CHAPTER-16-Definite Integrals

Exercise-16A

Q1

$\int_1^3 x^4\,dx = \left[\frac{x^5}{5}\right]$

$= \frac{3^5}{5} - \frac{1}{5}$

$\frac{243-1}{5}$

$= \frac{243}{5}$

Q2

$\int_1^4 \sqrt{x}dx = \left[\frac{2}{3}x^{\frac{3}{2}}\right]$

$\frac{2}{3}\left[4^{\frac{3}{2}} - 1\right]$

$= \frac{14}{3}$

Q3

$\int_1^2 x^{-5}dx = \left[\frac{x^{-4}}{-4}\right]$

$\frac{2^{-4}}{-4} - \frac{1}{-4}$

$\frac{16-1}{64}$

$\frac{15}{64}$

Q4

$\int_0^{16} x^{\frac{3}{4}}dx = \left[\frac{4}{7}x^{\frac{7}{4}}\right]$

$= \frac{4}{7}\left[16^{\frac{7}{4}} - 1\right]$

$= \frac{512}{7}$

Q5

$\int_{-4}^{-1} \frac{dx}{x} = -[logx]$

=[log(-1)-log(-4)]

=-[log(-4)-log(-1)]

$-\left[log\left(\frac{-4}{-1}\right)\right]$

=-log 4

Q6

$\int_1^4 \frac{dx}{\sqrt{x}} = [2\sqrt{x}]$

=[2√4-2]

=[4-2]

=2

Q7

$\int_0^1 \frac{dx}{\sqrt[3]{x}} = \left[\frac{3}{2}x^{\frac{2}{3}}\right]$

$= \left[\frac{3}{2}1^{\frac{4}{3}} - 0\right]$

$= \frac{3}{2}$

Q8

$\int_1^8 \frac{dx}{x^{\frac{2}{3}}} = \left[\frac{3}{1}x^{\frac{1}{3}}\right]$

$= \left[3(8)^{\frac{1}{3}} - 3(1)^{\frac{1}{3}}\right]$

=[6-3]

=3

Q9

$\int_2^4 3dx = 3[x]$

=3[4-2]

=6

Q10

$\int_0^1 \frac{dx}{1+x^2} = [tan^{-1}x]$

=[$\tan^{-1}$ 1-$\tan^{-1}$ 0]

=π/4

Q11

$\int_0^\infty \frac{dx}{1+x^2} = [tan^{-1}]$

=[$\tan^{-1}$ ∞-$\tan^{-1}$ 0]

=π/2

Q12

$\int_0^1 \frac{dx}{\sqrt{1-x^2}} = [sin^{-1}x]$

$=[\sin^{-1} 1-\sin^{-1} 0]$

$= \frac{\pi}{2}$

Q13

$\int_0^{\frac{\pi}{6}} sec^2x\ dx = [tanx]$

$= \left[tan\left(\frac{\pi}{6}\right) - tan\ 0\right]$

$= \frac{1}{\sqrt{3}}$

Q14

$\int_{\frac{-\pi}{4}}^{\frac{\pi}{4}} cosec^2x\ dx = [-cotx]$

$\left[-cot\left(\frac{\pi}{4}\right) + cot\left(-\frac{\pi}{4}\right)\right]$

$\left[-cot\left(\frac{\pi}{4}\right) - cot\left(\frac{\pi}{4}\right)\right]$

=-2

Q15

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} cot^2 x dx = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (cosec^2 x -) dx$$

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (cosec^2 x - 1) dx = [-cotx - x]$$

$$= \left[-cot\left(\frac{\pi}{2}\right) - \frac{\pi}{2} + cot\left(\frac{\pi}{4}\right) + \frac{\pi}{4}\right]$$

$$\left[0 - \frac{\pi}{4} + 1\right]$$

$$\left[1 - \frac{\pi}{4}\right]$$

Q16

$$\int_0^{\frac{\pi}{4}} tan^2 x dx = \int_0^{\frac{\pi}{4}} (sec^2 x - 1) dx$$

$$\int_0^{\frac{\pi}{4}} (sec^2 x - 1) dx = [tanx - x]$$

$$\left[tan\left(\frac{\pi}{4}\right) - \frac{\pi}{4} - tan(0) - 0\right] = \left[1 - \frac{\pi}{4}\right]$$

Q17

$$\int_0^{\frac{\pi}{2}} sin^2 x dx = \int_0^{\frac{\pi}{2}} \frac{1}{2}(1 - co2x)dx$$

$$= \frac{1}{2}\left[x - \frac{sin2x}{2}\right]$$

$$= \frac{1}{2}\left[\frac{\pi}{2} - \frac{sin\pi}{2} - 0 + \frac{sin0}{2}\right]$$

$$= \frac{\pi}{4}$$

Q18

$$\int_{\frac{\pi}{2}}^{\frac{\pi}{4}} cos^2 x dx = \int_0^{\frac{\pi}{2}} (1 + co2x)dx$$

$$\frac{1}{2}\left[x + \frac{sin2x}{2}\right]$$

$$= \frac{1}{2}\left[\frac{\pi}{4} + \frac{sin\left(\frac{\pi}{2}\right)}{2} - 0 - \frac{sin0}{2}\right]$$

$$= \frac{\pi}{8} + \frac{1}{4}$$

Q19

$\int_0^{\frac{\pi}{3}} tanxdx = log|secx|$

$= log\left|sec\left(\frac{\pi}{3}\right)\right| - ln|cos0|$

=log|2|-log|1|

=log2

Q20

$\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} cosecxdx = -log|cosecx + cotx|$

$= -log\left|cosec\left(\frac{\pi}{4}\right) + cot\left(\frac{\pi}{4}\right)\right| + log\left|cosec\left(\frac{\pi}{6}\right) + cot\left(\frac{\pi}{6}\right)\right|$

=-log|√2+1|+log|2+√3|

Q21

$\int_0^{\frac{\pi}{3}} cos^3x\, dx = \frac{1}{4}\int_0^{\frac{\pi}{3}}(3cosx + cos3x)dx$

$\frac{1}{4}\int_0^{\frac{\pi}{3}}(3cosx - cos3x)dx = \frac{1}{4}\left[3sinx + \frac{sin3x}{3}\right]$

$\frac{1}{4}\left[3sin\left(\frac{\pi}{3}\right) + \frac{sin\pi}{3}\right] - \frac{1}{4}\left[3sin0 + \frac{sin0}{3}\right]$

$\frac{1}{4}\left[\frac{3\sqrt{3}}{2}\right]$

$= \frac{3\sqrt{3}}{8}$

Q22

$\int_0^{\frac{\pi}{2}} sin^3x\, dx = \frac{1}{4}\int_0^{\frac{\pi}{2}}(3sinx - sin3x)dx$

$\frac{1}{4}\int_0^{\frac{\pi}{2}}(3sinx - sin3x)dx = \frac{1}{4}\left[-3cosx + \frac{cos3x}{3}\right]$

$= \frac{1}{4}\left[-3cos\left(\frac{\pi}{2}\right) + \frac{cos\left(\frac{3\pi}{2}\right)}{3}\right] - \frac{1}{4}\left[-3cos0 + \frac{cos0}{3}\right]$

$= \frac{1}{4}\left[\frac{9-1}{3}\right]$

$= \frac{2}{3}$

Q23

$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{(1-3cosx)}{sin^2x}\, dx = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}}(cosec^2(x) - 3cosec(x)cot(x)dx$

$= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\big(cosec^2(x) - 3cosec(x)cot(x)\big)dx$

Q24

$\int_0^{\frac{\pi}{4}} \sqrt{1+cos2x}\, dx = \int_0^{\frac{\pi}{4}} \sqrt{2cos^2 x}\, dx$

$= \sqrt{2}[sinx]$

$\sqrt{2}\left[sin\left(\frac{\pi}{4}\right) - sin0\right]$

$\sqrt{2}\left[\frac{1}{\sqrt{2}}\right]$

=1

Q25

$\int_0^{\frac{\pi}{4}} \sqrt{1-sin2x}\, dx = \int_0^{\frac{\pi}{4}} \sqrt{sin^2 x + cos^2 x - 2sinxcos}\, dx$

$\int_0^{\frac{\pi}{4}} (cosx - sinx)dx$

=[sin x + cos x]

$= \left[cos\left(\frac{\pi}{4}\right) + sin\left(\frac{\pi}{4}\right) - cos0 - sin0\right]$

$= \left[+\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - 1\right]$

=[√2-1]

Q26

$$\int_{\frac{-\pi}{4}}^{\frac{\pi}{4}} \frac{dx}{1+sinx} = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{sec^2\left(\frac{x}{2}\right)}{\left(tan^2\left(\frac{x}{2}\right)+1\right)} dx$$

Let

$$u = \left(tan\left(\frac{x}{2}\right) + 1\right)$$

$$dx = \frac{2}{sec^2\left(\frac{x}{2}\right)} du$$

$$\int_{\frac{-\pi}{4}}^{\frac{\pi}{4}} \frac{dx}{1+sinx} = 2\int_{\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1}{u^2} du$$

$$= -\frac{2}{u}$$

$$= -\frac{2}{tan\left(\frac{x}{2}\right)+1}$$

=2

Q27

$$\int_0^{\frac{\pi}{4}} \frac{dx}{1+cos2x} = \int_0^{\frac{\pi}{4}} \frac{dx}{2cos^2x}$$

$$\int_0^{\frac{\pi}{4}} \frac{dx}{2cos^2x} = \int_0^{\frac{\pi}{4}} \frac{1}{2} sec^2x\, dx$$

$$\int_0^{\frac{\pi}{4}} \frac{1}{2} sec^2x dx = \frac{1}{2}[tanx]$$

$$= \frac{1}{2}\left[tan\left(\frac{\pi}{4}\right) - tan0\right]$$

$$-\frac{1}{2}[1] \quad = \frac{1}{2}$$

Q28

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{dx}{1-cos2x} = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{dx}{2sin^2x}$$

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{dx}{2sin^2x} = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{1}{2} cosec^2 x dx$$

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{1}{2} cosec^2 x dx = \frac{1}{2}[cotx]$$

$$= \frac{1}{2}\left[cot\left(\frac{\pi}{4}\right) - cot0\right]$$

$$= \frac{1}{2}[1]$$

$$= \frac{1}{2}$$

Q29

$$\int_0^{\frac{\pi}{4}} sin2xsin3xdx = \frac{1}{2}\int_0^{\frac{\pi}{4}}(cosx - cos5x)dx$$

$$-\frac{1}{2}\int_0^{\frac{\pi}{4}}(cox - cos5x)dx$$

$$= \frac{1}{2}\left[sinx - \frac{sin5x}{5}\right]$$

$$= \frac{1}{2}\left[sin\left(\frac{\pi}{4}\right) - \frac{sin\left(\frac{5\pi}{4}\right)}{5}\right] - \frac{1}{2}\left[sin(0) - \frac{sin(0)}{5}\right]$$

