & -1 & -1 & 1 & -1 & 0 \\ 0 & 2 & 0 & -\frac{3}{4} & \frac{6}{4} & -\frac{1}{4} \\ 0 & 0 & 1 & -\frac{5}{4} & \frac{6}{4} & \frac{1}{4} \end{array}\right)$$

$$\xrightarrow{R_2 \to \frac{1}{2}R_2} \left(\begin{array}{ccc|ccc} 1 & -1 & -1 & 1 & -1 & 0 \\ 0 & 1 & 0 & -\frac{3}{8} & \frac{6}{8} & -\frac{1}{8} \\ 0 & 0 & 1 & -\frac{5}{4} & \frac{6}{4} & \frac{1}{4} \end{array}\right)$$

$$\xrightarrow{R_1 \to R_1 + R_2} \left(\begin{array}{ccc|ccc} 1 & 0 & -1 & \frac{5}{8} & -\frac{2}{5} & -\frac{1}{8} \\ 0 & 1 & 0 & -\frac{3}{8} & \frac{6}{8} & -\frac{1}{8} \\ 0 & 0 & 1 & -\frac{5}{4} & \frac{6}{4} & \frac{1}{4} \end{array}\right)$$

$$\xrightarrow{R_1 \to R_1 + R_3} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{-5}{8} & \frac{10}{8} & \frac{1}{8} \\ 0 & 1 & 0 & -\frac{3}{8} & \frac{6}{8} & -\frac{1}{8} \\ 0 & 0 & 1 & -\frac{5}{4} & \frac{6}{4} & \frac{1}{4} \end{array}\right)$$

Thus we have the inverse of the matrix as $A^{-1} = \begin{pmatrix} \frac{-5}{8} & \frac{10}{8} & \frac{1}{8} \\ -\frac{3}{8} & \frac{6}{8} & -\frac{1}{8} \\ -\frac{5}{4} & \frac{6}{4} & \frac{1}{4} \end{pmatrix}$

Q12

We have $A = \begin{pmatrix} 1 & 3 & -2 \\ -3 & 0 & -1 \\ 2 & 1 & 0 \end{pmatrix}$.

To get the inverse we will proceed by augmented matrix with elementary row transformation process is as follow:

$$(A|I) \to (I|A^{-1})$$

Let us proceed

$$\left(\begin{array}{ccc|ccc} 1 & 3 & -2 & 1 & 0 & 0 \\ -3 & 0 & -1 & 0 & 1 & 0 \\ 2 & 1 & 0 & 0 & 0 & 1 \end{array}\right)$$

$$\xrightarrow{R_3 \to R_3 - 2R_1} \left(\begin{array}{ccc|ccc} 1 & 3 & -2 & 1 & 0 & 0 \\ -3 & 0 & -1 & 0 & 1 & 0 \\ 0 & -5 & 4 & -2 & 0 & 1 \end{array}\right)$$

$$\xrightarrow{R_2 \to R_2 + 3R_1} \left(\begin{array}{ccc|ccc} 1 & 3 & -2 & 1 & 0 & 0 \\ 0 & 9 & -7 & 3 & 1 & 0 \\ 0 & -5 & 4 & -2 & 0 & 1 \end{array}\right)$$

$$\xrightarrow{R_2 \to R_2 + R_3} \left(\begin{array}{ccc|ccc} 1 & 3 & -2 & 1 & 0 & 0 \\ 0 & 4 & -3 & 1 & 1 & 1 \\ 0 & -5 & 4 & -2 & 0 & 1 \end{array}\right)$$

$$\xrightarrow{R_3 \to -R_3} \left(\begin{array}{ccc|ccc} 1 & 3 & -2 & 1 & 0 & 0 \\ 0 & 4 & -3 & 1 & 1 & 1 \\ 0 & 5 & -4 & 2 & 0 & -1 \end{array}\right)$$

$$\xrightarrow{R_3 \to R_3 - R_2} \left(\begin{array}{ccc|ccc} 1 & 3 & -2 & 1 & 0 & 0 \\ 0 & 4 & -3 & 1 & 1 & 1 \\ 0 & 1 & -1 & 1 & -1 & -2 \end{array}\right)$$

$$\xrightarrow{R_2 \to R_2 - 4R_3} \left(\begin{array}{ccc|ccc} 1 & 3 & -2 & 1 & 0 & 0 \\ 0 & 0 & 1 & -3 & 5 & 9 \\ 0 & 1 & -1 & 1 & -1 & -2 \end{array}\right)$$

$$\xrightarrow{R_2 \leftrightarrow R_3} \left(\begin{array}{ccc|ccc} 1 & 3 & -2 & 1 & 0 & 0 \\ 0 & 1 & -1 & 1 & -1 & -2 \\ 0 & 0 & 1 & -3 & 5 & 9 \end{array}\right)$$

$$\xrightarrow{R_2 \to R_2 + R_3} \left(\begin{array}{ccc|ccc} 1 & 3 & -2 & 1 & 0 & 0 \\ 0 & 1 & 0 & -2 & 4 & 7 \\ 0 & 0 & 1 & -3 & 5 & 9 \end{array}\right)$$

$$\xrightarrow{R_1 \to R_1 + 2R_3} \left(\begin{array}{ccc|ccc} 1 & 3 & 0 & -5 & 10 & 18 \\ 0 & 1 & 0 & -2 & 4 & 7 \\ 0 & 0 & 1 & -3 & 5 & 9 \end{array}\right)$$

$$\xrightarrow{R_1 \to R_1 - 3R_2} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -2 & -3 \\ 0 & 1 & 0 & -2 & 4 & 7 \\ 0 & 0 & 1 & -3 & 5 & 9 \end{array}\right)$$

Thus we have the inverse of the matrix as

$$A^{-1} = \begin{pmatrix} 1 & -2 & -3 \\ -2 & 4 & 7 \\ -3 & 5 & 9 \end{pmatrix}$$

Q13

We have $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{pmatrix}$.

To get the inverse we will proceed by augmented matrix with elementary row transformation process is as follow:

$$(A|I) \to (I|A^{-1})$$

Let us proceed

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 2 & 5 & 7 & 0 & 1 & 0 \\ -2 & -4 & -5 & 0 & 0 & 1 \end{array}\right)$$

$$\xrightarrow{R_2 \to R_2 - 2R_1} \left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 1 & -2 & 1 & 0 \\ -2 & -4 & -5 & 0 & 0 & 1 \end{array}\right)$$

$$\xrightarrow{R_3 \to R_3 + 2R_1} \left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 1 & -2 & 1 & 0 \\ 0 & 0 & 1 & 2 & 0 & 1 \end{array}\right)$$

$$\xrightarrow{R_2 \to R_2 - R_3} \left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 0 & -4 & 1 & -1 \\ 0 & 0 & 1 & 2 & 0 & 1 \end{array}\right)$$

$$\xrightarrow{R_1 \to R_1 - 2R_2} \left(\begin{array}{ccc|ccc} 1 & 0 & 3 & 9 & -2 & 2 \\ 0 & 1 & 0 & -4 & 1 & -1 \\ 0 & 0 & 1 & 2 & 0 & 1 \end{array}\right)$$

$$\xrightarrow{R_1 \to R_1 - 3R_3} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & -2 & -1 \\ 0 & 1 & 0 & -4 & 1 & -1 \\ 0 & 0 & 1 & 2 & 0 & 1 \end{array}\right)$$

Thus we have the inverse of the matrix as

$$A^{-1} = \begin{pmatrix} 3 & -2 & -1 \\ -4 & 1 & -1 \\ 2 & 0 & 1 \end{pmatrix}$$

Q14

We have $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{pmatrix}$.