$= \frac{1}{2}\left[\frac{1}{\sqrt{2}} + \frac{1}{5\sqrt{2}}\right]$

$= \frac{3}{5\sqrt{2}}$

Q30

$\int_0^{\frac{\pi}{6}} cosxcos2xdx = \frac{1}{2}\int_0^{\frac{\pi}{6}}(cos3x + cosx)dx$

$= \frac{1}{2}\left[\frac{sin3x}{3} + sinx\right]$

$= \frac{1}{2}\left[\frac{sin\left(\frac{\pi}{2}\right)}{3} + sin\left(\frac{\pi}{6}\right)\right] - 0$

$= \frac{1}{2}\left[\frac{1}{3} + \frac{1}{2}\right]$

$= \frac{5}{12}$

Q31

$\int_0^{\pi} sin2xcos3xdx = \frac{1}{2}\int_0^{\pi}(sin5x - sinx)dx$

$= \frac{1}{2}\left[\frac{-cos5x}{5} + cosx\right]$

$= \frac{1}{2}\left[-\frac{cos(5\pi)}{5} + cos(\pi)\right] - \frac{1}{2}\left[-\frac{cos(0)}{5} + cos(0)\right]$

$= \frac{1}{2}\left[\frac{-(-1)}{5} - 1\right] - \frac{1}{2}\left[-\frac{1}{5} + 1\right]$

$= \frac{1}{2}\left[\frac{-4}{5} - \frac{4}{5}\right]$

$= \frac{1}{2}2\left(-\frac{4}{5}\right)$

$= -\frac{4}{5}$

Q32

$\int_0^{\frac{\pi}{2}} \sqrt{1 + sin(x)}dx = \int_0^{\frac{\pi}{2}} \sqrt{2}cos\left(\frac{2x-\pi}{4}\right)dx$

$= 2^{\frac{2}{3}} sin\left(\frac{2x-\pi}{4}\right)$

$= 2^{\frac{2}{3}}\left(0 - sin\left(-\frac{\pi}{4}\right)\right)$

$= \frac{2\sqrt{2}}{\sqrt{2}}$

=2

Q33

$\int_0^{\frac{\pi}{2}} \sqrt{1 + cos(x)}dx = \int_0^{\frac{\pi}{2}} \sqrt{2}cos\left(\frac{x}{2}\right)dx$

$= 2^{\frac{3}{2}} sin\left(\frac{x}{2}\right)$

$= 2^{\frac{3}{2}}\left(sin\left(\frac{\pi}{4}\right) - 0\right)$

$= \frac{2\sqrt{2}}{\sqrt{2}}$

=2

Q34

$\int_0^2 \left\{\frac{(x^4+1)}{x^2+1}\right\} dx = \int_0^2 \frac{x^4+2-1}{x^2+1} dx$

$= \int_0^2 \frac{x^4-1}{x^2+1} dx + \int_0^2 \frac{2}{x^2+1} dx$

$= \int_0^2 \frac{(x^2-1)(x^2+1)}{x^2+1} dx + \int_0^2 \frac{2}{x^2+1} dx$

$= \int_0^2 (x^2 - 1)dx + 2tan^{-1}x$

$= \left[\frac{x^3}{3} - x + 2tan^{-1}x\right]_0^2$

$= \frac{2}{3} + 2tan^{-1}2$

Q35

$\int_1^2 \frac{dx}{(x+1)(x+2)} = \int_1^2 \frac{(x+2)-(x+1)}{(x+1)(x+2)} dx$

$= \int_1^2 \frac{1}{(x+1)} dx - \int_1^2 \frac{1}{(x+2)} dx$

$= [log(x+1) - log(x+2)]_1^2$

=2log3-3log2

Q36

$\int_1^2 \frac{x+3}{x(x+2)} dx = \int_1^2 \frac{3}{2x} dx - \int_1^2 \frac{1}{x+2} dx$

$= \frac{3}{2} logx - log(x+2)$

$= \frac{1}{2}(log2 + log3)$

Q37

$\int_3^4 \frac{dx}{x^2-4} = \int_3^4 \frac{1}{(x-2)(x+2)} dx$

$= \int_3^4 \frac{1}{4(x-1)} dx - \int_3^4 \frac{1}{4(x+2)} dx$

$= \frac{1}{4} log(x-2) - \frac{1}{4} log(x+2)$

$= \frac{1}{4} log3 - \frac{1}{4} log1 - \frac{1}{4} log6 + \frac{1}{4} log5$

$= \frac{1}{4}\left(log5 - log\left(\frac{6}{2}\right)\right)$

$= \frac{1}{4}(log5 - log3)$

Q38

$$\int \frac{dx}{\sqrt{x^2+2x+3}} = \int \frac{dx}{\sqrt{(x+1)^2+2}}$$

Substitute:

$$\frac{x+1}{\sqrt{2}} = u$$

$$\therefore dx = \sqrt{2}du$$

$$= \int \frac{\sqrt{2}du}{\sqrt{2u^2+2}}$$

$$= log(\sqrt{u^2+1}+u)$$

Undo substitution:

$$u = \frac{x+1}{\sqrt{2}}$$

$$\therefore \int_0^4 \frac{dx}{\sqrt{x^2+4x+3}} = log\left(\sqrt{(x+1)^2+2}+x+1\right)$$

$$= log\left(\sqrt{(4+1)^2+2}+4+1\right) - log(\sqrt{(0+1)^2+2}+0+1)$$

$$= log\left(5+3\sqrt{3}\right) - log(1+\sqrt{3})$$

$$= log\left(\frac{5+3\sqrt{3}}{1+\sqrt{3}}\right)$$

Q39

$$\int \frac{dx}{\sqrt{x^2+4x+3}} = \int \frac{dx}{\sqrt{(x+2)^2-1}}$$

Substitute:

x+2=u

∴ dx=du

$= \int \frac{du}{\sqrt{u^2-1}}$

$= log(\sqrt{u^2-1}+u$

Undo substitution:

$u = x + 2$

$\therefore \int_1^2 \frac{dx}{\sqrt{x^2+4x+3}} = log\left(\sqrt{(x+2)^2-1}+x+2\right)$

$= log\left(\sqrt{(2+2)^2-1}+2+2\right) - log(\sqrt{(1+2)^2-1}+1+2)$

=log(4+√15)-log(3+√8)

Q40

$\int_0^1 \frac{1}{2x^2+x+1}dx = \int_0^1 \frac{1}{\left(\left(\sqrt{2x}+\frac{1}{2^{\frac{3}{2}}}\right)2+\frac{7}{8}\right)}dx$

Substitute 4x+1√7=u

$\therefore$ dx $= \frac{\sqrt{7}}{4}du$

Now solving:

$$\int \left(\frac{1}{u^2}+1\right) du = tan^{-1}u$$

$$\frac{2}{\sqrt{7}}\int \frac{1}{u^2+1} du = \frac{2}{\sqrt{7t}} tan^{-1}u$$

$$\therefore \int_0^1 \frac{1}{2x^2+x+1} dx = \frac{2}{\sqrt{7}} tan^{-1}\left(\frac{4x+1}{\sqrt{7}}\right)$$

$$= \frac{2}{\sqrt{7}} tan^{-1}\left(\frac{4+1}{\sqrt{7}}\right) - \frac{2}{\sqrt{7}} tan^{-1}\left(\frac{1}{\sqrt{7}}\right)$$

$$= \frac{2}{\sqrt{7}}\left\{tan^{-1}\left(\frac{5}{\sqrt{7}}\right) - tan^{-1}\left(\frac{1}{\sqrt{7}}\right)\right\}$$

Q41

$$\int_0^{\pi}(acos^2x + bsin^2x)dx = \int_0^{\frac{\pi}{2}}\left[\frac{a}{2}(cos2x+1) + \frac{b}{2}(1 - cos2x)\right] dx$$

$$= \left[\frac{a}{2}\left(\frac{sin2x}{2}+x\right) + \frac{b}{2}\left(x - \frac{sin2x}{2}\right)\right]$$

$$= \left[\frac{a}{2}\left(\frac{sin\pi}{2}+\frac{\pi}{2}\right) + \frac{b}{2}\left(\frac{\pi}{2} - \frac{sin\pi}{2}\right) - \frac{a}{2}\left(\frac{sin0}{2}+0\right) - \frac{b}{2}\left(0 - \frac{sin0}{2}\right)\right]$$

$$= \left[\frac{a}{2}\left(0+\frac{\pi}{2}\right) + \frac{b}{2}\left(\frac{\pi}{2}-0\right) - \frac{a}{2}(0+0) - \frac{b}{2}(0-0)\right]$$

$$= \frac{\pi}{4}(a+b)$$

Q42

$$\int_{\frac{\pi}{3}}^{\frac{\pi}{4}}(tanx+cotx)^2dx=\int_{\frac{\pi}{3}}^{\frac{\pi}{4}}\left(\frac{tan^2x+1}{tanx}\right)^2dx$$

$$\int_{\frac{\pi}{3}}^{\frac{\pi}{4}}\left(\frac{tan^2x+1}{tanx}\right)^2dx=\int_{\frac{\pi}{3}}^{\frac{\pi}{4}}\frac{sec^2x(tan^2x+1)}{tan^2x}dx$$

Substitute:

tan(x)=u

$$\therefore dx=\frac{1}{sec^2(x)}du$$

$$\therefore=\int\frac{(u^2+1)}{u^2}du$$

$$\therefore=u-\frac{1}{u}$$

$$\int_{\frac{\pi}{3}}^{\frac{\pi}{4}}\left(\frac{tan^2x+1}{tanx}\right)^2dx=[tan(x)-cot(x)]$$

$$=\left[tan\left(\frac{\pi}{4}\right)-cot\left(\frac{\pi}{4}\right)-tan\left(\frac{\pi}{3}\right)+cot\left(\frac{\pi}{3}\right)\right]$$

$$=\left[1-1-\sqrt{3}+\frac{1}{\sqrt{3}}\right]$$

$$=-\frac{2}{\sqrt{3}}$$

Q43

By reduction formula:

$$\int_0^{\frac{\pi}{2}}cos^4xdx=\frac{cos^3(x)sin(x)}{4}+\frac{3}{4}\int cos^2xdx$$

We know that,

$\int cos^2 x dx = \frac{1}{2}\left[\frac{sin2x}{2} + x\right]$ Type equation here.

$\int_0^{\frac{\pi}{2}} cos^4 x dx = \frac{cos^3(x)sin(x)}{4} + \frac{3}{8}\left[\frac{sin2x}{2} + x\right]$

$= \frac{cos^3\left(\frac{\pi}{2}\right)sin\left(\frac{\pi}{2}\right)}{4} + \frac{3}{8}\left[\frac{sin\pi}{2} + \frac{\pi}{2}\right] - \frac{cos^3(0)sin(0)}{4} - \frac{3}{8}\left[\frac{sin0}{2} + 0\right]$

$= 0 + \frac{3}{8}\left[0 + \frac{\pi}{2}\right] - 0 - \frac{3}{8}[0 + 0] = \frac{3\pi}{16}$

Q44

Assume that a≠0.

$\int_0^2 \frac{1}{-x^2+ax+a^2} dx = -\int_0^2 \frac{1}{x^2-ax-a^2} dx$

$= \int_0^2 \frac{4}{(2x+(-\sqrt{5}-1)a)(2x+(\sqrt{5}-1)a)} dx$

$= \int_0^2 \left(\frac{2}{\sqrt{5}\,a(2x+(-\sqrt{5}-1)a)} - \frac{2}{\sqrt{5}\,a(2x+(\sqrt{5}-1)a)}\right) dx$

Now,

$\int \frac{1}{2x+(-\sqrt{5}-1)a} dx$

Substitute:

u=2x+(-√5-1)a

$\therefore dx = \frac{1}{2}du$

$= \frac{1}{2}\int \frac{1}{u}du$

$= \frac{1}{2}logu$

Undo substitution:

$u = 2x + \left(-\sqrt{5} - 1\right)a$

$\therefore \int \frac{1}{2x+\left(-\sqrt{5}-1\right)a}dx = \frac{1}{2}log(2x + \left(-\sqrt{5} - 1\right)a)$

Now,

$\int \frac{1}{2x+\left(\sqrt{5}-1\right)a}dx$

Substitute:

$u = 2x + \left(\sqrt{5} - 1\right)a$

$\therefore dx = \frac{1}{2}du$

$= \frac{1}{2}\int \frac{1}{u}du$

$= \frac{1}{2}logu$

Undo substitution

$u = 2x + \left(\sqrt{5} - 1\right)a$

$\therefore \int \frac{1}{2x+\left(\sqrt{5}-1\right)a}dx = \frac{1}{2}log(2x + \left(\sqrt{5} - 1\right)a)$

$$\frac{2}{\sqrt{5}\,a}\int_0^2 \frac{1}{(2x+(-\sqrt{5}-1)a)}\,dx - \frac{2}{\sqrt{5}a}\int_0^2 \frac{1}{2x+(\sqrt{5}-1)a}\,dx$$

$$= \frac{log(2x+(-\sqrt{5}-1)a)}{\sqrt{5}a} - \frac{log(2x+(\sqrt{5}-1)a)}{\sqrt{5}a}$$

$$-\int_0^2 \frac{1}{x^2-ax-a^2}\,dx = \frac{\log(2x+(\sqrt{5}-1)a)}{\sqrt{5}a} - \frac{log(2x+(-\sqrt{5}-1)a)}{\sqrt{5}a}$$

$$= \frac{log(4+(\sqrt{5}-1)a)}{\sqrt{5}a} - \frac{log(4+(-\sqrt{5}-1)a)}{\sqrt{5}a} - \frac{\log(0+(\sqrt{5}-1)a)}{\sqrt{5}a} +$$

$$\frac{log(0+(-\sqrt{5}-1)a)}{\sqrt{5}a}$$

Q45 $= \frac{1}{\sqrt{5}\,a} log\left(\frac{7+3\sqrt{5}}{2}\right)$

$$\int_{\frac{1}{4}}^{\frac{1}{2}} \frac{dx}{\sqrt{x-x^2}} = \int_{\frac{1}{4}}^{\frac{1}{2}} \frac{1}{\sqrt{\frac{1}{4}-\left(x-\frac{1}{2}\right)^2}}$$

Substitute:

2x-1=u

$\therefore dx = \frac{1}{2}du$

$$\int \frac{1}{\sqrt{1-u^2}}du = sin^{-1}(u)$$

Undo Substitution:

u=2x-1

$\therefore$=$\sin^{-1}$ (2x-1)

$$\int_{\frac{1}{4}}^{\frac{1}{2}} \frac{dx}{\sqrt{x-x^2}} = sin^{-1}(2x-1)$$

$= sin^{-1}(1-1) - sin^{-1}(\frac{1}{2}-1)$

$= \frac{\pi}{6}$

Q46

$\int_0^1 \sqrt{x-x^2}\,dx = \int_0^1 \sqrt{\frac{1}{4}-\left(x-\frac{1}{2}\right)^2}\,dx$

$= \frac{1}{2}\int_0^1 \sqrt{1-(2x-1)^2}\,dx$

Substitute:

2x-1=u

$\therefore dx = \frac{1}{2}du$

$\therefore \frac{1}{2}\int \sqrt{1-u^2}\,du$

Substitute:

u=sin(v)

$\therefore \sin^{-1}(u)=v$

$\therefore du=\cos(v)dv$

$= \int cos(v)\sqrt{a-sin^2(v)}dv$

$= \int cos^2(v)dv$

We know that,

$\int cos^2(v)dv = \frac{1}{2}\left[\frac{\sin(2v)}{2} + v\right]$

Undo Substitution:

$v=\sin^{-1}(u)$

$\sin(\sin^{-1}(u))=u$

$cos\left(sin^{-1}(u)\right) = \sqrt{1-u^2}$

$= \frac{sin^{-1}(u)}{2} + \frac{u\sqrt{1-u^2}}{2}$

Undo Substitution:

u=2x-1

$\therefore = \frac{sin^{-1}(2x-1)}{4} + \frac{(2x-1)\sqrt{1-(2x-1)^2}}{4}$

$\frac{1}{2}\int_0^1 \sqrt{1-(2x-1)^2}\,dx = \frac{sin^{-1}(2x-1)}{8} + \frac{(2x-1)\sqrt{1-(2x-1)^2}}{8}$

$= \frac{sin^{-1}(2-1)}{8} + \frac{(2-1)\sqrt{1-(2-1)^2}}{8} - \frac{sin^{-1}(0-1)}{8} - \frac{(0-1\sqrt{1-(0-1)^2}}{8}$

$= \frac{\pi}{16} + 0 - \frac{\pi}{8} - 0$ Type equation here.

$= \frac{\pi}{8}$

Q47

$\int_1^3 \frac{1}{X^2(X+1)}dx$

Perform partial fraction decomposition:

$$\int_1^3 \frac{1}{x^2(x+1)}dx = \int_1^3 \left(\frac{1}{x+1} - \frac{1}{x} + \frac{1}{x^2}\right)dx$$

$$= \left[log(x+1) - log(x) - \frac{1}{x}\right]$$

$$= \left[log(4) - log(3) - \frac{1}{3} - log(2) + log(1) + \frac{1}{1}\right]$$

$$= log(2) - log(3) + \frac{2}{3}$$

Q48

$$\int_1^2 \frac{1}{x(2x+1)^2}dx = \int_1^2 \left(-\frac{2}{(2x+1} - \frac{2}{(2x+1)^2} + \frac{1}{x}\right)dx$$

$$= -2\int_1^2 \frac{1}{2x+1}dx - 2\int_1^2 \frac{1}{(2x+1)^2}dx + \int_1^2 \frac{1}{x}dx$$

$$= -2\left[\frac{1}{2}log(2x+1)\right] - 2\left[\frac{-1}{2(2x+1)}\right] + [log(x)]$$

$$= -[log(5)] + \left[\frac{1}{(5)}\right] + [log(2)] + [log(3)] - \left[\frac{1}{(3)}\right] + [log(1)]$$

$$= log(6) - log(5) - \frac{2}{15}$$

Q49

$$\int_0^1 xe^x dx = \int_0^1 (x-1+1)e^x dx$$

=[(x-1)e^x]

$= [(x-1)e^x]$

=[(1-1) e^1-(0-1) e^0]

$[(1-1)e^1-(0-1)e^0]$

=1

Q50

$\int_0^{\frac{\pi}{2}} x^2 cos(x)dx = x^2 sin(x) - \int 2xsin(x)dx$

$\int_0^{\frac{\pi}{2}} x^2 cos(x)dx = [x^2 sin(x) - 2sin(x) - 2xcos(x)]$

$= [\left(\frac{\pi}{2}\right)^2 sin\left(\frac{\pi}{2}\right) - 2sin\left(\frac{\pi}{2}\right) - \pi cos\left(\frac{\pi}{2}\right) - (0)^2 sin(0) + 2sin(0) +$

0]

$= \left[\frac{\pi^2}{4} - 2 - 0 - 0 + 0 + 0\right]$

$= \left(\frac{\pi^2}{4} - 2\right)$

Q51

From integrate by parts:

$\int_0^{\frac{\pi}{4}} x^2 sin(x)dx = -x^2 cos(x) - \int -2xcos(x)dx$

From integrate by parts:

$$\int_0^{\frac{\pi}{4}} x^2 cos(x)dx = [-x^2 cos(x) + 2xsin(x) + 2cos(x)]$$

$$= [2xsin(x) + (2 - x^2)cos(x)]$$

$$= [\frac{\pi}{2} sin\left(\frac{\pi}{4}\right) + \left(2 - \frac{\pi^2}{16}\right) cos\left(\frac{\pi}{4}\right) - 2(0)sin(0) - (2 - 0)cos(0)]$$

$$= \left[\frac{\pi}{2\sqrt{2}} + \frac{2}{\sqrt{2}} - \frac{\pi^2}{16\sqrt{2}} + 0 - 0 - 2\right]$$

$$= \sqrt{2} + \frac{\pi}{2\sqrt{2}} - \frac{\pi^2}{16\sqrt{2}} - 2$$

Q52

$$\int_0^{\frac{\pi}{2}} x^2 cos(2x)dx = \frac{x^2 sin(2x)}{2} - \int xsin(x)dx$$

$$\int_0^{\frac{\pi}{2}} x^2 cos(x)dx = \left[\frac{x^2 \sin(2x)}{2} - \frac{sin(2x)}{4} + \frac{xcos(2x)}{2}\right]$$

$$\left[\frac{\left(\frac{\pi}{2}\right)^2 sin(\pi)}{2} - \frac{sin(\pi)}{4} + \frac{\left(\frac{\pi}{2}\right)cos(\pi)}{2} - \frac{(0)^2 sin(0)}{2} + \frac{sin(0)}{4} - \frac{(0)cos(0)}{2}\right]$$

$$= [0 - 0 - \frac{\pi}{4} - 0 + 0 - 0]$$

$$= -\frac{\pi}{4}$$

Q53

$$\int_0^{\frac{\pi}{2}} x^3 sin(3x)dx = -\frac{x^3 c0s(3x)}{3} - \int -x^2 cos(3x)dx$$

$$= -\frac{x^3 cos(3x)}{3} + \frac{x^2 sin(3x)}{3} - \int \frac{2xsinx(3x)}{3} dx$$

$$= -\frac{x^3 cos(3x)}{3} + \frac{x^2 sin(3x)}{3} + \frac{2xcos(3x)}{9} + \frac{2}{3}\int -\frac{cos(3x)}{3} dx$$

$$= -\frac{x^3 cos(3x)}{3} + \frac{x^2 sin(3x)}{3} + \frac{2xcos(3x)}{9} - \frac{2sin(3x)}{27}$$

$$= -0 + \frac{\left(\frac{\pi}{2}\right)^2 sin\left(\frac{3\pi}{2}\right)}{3} + 0 - \frac{2sin\left(\frac{3\pi}{2}\right)}{27} + 0 - 0 - 0 + 0$$

$$-\left(\frac{2}{27} - \frac{\pi^2}{12}\right)$$

Q54

$$\int_0^{\frac{\pi}{2}} x^2 cos^2 x dx = \int_0^{\frac{\pi}{2}} \frac{x^2}{2}(cos(2x) + 1)dx$$

$$= \int_0^{\frac{\pi}{2}} \left(\frac{x^2}{2} cos(2x) + \frac{x^2}{2}\right) dx$$

$$\int_0^{\frac{\pi}{2}} \left(\frac{x^2}{2} cos(2x) + \frac{x^2}{2}\right) dx = \frac{x^2 sin(2x)}{2} - \int xsin(2x)dx + \frac{x^3}{6}$$

$$= \frac{x^2 sin(2x)}{2} + \frac{xcos(2x)}{4} + \int -\frac{cos(2x)}{2} dx + \frac{x^3}{6}$$

$$= \frac{x^2 sin(2x)}{2} + \frac{xcos(2x)}{4} - \frac{sin(2x)}{4} + \frac{x^3}{6}$$

$$= \frac{x^2 sin(2x)}{2} + \frac{xcos(2x)}{4} - \frac{sin(2x)}{4} + \frac{x^3}{6}$$

$$= 0 + \frac{\frac{\pi}{2}cos(\pi)}{4} - 0 + \frac{\left(\frac{\pi}{2}\right)^3}{6} - 0 - 0 + 0 - 0$$

$$= \left(\frac{\pi^3}{48} - \frac{\pi}{8}\right)$$

Q55

$$\int_1^2 log(x)dx = xlog(x) - (x)$$

$$= 2log(2) - (2) - 1log(1) + (1)$$

$$= 2log(2) - 1$$

Q56

$$\int_1^3 \frac{log(x)}{(1+x)^2} dx = -\frac{lod(x)}{1+x} - \int \left(-\frac{1}{x(1+x)}\right) dx$$

Now,

$$\int \left(-\frac{1}{x(1+x)}\right) dx = -\int \left(\frac{1}{x^2\left(\frac{1}{x}+1\right)}\right) dx$$

Let,

$$\frac{1}{x} + 1 = u$$

$\therefore dx=-x^2\ du$

$\therefore -\int \left(\frac{1}{x^2\left(\frac{1}{x}+1\right)}\right) dx = \int \frac{1}{u} du$

$= log(u)$

Undo substitution:

$u = \frac{1}{x} + 1$

$\int_1^3 \frac{log(x)}{(1+x)^2} dx = -\frac{log(x)}{1+x} + log\left(\frac{1}{x}+1\right)$

$= -\frac{log(3)}{4} + log\left(\frac{4}{3}\right) + \frac{log(1)}{2} - log(2)$

$= -\frac{log(3)}{4} + log(4) + log(3) - log2$

$= \frac{3}{4} log3 - log2$

Q57

Let,

log(x)=u

$\rightarrow x=e^u$

$\rightarrow dx=e^u\, du$

$\int \left\{\frac{1}{u} - \frac{1}{u^2}\right\} e^u du = \frac{e^u}{u}$

Undo substitution:

$u = log(x)$

$\int_0^{e^2} \left\{\frac{1}{\log(x)} - \frac{1}{log(x)^2}\right\} dx = \frac{x}{log(x)}$

$= \frac{e^2}{log(e^2)} - 0$

$= \frac{e^2}{2}$

Q58

$\int_1^e \left(\frac{(1+xlog(x))}{x}\right) dx - \int_1^e e^x \left(\frac{1}{x} + log(x)\right) dx$

=log(x) e^x

=log(e) e^e-log(1) e^1

=e^e

Q59

$\int_0^1 \frac{xe^x}{(1+x)^2} dx$

From Integrates by parts:

$= -\frac{xe^x}{x+1} - \int \frac{-xe^x - e^x}{x+1} dx$ Type equation here.

$\therefore \int \frac{-xe^x - e^x}{x+1} dx = \int -e^x dx$

=-e^x

$\int_0^1 \frac{xe^x}{(x+1)^2} dx = \left[-\frac{xe^x}{x+1} - e^x\right]$

$= \left[-\frac{1e^1}{1+1} - e^1 - \frac{0}{1+0} + e^0\right]$

$= \left[-\frac{e}{2} + e + 0 - 1\right]$

$= \left[\frac{e}{2} - 1\right]$

Q60

$\int_0^{\frac{\pi}{2}} 2tan^3 x dx = 2\int_0^{\frac{\pi}{2}} tan^2 x tanx dx$

$= 2\int_0^{\frac{\pi}{2}} tan^2 x tanx dx$

$= 2\int_0^{\frac{\pi}{2}} (sec^2 x - 1)tanx dx$

Substitute:

$sec(x) = u$

$\therefore dx = \frac{1}{sec(x)tan(x)} du$

$= 2\int_0^{\frac{\pi}{2}} \frac{(u^2-1)}{u} du$

$= 2\int_0^{\frac{\pi}{2}} \left(u - \frac{1}{u}\right) du$

$= 2\int_0^{\frac{\pi}{2}} \left(u - \frac{1}{u}\right) du$

$= 2\left[\frac{u^2}{2} - logu\right]$

Undo substitution:

$u = sec(x)$

$\therefore \int_0^{\frac{\pi}{2}} 2tan^3 x dx = 2\left[\frac{sec^2 x}{2} - log(secx)\right]$

$= 2\left[\frac{sec^2\left(\frac{\pi}{2}\right)}{2} - log\left(sec\left(\frac{\pi}{2}\right)\right) - \frac{sec^2(0)}{2} + log(sec(0))\right]$

$= 2\left[\frac{1}{2} - log(1)\right]$

=1-log2

Q61

$\int_1^2 \frac{5x^2}{(x^2+4x+3)} dx = 5[\int_1^2 \frac{x^2}{(x+3)(x+1)} dx]$

$= 5[\int_1^2 (1 - \frac{9}{2(x+3)} + \frac{1}{2(x+1)} dx]$

$= 5\left[x - \frac{9}{2}log(x+3) + \frac{1}{2}log(x+1)\right]_1^2$

$= 5\left[2 - \frac{9}{2}log5 + \frac{1}{2}log3 - 1 + \frac{9}{2}log4 - \frac{1}{2}log2\right]$

$= 5[1 - \frac{9}{2}log\left(\frac{5}{4}\right) + \frac{1}{2}log\left(\frac{3}{2}\right)$

$= 5 - \frac{5}{2}\left(9log\left(\frac{5}{4}\right) - log\left(\frac{3}{2}\right)\right)$

Exercise 16 B

Q1

Let

$I = \int_0^1 \frac{1}{2x-3} dx$

Let 2x-3=t

$\Rightarrow$ 2dx=dt.

Hence,

$I = \frac{1}{2}\int_0^1 \frac{1}{t} dt = \frac{1}{2} log_e|t|$

$= \frac{1}{2} log_e|2x-3|_0^1$

$\Rightarrow I = \frac{1}{2} log_e 1 - \frac{1}{2} log_e 3 = \frac{1}{2} log_e \frac{1}{3}$

$= -\frac{1}{2} log_e 3$

$(Since\ log_a \frac{1}{b} = -log_a b)$

Q2

Let

$I = \int_0^1 \frac{2x}{1+x^2} dx$

Let $1+x^2=t$

$\Rightarrow 2xdx=dt.$

when x=0, t=1

and

when x=1, t=2

Hence,

$I = \int_1^2 \frac{1}{t} dt = log_e|t|_1^2$

$= log_e 2 - log_e 1$

$= log_e 2$

Q3

Let

$I = \int_1^2 \frac{3x}{9x^2-1} dx$

Let $9x^2-1=t$

$\Rightarrow 18xdx=dt.$

Also,

when x=1, t=8

and

when x=2, t=35.

Hence,

$$I = \frac{1}{6}\int_8^{35} \frac{1}{t}dt = \frac{1}{6}log_e t|_8^{35} = \frac{1}{6}(log_e 35 - log_e 8)$$

Q4

Let

$$I = \int_0^1 \frac{tan^{-1}x}{1+x^2}dx$$

Let $\tan^{-1}x=t$

$\Rightarrow$.

$$\Rightarrow \frac{1}{1+x^2}dx = dt$$

Also, when x=0, t=0

and when x=1, $t = \frac{\pi}{4}$

$$t = \frac{\pi}{4}$$

Hence,

$$I = \int_0^{\frac{\pi}{4}} t\, dt = \frac{1}{2}t^2|_0^{\frac{\pi}{4}} = \frac{\pi^2}{32}$$

Q5

Let

$$I = \int_0^1 \frac{e^x}{1+e^{2x}}dx$$

Let $e^x=t$

$\Rightarrow e^x dx=dt.$

when x=0, t=1

and

when x=1, t=e.

Hence,

$$I = \int_1^e \frac{1}{1+t^2} dt = tan^{-1}t|_1^e$$

$$= tan^{-1}e - \frac{\pi}{4}$$

Q6

Let

$$I = \int_0^1 \frac{2x}{1+x^4} dx$$

Let $x^2=t$

$\Rightarrow 2xdx=dt.$

when x=0, t=0

and

when x=1, t=1.