To get the inverse we will proceed by augmented matrix with elementary row transformation process is as follow:

$$(A|I) \to (I|A^{-1})$$

Let us proceed

$$\left(\begin{array}{ccc|ccc} 3 & 0 & -1 & 1 & 0 & 0 \\ 2 & 3 & 0 & 0 & 1 & 0 \\ 0 & 4 & 1 & 0 & 0 & 1 \end{array}\right) \xrightarrow{R_1 \to R_1 - R_2} \left(\begin{array}{ccc|ccc} 1 & -3 & -1 & 1 & -1 & 0 \\ 2 & 3 & 0 & 0 & 1 & 0 \\ 0 & 4 & 1 & 0 & 0 & 1 \end{array}\right)$$

$$\xrightarrow{R_2 \to R_2 - 2R_1} \left(\begin{array}{ccc|ccc} 1 & -3 & -1 & 1 & -1 & 0 \\ 0 & 9 & 2 & -2 & 3 & 0 \\ 0 & 4 & 1 & 0 & 0 & 1 \end{array}\right)$$

$$\xrightarrow{R_2 \to R_2 - 2R_3} \left(\begin{array}{ccc|ccc} 1 & -3 & -1 & 1 & -1 & 0 \\ 0 & 1 & 0 & -2 & 3 & -2 \\ 0 & 4 & 1 & 0 & 0 & 1 \end{array}\right)$$

$$\xrightarrow{R_3 \to R_3 - 4R_2} \left(\begin{array}{ccc|ccc} 1 & -3 & -1 & 1 & -1 & 0 \\ 0 & 1 & 0 & -2 & 3 & -2 \\ 0 & 0 & 1 & 8 & -12 & 9 \end{array}\right)$$

$$\xrightarrow{R_1 \to R_1 + R_3} \left(\begin{array}{ccc|ccc} 1 & -3 & 0 & 9 & -13 & 9 \\ 0 & 1 & 0 & -2 & 3 & -2 \\ 0 & 0 & 1 & 8 & -12 & 9 \end{array}\right)$$

$$\xrightarrow{R_1 \to R_1 + 3R_2} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & -4 & 3 \\ 0 & 1 & 0 & -2 & 3 & -2 \\ 0 & 0 & 1 & 8 & -12 & 9 \end{array}\right)$$

Thus we have the inverse of the matrix as

$$A^{-1} = \begin{pmatrix} 3 & -2 & -1 \\ -4 & 1 & -1 \\ 2 & 0 & 1 \end{pmatrix}$$

Q15

We have $A = \begin{pmatrix} -1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{pmatrix}$.

To get the inverse we will proceed by augmented matrix with elementary row transformation process is as follow:

$$(A|I) \rightarrow (I|A^{-1})$$

Let us proceed

$$\begin{pmatrix} -1 & 1 & 2 & 1 & 0 & 0 \\ 1 & 2 & 3 & 0 & 1 & 0 \\ 3 & 1 & 1 & 0 & 0 & 1 \end{pmatrix} \xrightarrow{R_1 \to R_1 + R_2} \begin{pmatrix} 0 & 3 & 5 & 1 & 1 & 0 \\ 1 & 2 & 3 & 0 & 1 & 0 \\ 3 & 1 & 1 & 0 & 0 & 1 \end{pmatrix}$$

$$\xrightarrow{R_3 \to R_3 - 3R_2} \begin{pmatrix} 0 & 3 & 5 & 1 & 1 & 0 \\ 1 & 2 & 3 & 0 & 1 & 0 \\ 0 & -5 & -8 & 0 & -3 & 1 \end{pmatrix}$$

$$\xrightarrow{R_1 \leftrightarrow R_2} \begin{pmatrix} 1 & 2 & 3 & 0 & 1 & 0 \\ 0 & 3 & 5 & 1 & 1 & 0 \\ 0 & -5 & -8 & 0 & -3 & 1 \end{pmatrix}$$

$$\xrightarrow{R_2 \to R_2 + R_3} \begin{pmatrix} 1 & 2 & 3 & 0 & 1 & 0 \\ 0 & -2 & -3 & 1 & -2 & 1 \\ 0 & -5 & -8 & 0 & -3 & 1 \end{pmatrix}$$