Hence,

$I = \int_0^1 \frac{1}{1+t^2} dt$

$= tan^{-1}t|_0^1$

$= \frac{\pi}{4}$

Q7

Let

$I = \int_0^1 xe^{x2} dx$

Let $x^2=t$

$\Rightarrow$ 2xdx=dt.

Also,

when x=0, t=0

and

when x=1, t=1.

Hence,

$I = \frac{1}{2}\int_0^1 e^t dt$

$= \frac{1}{2}e^t|_0^1$

$= \frac{1}{2}(e-1)$

Q8

Let

$I = \int_1^2 \frac{e^{\frac{1}{x}}}{x^2} dx$

Let

$\frac{1}{x} = t$

$\Rightarrow$.

$\Rightarrow \frac{-1}{x^2} dx = dt$

when x=1, t=1

and

when x=2, .

$t = \frac{1}{2}$

Hence,

$I = -\int_1^{\frac{1}{2}} e^t dt$

$= -e^t|_1^{\frac{1}{2}}$

$= e - \sqrt{e}$

Q9

Let

$I = \int_0^{\frac{\pi}{6}} \frac{cosx}{3+4sinx} dx$

Let 3+4sinx=t

⇒ 4cosxdx=dt.

when x=0, t=3

and

when, t=5.

$x = \frac{\pi}{6}$

Hence,

$I = \frac{1}{4}\int_3^5 \frac{1}{t} dt$

$= \frac{1}{4} log_e t|_3^5$

$= \frac{1}{4}(log_e 5 - log_e 3)$

Q10

Let

$I = \int_0^{\frac{\pi}{2}} \frac{sinx}{1+cos^2 x} dx$

Let cos x=t

⇒ -sin x dx=dt.

when x=0, t=1

and

when , t=0.

$x = \frac{\pi}{2}$

Hence,

$I = -\int_1^0 \frac{1}{1+t^2} dt$

$= -tan^{-1}t|_1^0$

$= \frac{\pi}{4}$

Q11

Let

$I = \int_0^1 \frac{1}{e^x+e^{-x}} dx = \int_0^1 \frac{e^x}{1+e^{2x}} dx$

Let e^x=t

$\Rightarrow e^x dx$=dt.

when x=0, t=1

and

when x=1, t=e.

Hence,

$I = \int_1^e \frac{1}{1+t^2} dt$

$= tan^{-1}t|_1^e$

$= tan^{-1}e - \frac{\pi}{4}$

Q12

Let

$I = \int_{\frac{1}{e}}^{e} \frac{1}{x(log_e x)^{\frac{1}{3}}} dx$

Let

$log_e x = t$

$\Rightarrow .$

$\Rightarrow \frac{1}{x} dx = dt$

when , t=-1

$x = \frac{1}{e}$

and

when x=e, t=1.

Hence,

$I = \int_{-1}^{1} \frac{1}{\frac{1}{t3}} dt$

$= \frac{3}{2} t^{\frac{2}{3}} \Big|_{-1}^{1}$

$= \frac{3}{2}(1-1)$

=0

Q13

Let

$$I = \int_0^1 \sqrt{\frac{tan^{-1}x}{1+x^2}}\, dx$$

Let $\tan^{-1}x=t$

$$\Rightarrow \frac{1}{1+x^2}dx = dt$$

when x=0, t=0

and

when x=1,

$$t = \frac{\pi}{4}$$

Hence,

$$I = \int_0^{\frac{\pi}{4}} \sqrt{t}\, dt$$

$$= \frac{2}{3}t^{\frac{3}{2}}\Big|_0^{\frac{\pi}{4}}$$

$$= \frac{\pi^{\frac{3}{2}}}{12}$$

Q14

Let

$$I = \int_0^{\frac{\pi}{2}} \frac{sinx}{\sqrt{1+cosx}}dx$$

Let 1+cos x=t

⇒ -sin x dx=dt.

Also, when x=0, t=2

and

when $x = \frac{\pi}{2}$, t=1

Hence,

$I = -\int_2^1 \frac{1}{\sqrt{t}} dt$

$= -2\sqrt{t}\Big|_2^1$

=2(√2-1)

Q15

Let

$I = \int_0^{\frac{\pi}{2}} \sqrt{sinx} cos^5 x dx$

Let sinx=t

⇒ cos x dx=dt.

Also,

when x=0, t=0

and

when

$x = \frac{\pi}{2}$, t=1.

Consider $\cos^5 x=\cos^4 x\times\cos x=(1-\sin^2 x)^2\times\cos x$ (Using $\sin^2 x+\cos^2 x=1$)

Hence,

$$I = \int_0^1 \sqrt{x}(1-x^2)^2 dx$$

$$= \int_0^1 \sqrt{x} dx + \int_0^1 x^{\frac{9}{2}} dx - 2\int_0^1 x^{\frac{5}{2}} dx$$

$$\Rightarrow I = \frac{2}{3}t^{\frac{3}{2}}\Big|_0^1 + \frac{2}{11}t^{\frac{11}{2}}\Big|_0^1 - \frac{4}{7}t^{\frac{7}{2}}\Big|_0^1$$

$$= \frac{2}{3} + \frac{2}{11} - \frac{4}{7}$$

$$= \frac{64}{231}$$

Q16

Let

$$I = \int_0^{\frac{\pi}{2}} \frac{sinxcosx}{1+sin^4 x} dx$$

Let $\sin^2 x=t$

$\Rightarrow 2\sin x \cos x=dt.$

Also,

when x=0, t=0

and

when $x = \frac{\pi}{2}$ t=1.

Hence,

$$I = \frac{1}{2}\int_0^{\frac{\pi}{2}} \frac{1}{1+t^2} dt$$

$= \frac{1}{2} tan^{-1} t|_0^1$

$= \frac{\pi}{8}$

Q17

Let

$I = \int_0^a \sqrt{a^2 - x^2} dx$

Let x=a sin t

⇒ a cos t dt=dx.

Also,

when x=0, t=0

and

when x=a, $t = \frac{\pi}{2}$

Hence,

$I = \int_0^{\frac{\pi}{2}} \sqrt{a^2 - a^2 sin^2 t}\, acost\, dt = a^2 \int_0^{\frac{\pi}{2}} cos^2 t dt$

$cos^2 t = \frac{1+cos2t}{2}$

$I = \frac{a^2}{2} \int_0^{\frac{\pi}{2}} (1 + cos2t) dt$

$= \frac{a^2}{2} \left(t + \frac{sin2t}{2}\right) \Big|_0^{\frac{\pi}{2}}$

$= \frac{\pi a^2}{4}$

Q18

Let

$I = \int_0^{\sqrt{2}} \sqrt{2 - x^2} dx$

Consider,

$I = \int_0^a \sqrt{a^2 - x^2} dx$

Let x=a sin t

⇒ a cos t dt=dx.

Also, when x=0, t=0

and when x=a, .

$t = \frac{\pi}{2}$

Hence,

$I = \int_0^{\frac{\pi}{2}} \sqrt{a^2 sin^2 t}\, acost\, dt = a^2 \int_0^{\frac{\pi}{2}} cos^2 t dt$

Using , we get

$cos^2 t = \frac{1+cos2t}{2}$

$I = \frac{a^2}{2} \int_0^{\frac{\pi}{2}} (1 + cos2t) dt$

$= \frac{a^2}{2} t + \frac{sin2t}{2} \Big|_0^{\frac{\pi}{2}}$

$= \frac{\pi a^2}{4}$

Here

$a = \sqrt{2}$

, hence

$I = \frac{\pi}{2}$

Q19

Let

$I = \int_0^a \frac{x^4}{\sqrt{a^2-x^2}} dx$

Let x=a sin t

⇒ a cos t dt=dx.

Also, when x=0, t=0

and when x=a, $t = \frac{\pi}{2}$

Hence,

$I = \int_0^{\frac{\pi}{2}} \frac{a^4 sin^4 t}{\sqrt{a^2-a^2 sin^2 t}} acostdt$

$= a^4 \int_0^{\frac{\pi}{2}} sin^4 t dt$

Using, we get

$sin^2 t = \frac{1-cos2t}{2}$

$I = a^4 \int_0^{\frac{\pi}{2}} \left(\frac{1-cos2t}{2}\right)^2 dt$

$= \frac{a^4}{4} \int_0^{\frac{\pi}{2}} (1 + cos^2 2t - 2cos2t) dt$

$$I = \frac{a^4}{4}\left(t\Big|_0^{\frac{\pi}{2}} - sin2t\Big|_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}}\left(\frac{1+cos4t}{2}\right)dt\right)\left(Using\ cos^2t = \frac{1+cos2t}{2}\right)$$

Hence,

$$I = \frac{\pi a^4}{8} + \frac{a^4}{4} \times \frac{t}{2}\Big|_0^{\frac{\pi}{2}} + \frac{a^4}{32}sin4t\Big|_0^{\frac{\pi}{2}} = \frac{3\pi a^4}{16}$$

Q20

Let

$$I = \int_0^a \frac{x}{\sqrt{a^2+x^2}}dx$$

Let $a^2+x^2=t^2$

$\Rightarrow$ x dx=t dt.