$$\xrightarrow{R_2 \to R_2 + R_1} \begin{pmatrix} 1 & 2 & 3 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & -1 & 1 \\ 0 & -5 & -8 & 0 & -3 & 1 \end{pmatrix}$$

$$\xrightarrow{R_1 \to -R_2 + R_1} \begin{pmatrix} 0 & 2 & 3 & -1 & 2 & -1 \\ 1 & 0 & 0 & 1 & -1 & 1 \\ 0 & -5 & -8 & 0 & -3 & 1 \end{pmatrix}$$

$$\xrightarrow{R_3 \to R_3 + 3R_1} \begin{pmatrix} 0 & 2 & 3 & -1 & 2 & -1 \\ 1 & 0 & 0 & 1 & -1 & 1 \\ 0 & 1 & 1 & -3 & 3 & -2 \end{pmatrix}$$

$$\xrightarrow{R_1 \to R_1 - 2R_3} \left(\begin{array}{ccc|ccc} 0 & 0 & 1 & 5 & -4 & 3 \\ 1 & 0 & 0 & 1 & -1 & 1 \\ 0 & 1 & 1 & -3 & 3 & -2 \end{array}\right)$$

$$\xrightarrow{R_3 \to R_3 - R_1} \left(\begin{array}{ccc|ccc} 0 & 0 & 1 & 5 & -4 & 3 \\ 1 & 0 & 0 & 1 & -1 & 1 \\ 0 & 1 & 0 & -8 & 7 & -5 \end{array}\right)$$

$$\xrightarrow{R_1 \leftrightarrow R_2} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 5 & -4 & 3 \\ 0 & 1 & 0 & -8 & 7 & -5 \end{array}\right)$$

$$\xrightarrow{R_3 \leftrightarrow R_2} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -1 & 1 \\ 0 & 1 & 0 & -8 & 7 & -5 \\ 0 & 0 & 1 & 5 & -4 & 3 \end{array}\right)$$

Thus we have the inverse of the matrix as

$$A^{-1} = \begin{pmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ 5 & -4 & 3 \end{pmatrix}$$

Exercise:5F

Q1

We have $a_{ij} = \frac{1}{2}(i - 2j)^2$.

Now

$a_{11} = \frac{(1-2(1))^2}{2} = \frac{1}{2}$, $a_{12} = \frac{(1-2(2))^2}{2} = \frac{9}{2}$;

$a_{21} = \frac{(2-2(1))^2}{2} = 0$, $a_{22} = \frac{(2-2(2))^2}{2} = 2$;

$a_{31} = \frac{(3-2(1))^2}{2} = \frac{1}{2}$, $a_{32} = \frac{(3-2(2))^2}{2} = \frac{1}{2}$.

Thus the required matrix of the order 3×2 with satisfying the given equation is given by

$$\begin{pmatrix} \frac{1}{2} & \frac{9}{2} \\ 0 & 2 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}.$$

Q2

We have $a_{ij} = \frac{1}{2}|-3i+j|^2$.

Now

$a_{11} = \frac{|-3(1)+1|}{2} = 1$, $a_{12} = \frac{|-3(1)+2|}{2} = \frac{9}{2}$, $a_{13} = \frac{|-3(1)+3|}{2} = \frac{9}{2}$;

$a_{21} = \frac{|-3(2)+1|}{2} = \frac{5}{2}$, $a_{22} = \frac{|-3(2)+2|}{2} = 2$, $a_{23} = \frac{|-3(2)+3|}{2} = \frac{3}{2}$;

Thus the required matrix of the order 3×2 with satisfying the given equation is given by

$$\begin{pmatrix} 1 & \frac{1}{2} & 0 \\ \frac{5}{2} & 2 & \frac{3}{2} \end{pmatrix}.$$

Q3

We have $\begin{pmatrix} x+2y & -y \\ 3x & 4 \end{pmatrix} = \begin{pmatrix} -4 & 3 \\ 6 & 4 \end{pmatrix}$.