Also, when x=0, t=a

and when x=a, .

$$t = \sqrt{2a}$$

Hence,

$$I = \int_a^{\sqrt{2a}} \frac{t}{\sqrt{t^2}}dt$$

$$= t|_a^{\sqrt{2a}}$$

=a(√2-1)

Q21

Let

$I = \int_0^2 x\sqrt{2-x}dx$

Using the property that , we get

$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

$I = \int_0^2 (2-x)\sqrt{x}dx$

$= \int_0^2 2\sqrt{x}dx - \int_0^2 x^{\frac{3}{2}}dx$

$= 2 \times \frac{2}{3}x^{\frac{3}{2}}\Big|_0^2 - \frac{2}{5}x^{\frac{5}{2}}\Big|_0^2$

Hence,

$I = 2\sqrt{2}\left(\frac{4}{3} - \frac{4}{5}\right)$

$= \frac{16}{15}\sqrt{2}$

Q22

Let

$I = \int_0^1 sin^{-1}(\frac{2x}{1+x^2})dx$

Let

$f(x) = sin^{-1}\left(\frac{2x}{1+x^2}\right)$

Let x=tanθ

⇒ θ=$\tan^{-1}$x

$\Rightarrow f(x) = sin^{-1}\left(\frac{2tan\theta}{1+tan^2\theta}\right)$

$= sin^{-1}\left(\frac{2tan\theta}{sec^2\theta}\right)$

$=\sin^{-1}(2\sin\theta\cos\theta)$

$=\sin^{-1}(\sin 2\theta)$

Hence $f(x)=2\theta$

$=2\tan^{-1}x$

Hence

$I = 2\int_0^1 1 \times tan^{-1}x dx$

Using integration by parts, we get

$I = 2xtan^{-1}x|_0^1 - \int_0^1 \frac{2x}{1+x^2} dx$ ---------(1)

$= \frac{\pi}{2} - \int_0^1 \frac{2x}{1+x^2} dx$

Let

$I' = \int_0^1 \frac{2x}{1+x^2} dx$

Let $1+x^2=t$

$\Rightarrow 2x\, dx=dt.$

Also, when x=0, t=1

and when x=1, t=2

Hence,

$I' = \int_1^2 \frac{1}{t} dt = log_e|t| \,|_1^2 = log_e 2 - log_e 1 = log_e 2$ ----------(2)

Substituting value of (2) in (1), we get

$I = \frac{\pi}{2} - log_e 2$

Q23

Let

$I = \int_0^{\frac{\pi}{2}} \sqrt{1 + cosx}\, dx$

Using , we get

$1 + cosx = 2cos^2 \frac{x}{2}$

$I = \sqrt{2} \int_0^{\frac{\pi}{2}} cos\left(\frac{x}{2}\right)\, dx$

$= 2\sqrt{2} sin\left(\frac{x}{2}\right) \Big|_0^{\frac{\pi}{2}}$ =2

Q24

Let

$I = \int_0^{\frac{\pi}{2}} \sqrt{1 + sinx}\, dx$

Using and

$sin^2 \frac{x}{2} + cos \frac{x}{2} = 1$

$sinx = 2sin \frac{x}{2} cos \frac{x}{2}$

$$I = \int_0^{\frac{\pi}{2}} \sqrt{\left(sin\left(\frac{x}{2}\right) + cos\left(\frac{x}{2}\right)\right)^2}\, dx$$

$$= \int_0^{\frac{\pi}{2}} \left(sin\left(\frac{x}{2}\right) + cos\left(\frac{x}{2}\right)\right) dx$$

$$= -2cos\left(\frac{x}{2}\right)\Big|_0^{\frac{\pi}{2}} + 2\sin\left(\frac{x}{2}\right)\Big|_0^{\frac{\pi}{2}}$$

=-(√2-2) +(√2) =2

Q25

Let

$$I = \int_0^{\frac{\pi}{2}} \frac{1}{a^2cos^2x + b^2sin^2x}\, dx$$

Dividing by $\cos^2 x$ in numerator and denominator, we get

$$I = \int_0^{\frac{\pi}{2}} \frac{sec^2x}{a^2 + b^2tan^2x}\, dx$$

Let tan x=t

$\Rightarrow \sec^2 x dx = dt$

$$I = \int_0^{\frac{\pi}{2}} \frac{1}{a^2 + b^2t^2}\, dt = \frac{1}{b^2}\int_0^{\frac{\pi}{2}} \frac{1}{\frac{a^2}{b^2} + t^2}\, dt$$

Let

$$t = \frac{a}{b}tan\theta = tanx$$

$$I = \frac{1}{B^2}\int_0^{\frac{\pi}{2}} \frac{\frac{a}{b}sec^2\theta}{\frac{a^2}{b^2}+\frac{a^2}{b^2}tan^2\theta}\, d\,\theta$$

$$= \frac{1}{ab}\theta$$

$$= \frac{1}{ab}tan^{-1}\left(\frac{b}{a}tanx\right)\Big|_0^{\frac{\pi}{2}}$$

$$= \frac{\pi}{2ab}$$

Q26

Let

$$I = \int_0^{\frac{\pi}{2}} \frac{1}{1+cos^2x}\, dx$$

Dividing by $\cos^2 x$ in numerator and denominator, we get

$$I = \int_0^{\frac{\pi}{2}} \frac{sec^2x}{sec^2x+tan^2x}\, dx = \int_0^{\frac{\pi}{2}} \frac{sec^2x}{1+2tan^2x}\, dx$$

Consider

$$I = \int_0^{\frac{\pi}{2}} \frac{sec^2x}{a^2+b^2tan^2x}\, dx$$

Let tan x=t

$\Rightarrow \sec^2 x dx = dt$

$$I = \int_0^{\frac{\pi}{2}} \frac{1}{a^2+b^2t^2}\, dt$$

$$= \frac{1}{b^2}\int_0^{\frac{\pi}{2}} \frac{1}{\frac{a^2}{b^2}+t^2}\, dt$$

Let

$t = \frac{a}{b}tan\theta$

=tan x

$I = \frac{1}{b^2}\int_0^{\frac{\pi}{2}} \frac{\frac{a}{b}sec^2\theta}{\frac{a^2}{b^2}+\frac{a^2}{b^2}tan^2\theta}\, d\,\theta$

$= \frac{1}{ab}\theta = \frac{1}{ab}tan^{-1}\left(\frac{b}{a}tanx\right)\Big|_0^{\frac{\pi}{2}}$

$= \frac{\pi}{2ab}$

Here, a=1 and b=√2

Hence,

$I = \frac{\pi}{2\sqrt{2}}$

Q27

Let

$I = \int_0^{\frac{\pi}{2}} \frac{1}{4+9cos^2x}\, dx$

Dividing by $\cos^2 x$ in numerator and denominator, we get

$I = \int_0^{\frac{\pi}{2}} \frac{sec^2x}{4sec^2x+9tan^2x}\, dx$

$= \int_0^{\frac{\pi}{2}} \frac{sec^2x}{4+13tan^2x}\, dx$

Consider

$I = \int_0^{\frac{\pi}{2}} \frac{sec^2x}{a^2+b^2tan^2x} dx$

Let tan x=t

$\Rightarrow sec^2xdx=dt$

$I = \int_0^{\frac{\pi}{2}} \frac{1}{a^2+b^2t^2} dt$

$= \frac{1}{b^2} \int_0^{\frac{\pi}{2}} \frac{1}{\frac{a^2}{b^2}+t^2} dt$

Let

$t = \frac{a}{b} tan\theta$

=tan x

$I = \frac{1}{b^2} \int_0^{\frac{\pi}{2}} \frac{\frac{a}{b}sec^2\theta}{\frac{a^2}{b^2}+\frac{a^2}{b^2}tan^2\theta} d\theta$

$= \frac{1}{ab} \theta$

$= \frac{1}{ab} tan^{-1}\left(\frac{b}{a}tanx\right)\Big|_0^{\frac{\pi}{2}}$

$= \frac{\pi}{2ab}$

Here, a=2 and b=√13

Hence,

$I = \frac{\pi}{4\sqrt{13}}$

Q28

Let

$$I = \int_0^{\frac{\pi}{2}} \frac{1}{5+4sinx}\,dx$$

$$sinx = \frac{2tan\left(\frac{x}{2}\right)}{1+tan^2\left(\frac{x}{2}\right)}$$

$$I = \int_0^{\frac{\pi}{2}} \frac{1}{5+4\frac{2tan\left(\frac{x}{2}\right)}{1+tan^2\left(\frac{x}{2}\right)}}\,dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{sec^2\left(\frac{x}{2}\right)}{5+5tan^2\left(\frac{x}{2}\right)+8tan\left(\frac{x}{2}\right)}\,dx$$

Let

$$tan\left(\frac{x}{2}\right) = t$$

$$\frac{1}{2}sec^2\left(\frac{x}{2}\right)dx = dt$$

when x=0, t=0 and when , t=1.

$$x = \frac{\pi}{2}$$

Hence,

$$I = \int_0^1 \frac{2}{5+5t^2+8t}\,dt$$

$$= \frac{2}{5}\int_0^1 \frac{1}{t^2+\frac{8}{5}t+\frac{16}{25}+\frac{9}{25}}\,dt$$

$= \frac{2}{5}\int_0^1 \frac{1}{\left(t+\frac{4}{5}\right)^2+\frac{9}{25}}dt$

Let

$t + \frac{4}{5} = u$

$\Rightarrow$ dt=du.

When t=0, $u = \frac{4}{5}$ and when t=1, $u = \frac{9}{5}$

$I = \frac{2}{5}\int_{\frac{4}{5}}^{\frac{9}{5}} \frac{1}{(u)^2+\frac{9}{25}}du$

$= \frac{2}{5} \times \frac{5}{3} tan^{-1}\left(\frac{5x}{3}\right)\Big|_{\frac{4}{5}}^{\frac{9}{5}}$

$= \frac{2}{3}\left(tan^{-1}3 - tan^{-1}\left(\frac{4}{3}\right)\right)$

$= \frac{2}{3} \times tan^{-1}\left(\frac{3-\frac{4}{3}}{5}\right)$

$= \frac{2}{3} tan^{-1}\left(\frac{1}{3}\right)$

$\left(Using\ tan^{-1}x - tan^{-1}y = tan^{-1}\left(\frac{x-y}{1+xy}\right)\right)$

Q29

Let

$I = \int_0^{\pi} \frac{1}{6-cosx}dx$

Using , we get

$cosx = \frac{1-tan^2(\frac{x}{2})}{1+tan^2(\frac{x}{2})}$

$I = \int_0^{\pi} \frac{1}{6-\frac{1-tan^2(\frac{x}{2})}{1+tan^2(\frac{x}{2})}} dx$

$= \int_0^{\pi} \frac{sec^2(\frac{x}{2})}{5+7tan^2(\frac{x}{2})} dx$

Let

$tan\left(\frac{x}{2}\right) = t$

$\Rightarrow \frac{1}{2} sec^2\left(\frac{x}{2}\right) dx = dt$

when x=0, t=0 and when x=π, t=∞.

Hence,

$I = \int_0^{\infty} \frac{2}{5+7t^2} dt$

$= \frac{2}{7} \int_0^{\infty} \frac{1}{t^2+\frac{5}{7}} dt$

$= \frac{2}{7} \times \sqrt{\frac{7}{5}} ta\, n^{-1}\left(\sqrt{\frac{7}{5}} x\right)\Big|_0^{\infty}$

$\Rightarrow I = \frac{2}{\sqrt{35}}\left(\frac{\pi}{2} - 0\right)$

$= \frac{\pi}{\sqrt{35}}$

Q30

Let

$$I = \int_0^{\pi} \frac{1}{5+4cosx}\,dx$$

Using, we get

$$cosx = \frac{1-tan^2\left(\frac{x}{2}\right)}{1+tan^2\left(\frac{x}{2}\right)}$$

$$I = \int_0^{\pi} \frac{1}{5+4\times\frac{1-tan^2\left(\frac{x}{2}\right)}{1+tan^2\left(\frac{x}{2}\right)}}\,dx$$

$$= \int_0^{\pi} \frac{sec^2\left(\frac{x}{2}\right)}{9+tan^2\left(\frac{x}{2}\right)}\,dx$$

Let

$$tan\left(\frac{x}{2}\right) = t$$

$$\Rightarrow \frac{1}{2}sec^2\left(\frac{x}{2}\right)dx = dt$$

when x=0, t=0 and when x=π, t=∞.

Hence,

$$I = \int_0^{\infty} \frac{2}{9+t^2}\,dt$$

$$= 2\int_0^{\infty} \frac{1}{9+t^2}\,dt$$

$$= 2\times\frac{1}{3}tan^{-1}\left(\frac{x}{3}\right)\Big|_0^{\infty}$$

$$\Rightarrow I = \frac{2}{3}\left(\frac{\pi}{2}-0\right) = \frac{\pi}{3}$$

Q31

Let

$$I = \int_0^{\frac{\pi}{2}} \frac{1}{cosx+2sinx}\,dx$$

Using

$$sinx = \frac{2tan\left(\frac{x}{2}\right)}{1+tan^2\left(\frac{x}{2}\right)}$$

And

$$cosx = \frac{1-tan^2\left(\frac{x}{2}\right)}{1+tan^2\left(\frac{x}{2}\right)}$$

we get

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{1}{\frac{1-tan^2\left(\frac{x}{2}\right)}{1+tan^2\left(\frac{x}{2}\right)}+2\frac{2tan\left(\frac{x}{2}\right)}{1+tan^2\left(\frac{x}{2}\right)}}\,dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{sec^2\left(\frac{x}{2}\right)}{1-tan^2\left(\frac{x}{2}\right)+4tan\left(\frac{x}{2}\right)}\,dx$$

Let

$$tan\left(\frac{x}{2}\right) = t$$

$$\Rightarrow \frac{1}{2}sec^2\left(\frac{x}{2}\right)dx = dt$$

when x=0, t=0

and when , t=1.

$$x = \frac{\pi}{2}$$

Hence,

$I = \int_0^1 \frac{2}{1-t^2+4t} dt$

$= -2\int_0^1 \frac{1}{t^2-4t+4-5} dt$

$-2\int_0^1 \frac{1}{(t-2)^2-5} dt$

Let t-2=u

$\Rightarrow$ dt=du.

Also, when t=0, u=-2

and when t=1, u=-1.

$\Rightarrow I = -2\int_{-2}^{-1} \frac{1}{u^2-5} dt$

$= -2 \times \frac{1}{2\sqrt{5}} log_e \left|\frac{x-\sqrt{5}}{x+\sqrt{5}}\right| \Big|_{-2}^{-1}$

$\left(Using \int \frac{1}{x^2-a^2} dx = \frac{1}{2a} log_e \left|\frac{x-a}{x+a}\right|\right)$

Hence,

$I = -\frac{1}{\sqrt{5}}\left(log_e \left|\frac{-1-\sqrt{5}}{-1+\sqrt{5}}\right| - log_e \left|\frac{-2-\sqrt{5}}{-2+\sqrt{5}}\right|\right)$

$= \frac{-1}{\sqrt{5}}\left(log_e \left|\frac{\sqrt{5}+1}{\sqrt{5}-1}\right| \times \left|\frac{\sqrt{5}-2}{2+\sqrt{5}}\right|\right)$

$\left(Using\ log_e a - log_e b = log_e \frac{a}{b}\right)$

$\Rightarrow I = \frac{-1}{\sqrt{5}}\left(log_e \left|\frac{3-\sqrt{5}}{3+\sqrt{5}}\right|\right)$

$= \frac{-2}{\sqrt{5}}\left(log_e\left(\frac{3-\sqrt{5}}{2}\right)\right)$ (Using $log_e a^b = b log_e a$)

Q32

Let

$I = \int_0^{\pi} \frac{1}{3+cosx+2sinx}\,dx$

Using

$sinx = \frac{2tan\left(\frac{x}{2}\right)}{1+tan^2\left(\frac{x}{2}\right)}$

$cosx = \frac{1-tan^2\left(\frac{x}{2}\right)}{1+tan^2\left(\frac{x}{2}\right)}$

we get

$\Rightarrow I = \int_0^{\pi} \frac{1}{3+\frac{1-tan^2\left(\frac{x}{2}\right)}{1+tan^2\left(\frac{x}{2}\right)}+2\frac{2tan\left(\frac{x}{2}\right)}{1+tan^2\left(\frac{x}{2}\right)}}\,dx$

$= \int_0^{\pi} \frac{sec^2\left(\frac{x}{2}\right)}{4+2tan^2\left(\frac{x}{2}\right)+4tan\left(\frac{x}{2}\right)}\,dx$

Let

$tan\left(\frac{x}{2}\right) = t$

$\Rightarrow \frac{1}{2}sec^2\left(\frac{x}{2}\right)dx = dt$

when x=0, t=0

and when

$x = \pi$, t=∞.

Hence,

$I = \int_0^{\infty} \frac{1}{(t+1)^2+1} dt$

Let t+1=u

⇒ dt=du.

Also, when t=0, u=1

and when t=∞, u=∞.

$I = \int_1^{\infty} \frac{1}{u^2+1} dt$

$= tan^{-1}u|_1^{\infty}$

$= \frac{\pi}{2} - \frac{\pi}{4}$

$= \frac{\pi}{4}$

Q33

Let

$I = \int_0^{\frac{\pi}{4}} \frac{tan^3x}{1+cos2x} dx$

Using $1+\cos 2x=2\cos^2 x$, we get

$I = \frac{1}{2}\int_0^{\frac{\pi}{4}} tan^3 x sec^2 x dx$

Let tan x=t

$\Rightarrow \sec^2 x dx = dt.$

when x=0, t=0

and when , t=1.

$x = \frac{\pi}{4}$

$= \frac{1}{2}\int_0^1 t^3\, dt = \frac{t^4}{8}\Big|_0^1 = \frac{1}{8}$

Q34

Let

$I = \int_0^{\frac{\pi}{2}} \frac{sinxcosx}{cos^2x+3cosx+2} dx$

Let cos x=t

$\Rightarrow$ -sin x dx=dt.

Also, when x=0, t=1

and when , t=0.

$x = \frac{\pi}{2}$

Hence,

$I = -\int_1^0 \frac{t}{t^2+3t+2} dt$

$= -\int_1^0 \frac{2(t+1)-(t+2)}{(t+1)(t+2)} dt$

$= -\int_1^0 \frac{2}{t+2} dt + \int_1^0 \frac{1}{(t+1)} dt$

$\Rightarrow I = -2log_e(t+2)\lfloor_1^0 + log_e(t+1)\lfloor_1^0$

$= -2log_e 2 + 2log_e 3 - log_e 2$

Hence

$I = log_e 9 - log_e 8$

$(Using\ blog_e a = log_e a^b\ and\ log_e a + log_e b = log_e ab)$

Q35

Let

$I = \int_0^{\frac{\pi}{2}} \frac{sin2x}{sin^4x+cos^4x}dx$

Using sin 2x =2 sin x cos x, we get

$I = \int_0^{\frac{\pi}{2}} \frac{2sinxcosx}{cos^4x(tan^4x+1)}dx$

$= 2\int_0^{\frac{\pi}{2}} \frac{tanxsec^2x}{(tan^4x+1)}dx$

Let tan x=t

$\Rightarrow$ $sec^2xdx=dt$.

Also, when x=0, t=0

and when , t=∞, $x = \frac{\pi}{2}$

Hence,

$2\int_0^{\infty} \frac{t}{(t^4+1)}dt$

Let $x^2=t$

$\Rightarrow 2xdx=dt.$

Also, when x=0, t=0

and when x=∞, t=∞.

Hence,

$$I = \int_0^\infty \frac{1}{1+t^2}\,dt$$

$$= tan^{-1}t|_0^\infty$$

$$= \frac{\pi}{2}$$

Q36

Let

$$I = \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{\sqrt{1+cosx}}{(1-cosx)^{\frac{5}{2}}}\,dx$$

Using

$$1 + cosx = 2cosx^2\left(\frac{x}{2}\right)$$

$$1 - cosx = 2sin^2\left(\frac{x}{2}\right)$$

we get

$$I = \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{\sqrt{2}cos\left(\frac{x}{2}\right)}{4\sqrt{2}\left(sin\left(\frac{x}{2}\right)\right)^5}\,dx$$

$= \frac{1}{4}\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} cot\left(\frac{x}{2}\right) cosec^4\left(\frac{x}{2}\right) dx$

Let

$cot\left(\frac{x}{2}\right) = t$

$\Rightarrow -\frac{1}{2} cosec^2\left(\frac{x}{2}\right) dx = dt$

Also, when , $x = \frac{\pi}{3}$ $t = \sqrt{3}$

and when , t=1, $x = \frac{\pi}{2}$

Hence,

$I = -\frac{1}{2}\int_{\sqrt{3}}^{1} t(1+t^2)dt$

$= -\frac{1}{2}\frac{t^2}{2}\Big|_{\sqrt{3}}^{1} - \frac{1}{2}\frac{t^4}{4}\Big|_{\sqrt{3}}^{1}$

$= \frac{1}{2} + 1 = \frac{3}{2}$

Q37

Let

$I = \int_0^1 (cos^{-1}x)^2 dx$

Let x=cost ⇒ dx=-sin t dt.

Also, when x=0, $t = \frac{\pi}{2}$

and when x=1, t=0.

Hence,

$I = -\int_{\frac{\pi}{2}}^{0} t^2 sint\ dt$

Using integration by parts, we get

$I= -(t^2 \times -cost|_{\frac{\pi}{2}}^{0} + 2\int_{\frac{\pi}{2}}^{0} tcost\ dt)$

$= -(0 - 0 + 2t \times sint|_{\frac{\pi}{2}}^{0} - 2\int_{\frac{\pi}{2}}^{0} sint\ dt\ )$

$= -\left(-\pi + 2cost\ |_{\frac{\pi}{2}}^{0}\right)$

Hence, I=π-2

Q38

Let

$I = \int_0^1 x(tan^{-1}x)^2 dx$

Using integration by parts, we get

$I = \frac{(tan^{-1}x)^2 x^2}{2}\Big|_0^1 - \int_0^1 \frac{2tan^{-1}x}{1+x^2} \times \frac{x^2}{2} dx$

$= \frac{\pi^2}{32} - 0 - \int_0^1 \frac{tan^{-1}x}{1+x^2} \times (1 + x^2 - 1)dx$

$= \frac{\pi^2}{32} - \int_0^1 tan^{-1}x dx + \int_0^1 \frac{tan^{-1}x}{1+x^2} dx$

Let $\tan^{-1}x=t$

⇒ .