Now from the equality of matrices we can write

$$3x = 6\ ;\ -y = 3$$

Therefore $x = 2\ ; y = -3$.

Q4

We have $2\begin{pmatrix} 1 & 3 \\ 0 & x \end{pmatrix} + \begin{pmatrix} y & 0 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 5 & 6 \\ 1 & 8 \end{pmatrix}$.

We can use the addition rule and get

$$\begin{pmatrix} 2+y & 6 \\ 1 & 2x+2 \end{pmatrix} = \begin{pmatrix} 5 & 6 \\ 1 & 8 \end{pmatrix}$$

Now from the equality of matrices we can write

$$2 + y = 5\ ;\ 2x + 2 = 8$$

Therefore $x = 3\ ; y = 3$.

Q5

We have $x\begin{pmatrix} 2 \\ 3 \end{pmatrix} + y\begin{pmatrix} -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 10 \\ 5 \end{pmatrix}$.

We can use the addition rule and get

$$\begin{pmatrix} 2x - y \\ 3x + y \end{pmatrix} = \begin{pmatrix} 10 \\ 5 \end{pmatrix}$$

Now from the equality of matrices we can write

$$2x - y = 10\ ;\ 3x + y = 5$$

Therefore $x = 3\ ; y = -4$.

Q6

We have $\begin{pmatrix} x & 3x - y \\ 2x + z & 3y - w \end{pmatrix} = \begin{pmatrix} 3 & 2 \\ 4 & 7 \end{pmatrix}$.

Now from the equality of matrices we can write

$$x = 3; 2x + z = 4\ ;\ 3x - y = 2; 3y - w = 7$$

Therefore $x = 3\ ; y = 7; z = -2\ ; w = 14$.

Q7

We have $\begin{pmatrix} x & 6 \\ -1 & 2w \end{pmatrix} + \begin{pmatrix} 4 & x + y \\ z + w & 3 \end{pmatrix} = 3\begin{pmatrix} x & y \\ z & w \end{pmatrix}$.

We can use the addition rule and get

$$\begin{pmatrix} x + 4 & 6 + x + y \\ -1 + z + w & 2w + 3 \end{pmatrix} = \begin{pmatrix} 3x & 3y \\ 3z & 3w \end{pmatrix}$$

Now from the equality of matrices we can write

$$x + 4 = 3x\ ;\ 2w + 3 = 3w\ ; 6 + x + y = 3y\ ;\ -1 + z + w = 3z$$

Therefore $x = 2\ ; y = 4\ ; z = 1\ ; w = 3$.

Q8

We have $A = diag\ (3\ \ -2\ \ 5) = \begin{pmatrix} 3 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 5 \end{pmatrix}$; $B =$

$diag\ (1\ 3\ \ -4) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & -4 \end{pmatrix}$.

We can use the addition rule and get$A + B$ as follow:

$$A + B = \begin{pmatrix} 3 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 5 \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & -4 \end{pmatrix} = \begin{pmatrix} 4 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Q9

We have $\cos\theta \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} + \sin\theta \begin{pmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{pmatrix} = I$.

We can use the addition rule and scalar multiplication of matrix as follow:

$$\text{LHL} = \begin{pmatrix} \cos^2\theta & \sin\theta\cos\theta \\ -\sin\theta\cos\theta & \cos^2\theta \end{pmatrix}$$

$$+ \quad \begin{pmatrix} \sin^2\theta & -\sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= I$$

$= RHL$

Q10

We have $A = \begin{pmatrix} 1 & -5 \\ -3 & 2 \\ 4 & -2 \end{pmatrix}$; $B = \begin{pmatrix} 3 & 1 \\ 2 & -1 \\ -2 & 3 \end{pmatrix}$. $and\ A + B + C = O$

We can use the addition rule and get $C = O - (A + B)$ as follow:

$$A + B = \begin{pmatrix} 1 & -5 \\ -3 & 2 \\ 4 & -2 \end{pmatrix} + \begin{pmatrix} 3 & 1 \\ 2 & -1 \\ -2 & 3 \end{pmatrix} = \begin{pmatrix} 4 & -4 \\ -1 & 1 \\ 2 & 1 \end{pmatrix}$$

$$\text{And } C = O - (A + B) = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} - \begin{pmatrix} 4 & -4 \\ -1 & 1 \\ 2 & 1 \end{pmatrix} = \begin{pmatrix} -4 & 4 \\ 1 & -1 \\ -2 & -1 \end{pmatrix}$$

Q11

We have $A = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$.

We can use the addition rule and get $A + A^T = I_2$ as follow:

$$A + A^T = I$$

$$\begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} + \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 2\cos\alpha & 0 \\ 0 & 2\cos\alpha \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

From the equality of matrices we get

$2\cos\alpha = 1$

Therefore $\cos\alpha = \frac{1}{2}$

Thus finally we have

$\alpha \in (0, \pi)$, $\alpha = \frac{\pi}{3}$

Q12

We have $\begin{pmatrix} 2 & -3 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$.

We can use the multiplication rule as follow:

$$\begin{pmatrix} 2 & -3 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

$$\begin{pmatrix} 2x - 3y \\ x + y \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

From the equality of matrices we get

$2x - 3y = 1$; $x + y = 3$

Thus finally we have

$$x = 2 \; ; y = 1$$

Q13

We have $\begin{pmatrix} x & y \\ 3y & x \end{pmatrix}\begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$.

We can use the multiplication rule as follow:

$$\begin{pmatrix} x & y \\ 3y & x \end{pmatrix}\begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$$

$$\begin{pmatrix} x + 2y \\ 3y + 2x \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$$

From the equality of matrices we get

$x + 2y = 3$; $3y + 2x = 5$

Thus finally we have

$$x = 1 \; ; y = 1$$

Q14

We have $\begin{pmatrix} 4 & 5 \\ 1 & 8 \end{pmatrix}$.

The transpose of the matrix is an operation of making interchange of elements by the rule on positioned element a_{ji} shifted to new position a_{ji}.

The symmetric matrix is defined as similarity of transpose of matrix with it self. i.e, $A^T = A$.

The skew-symmetric matrix is defined as similarity of transpose of matrix with it self. i.e, $A^T = -A$.

$A^T = \begin{pmatrix} 4 & 5 \\ 1 & 8 \end{pmatrix}^T = \begin{pmatrix} 4 & 1 \\ 5 & 8 \end{pmatrix}$

Thus finally we have

$$A + A^T = \begin{pmatrix} 4 & 5 \\ 1 & 8 \end{pmatrix} + \begin{pmatrix} 4 & 1 \\ 5 & 8 \end{pmatrix} = \begin{pmatrix} 8 & 6 \\ 6 & 16 \end{pmatrix}$$

Here $(A + A^T)^T = \begin{pmatrix} 8 & 6 \\ 6 & 16 \end{pmatrix}^T = \begin{pmatrix} 8 & 6 \\ 6 & 16 \end{pmatrix} = A + A^T$.

Thus $A + A^T$ is symmetric.

Q15

We have $\begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix}$.

The transpose of the matrix is an operation of making interchange of elements by the rule on positioned element a_{ji}shifted to new position a_{ji}.

The symmetric matrix is defined as similarity of transpose of matrix with it self. i.e, $A^T = A$.

The skew-symmetric matrix is defined as similarity of transpose of matrix with it self. i.e, $A^T = -A$.

$$A^T = \begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix}^T = \begin{pmatrix} 2 & 4 \\ 3 & 5 \end{pmatrix}$$

Thus finally we have

$$A - A^T = \begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix} - \begin{pmatrix} 2 & 4 \\ 3 & 5 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

Here $(A - A^T)^T = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}^T = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = -\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = A - A^T$.

Thus $A + A^T$ is skew symmetric.

Q16

We have $A = \begin{pmatrix} 2 & -3 \\ 4 & 5 \end{pmatrix}, B = \begin{pmatrix} -1 & 2 \\ 0 & 3 \end{pmatrix}$ and $A + 2B + X = O$.

We can have the addition as

$$A + 2B = \begin{pmatrix} 2 & -3 \\ 4 & 5 \end{pmatrix} + 2\begin{pmatrix} -1 & 2 \\ 0 & 3 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 4 & 11 \end{pmatrix}$$

From the given equation we can rewrite and compute

$$X = O = (A + 2B) = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} - \begin{pmatrix} 0 & 1 \\ 4 & 11 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ -4 & -11 \end{pmatrix}$$

Q17

We have $A = \begin{pmatrix} 4 & 2 \\ 1 & 3 \end{pmatrix}, B = \begin{pmatrix} -2 & 1 \\ 3 & 2 \end{pmatrix}$ and $3A - 2B + X = O$.

We can have the addition as

$$3A + 2B = 3\begin{pmatrix} 4 & 2 \\ 1 & 3 \end{pmatrix} - 2\begin{pmatrix} -2 & 1 \\ 3 & 2 \end{pmatrix} = \begin{pmatrix} 16 & 4 \\ -3 & 5 \end{pmatrix}$$

From the given equation we can rewrite and compute

$$X = O = (A + 2B) = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} - \begin{pmatrix} 16 & 4 \\ -3 & 5 \end{pmatrix} = \begin{pmatrix} -16 & -4 \\ 3 & -5 \end{pmatrix}$$

Q18

We have $A = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}$.

The transpose of the matrix is an operation of making interchange of elements by the rule on positioned element a_{ij} shifted to new position a_{ji}.

(i) The transpose of A is given by

$$A^T = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}^T = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$$

(ii) The multiplication of matrices A and B is given by

$$A^T A = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}\begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$$

Q19

We have A and B are symmetric matrices. Therefore $A^T = A$ and $B^T = B$.

The transpose of the matrix is an operation of making interchange of elements by the rule on positioned element a_{ji}shifted to new position a_{ji}.

The symmetric matrix is defined as similarity of transpose of matrix with it self. i.e, $A^T = A$.

The skew-symmetric matrix is defined as similarity of transpose of matrix with it self. i.e, $A^T = -A$.

We have some properties of transpose of matrices:

(i) $(A + B)^T = A^T + B^T$(ii) $(AB)^T = B^T A^T$

Let us check

$$(AB - BA)^T = (AB)^T - (BA)^T = B^T A^T - A^T B^T = BA - AB$$
$$= -(AB - BA)$$

Thus we are having $(AB - BA)$ is skew symmetric.

Q20

We have $A = \begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix}$.

Now addition/subtraction of two matrices is possible if order of both the matrices are same and multiplication of two matrices is possible if number of columns in left matrix is equals to the number of rows in right matrix

Next let us discuss the order of the matrices which are given the order of matrix A is 2×2.

Thus the multiplications we can proceed as

(i) The multiplication A^2 is possible.

$$A^2 = AA = \begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 7 & 12 \\ 4 & 7 \end{pmatrix}$$

(ii) We have the equation $f(x) = x^2 - 4x + 1$.

$$f(A) = A^2 - 4A + I_2 = \begin{pmatrix} 7 & 12 \\ 4 & 7 \end{pmatrix} - 2\begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix} + 3\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} =$$

$$\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}.$$

Q21

The transpose of the matrix is an operation of making interchange of elements by the rule on positioned element a_{ji} shifted to new position a_{ji}.

The symmetric matrix is defined as similarity of transpose of matrix with it self. i.e, $A^T = A$.

The skew-symmetric matrix is defined as similarity of transpose of matrix with it self. i.e, $A^T = -A$.

Suppose A is symmetric and skew symmetric matrix. Therefore $A^T = A \; and \; A^T = -A$.

Therefore

$$A^T = -A = -A^T$$

Thus we have

$$2A^T = O$$

Finally we have

$$A^T = O.$$