$\frac{1}{1+x^2}dx = dt$

When x=0, t=0 and when x=1, .

$t = \frac{\pi}{4}$

Hence

$I = \frac{\pi^2}{32} - tan^{-1}x \times x\,|_0^1 + \int_0^1 \frac{x}{1+x^2}dx + \int_0^{\frac{\pi}{4}} tdt$

$= \frac{\pi^2}{32} - \frac{\pi}{4} + \frac{t^2}{2}\Big|_0^{\frac{\pi}{4}} + \int_0^1 \frac{x}{1+x^2}dx$

Let $1+x^2=y$

$\Rightarrow 2xdx=dy.$

Also, when x=0, y=1

and when x=1, y=2.

$I = \frac{\pi^2}{16} - \frac{\pi}{4} + \frac{1}{2}\int_1^2 \frac{1}{y}dy$

$= \frac{\pi}{4}\left(\frac{\pi}{4} - 1\right) + \frac{1}{2}log_e y|_1^2$

$= \frac{\pi}{4}\left(\frac{\pi}{4} - 1\right) + \frac{1}{2}log_e 2$

Q39

Let

$I = \int_0^1 sin^{-1}\sqrt{x}\,dx$

Let $\sqrt{x}=t$

$\Rightarrow \frac{1}{2\sqrt{x}}dx = dt$

or

dx=2tdt.

When, x=0, t=0

and when x=1, t=1.

Hence,

$I = 2\int_0^1 tsin^{-1}t\, dt$

Using integration by parts, we get

$I = 2(sin^{-1}t \times \frac{t^2}{2}|_0^1 - \int_0^1 \frac{1}{\sqrt{1-t^2}} \times \frac{t^2}{2}dt)$

$= \frac{\pi}{2} - \int_0^1 \frac{t^2}{\sqrt{1-t^2}}dt$

Let t=sin y

$\Rightarrow$ dt=cos y dy.

When t=0, y=0, when t=1, $y = \frac{\pi}{2}$

$I = \frac{\pi}{2} - \int_0^{\frac{\pi}{2}} sin^2 ydy$ ….. (1)

Using

$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

$I = \frac{\pi}{2} - \int_0^{\frac{\pi}{2}} cos^2 ydy$…..(2)

Adding (1) and (2), we get

$2I = \pi - \int_0^{\frac{\pi}{2}} dy$

$= \pi - \frac{\pi}{2}$

Hence,

$I = \frac{\pi}{4}$

Q40

Let

$I = \int_0^a sin^{-1}\sqrt{\frac{x}{a+x}}\,dx$

Let $x=a\tan^2 y$

$\Rightarrow dx=2a\tan y \sec^2 y\, dy.$

Also, when x=0, y=0

and when x=a,

$y = \frac{\pi}{4}$

Hence

$I = \int_0^{\frac{\pi}{4}} sin^{-1}\left(\sqrt{\frac{atan^2 y}{a+atan^2 y}}\right) 2a\ tan\ y\ sec^2\ y\ dy =$

$2a\int_0^{\frac{\pi}{4}} y\ tan\ y\ sec^2 y dy$

Using integration by parts, we get

$I = 2a(y\int_0^{\frac{\pi}{4}} tanysec^2ydy - \int_0^{\frac{\pi}{4}}(\int tanysec^2ydy)dy)$

Let tan y=t

$\Rightarrow$ $sec^2ydy=dt$.

Also, when y=0, t=0

and when y = $\frac{\pi}{4}$, t=1.

Also, $y=tan^{-1}t$

$\Rightarrow dy = \frac{dt}{1+t^2}$

$I = 2a\ (tan^{-1}\int\ tdt\ |_0^1 - \int_0^1\ (\int\ tdt\ )\ \frac{dt}{1+t^2}\ )$

$= 2a\left(\frac{tan^{-1}t\times t^2}{2}\Big|_0^1\right) - 2a\int_0^1 \frac{t^2}{2}\ \frac{dt}{1+t^2}$

$= \frac{a\pi}{4} - a\int_0^1 \frac{t^2}{1+t^2}dt$

Let

$I' = \int_0^1 \frac{t^2}{1+t^2}dt$

$= \int_0^1 \frac{1+t^2-1}{1+t^2}dt$

$= \int_0^1 dt - \int_0^1 \frac{1}{1+t^2}dt$

$= t|_0^1 - tan^{-1}|_0^1$

Hence

$I' = 1 - \frac{\pi}{4}$

Substituting value of I' in I, we get

$I = \frac{a\pi}{4} - a\left(1 - \frac{\pi}{4}\right)$

$= a\left(\frac{\pi}{4} - 1\right)$

Q41

Let

$I = \int_0^9 \frac{1}{1+\sqrt{x}} dx$

Let √x=u

$\Rightarrow \frac{1}{2\sqrt{x}} dx = du$

or dx=2udu.

$= \frac{1}{2u} dx$

Also, when x=0, u=0 and x=9, u=3.

Hence,

$I = \int_0^3 \frac{2u}{1+u} du$

$= 2\left(\int_0^3 \frac{u+1-1}{1+u} du\right)$

$= 2(\int_0^3 du - \int_0^3 \frac{1}{1+u} du)$

$I = 2u|_0^3 - log_e(1+u)|_0^3$

$= 6 - 2log_e 4$

$= 6 - 4log_e 2$

$(Using\ log_e a^b = b\ log_e a)$

Q42

Let

$I = \int_0^1 x^3\sqrt{1+3x^4}dx$

Let $1+3x^4=t$

$\Rightarrow 12x^3dx=dt.$

Also, when x=0, t=1 and when x=1, t=4.

$I = \frac{1}{12}\int_1^4 \sqrt{t}\ dt$

$= \frac{1}{12} \times \frac{2}{3} t^{\frac{3}{2}}\Big|_1^4 = \frac{7}{18}$

Q43

Let

$I = \int_0^1 \frac{1-x^2}{(1+x^2)^2} dx$

Let

$I' = \int_0^1 \frac{1}{(1+x^2)^2} dx$

Let x=tan t

$\Rightarrow dx=sec^2tdt.$

Also, when x=0, t=0 and when x=1, $t = \frac{\pi}{4}$

Hence,

$$I' = \int_0^{\frac{\pi}{4}} \frac{sec^2 t}{(1+tan^2 t)^2} dt$$

$$= \int_0^{\frac{\pi}{4}} cos^2 t dt$$

Using , we get

$$cos^2 t = \frac{1+cos2t}{2}$$

$$I' = \int_0^{\frac{\pi}{4}} \left(\frac{1+cos2t}{2}\right) dt$$

$$= \frac{t}{2}\Big|_0^{\frac{\pi}{4}} + \frac{sin2t}{4}\Big|_0^{\frac{\pi}{4}}$$

$$= \frac{\pi+2}{8}$$

Let

$$I'' = \int_0^1 \frac{x^2}{(1+x^2)^2} dx$$

$$= \int_0^1 x \times \frac{x}{(1+x^2)^2} dx$$

$$= x \int_0^1 \frac{x}{(1+x^2)^2} dx - \int_0^1 \left(\int \frac{x}{(1+x^2)^2} dx\right) dx$$

Let $1+x^2=t \Rightarrow 2xdx=dt$.

When x=0, t=1 and when x=1, t=2.

$$I'' = \sqrt{t-1} \times \frac{1}{2} \int_1^2 \frac{1}{t^2} dt - \int_1^2 \frac{\left(\frac{1}{2}\int \frac{1}{t^2} dt\right) dt}{2\sqrt{t-1}}$$

$= -\frac{\sqrt{t-1}}{2} \times \frac{1}{t}\Big|_1^2 + \int_1^2 \frac{dt}{4t\sqrt{t-1}}$

$= -\frac{1}{4} + \int_1^2 \frac{dt}{4t\sqrt{t-1}}$

Substituting $t=1+x^2$

$\Rightarrow 2xdx=dt.$

When t=1, x=0 and when t=2, x=1.

$I'' = -\frac{1}{4} + \int_0^1 \frac{2xdx}{4x(1+x^2)}$

$= -\frac{1}{4} + \frac{1}{4} tan^{-1}x|_0^1$

$= \frac{\pi-2}{8}$

Hence,

$I = \frac{\pi+2}{8} - \frac{\pi-2}{8} = \frac{1}{2}$

Q44

Let

$I = \int_1^2 \frac{1}{(x-1)\sqrt{x^2-1}} dx$

Let x=sect

$\Rightarrow$ dx=sec t tan t dt.

Also,

when x=1, t=0 and when x=2, $t = \frac{\pi}{3}$

Hence,

$I = \int_0^{\frac{\pi}{3}} \frac{secttant}{(sect+1)\sqrt{sec^2t-1}} dt$

$= \int_0^{\frac{\pi}{3}} \frac{sect}{(sect+1)} dt$

$= \int_0^{\frac{\pi}{3}} \frac{1}{(1+cost)} dt$

Using

$1 + cost = 2cos^2(\frac{t}{2})$

we get

$I = \frac{1}{2}\int_0^{\frac{\pi}{3}} sec^2\left(\frac{t}{2}\right) dt$

$= tan\frac{t}{2}\Big|_0^{\frac{\pi}{3}}$

$= \frac{1}{\sqrt{3}}$

Q 45

Let

$I = \int_0^{\frac{\pi}{2}} \left(\sqrt{tanx} + \sqrt{costx}\right) dx = \int_0^{\frac{\pi}{2}} \frac{sinx+cosx}{\sqrt{sinxcosx}} dx$

Let sin x- cos x=t

⇒ (cos x + sin x)dx=dt.

When x=0, t=-1 and , t=1, $x = \frac{\pi}{2}$

Also, $t^2=(\sin x - \cos x)^2$

$=\sin^2x+\cos^2x-2\sin x\cos x$

$=1-2\sin x\cos x$

or

$$sinxcosx = \frac{1-t^2}{2}$$

Hence

$$I = \sqrt{2}\int_{-1}^{1}\frac{1}{\sqrt{1-t^2}}dt$$

Let t=sin y

$\Rightarrow$ dt=cos y dy.

Also, when t=-1,

$$y = -\frac{\pi}{2}$$

and when t=1, .

$$y = \frac{\pi}{2}$$

$$I = \sqrt{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{cosy}{\sqrt{1-sin^2y}}dy$$

$$= \sqrt{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}dy = \pi\sqrt{2}$$

Q46

Let

$I = \int_2^3 \frac{2-x}{\sqrt{5x-6-x^2}}\,dx$

Let,

$2 - x = \frac{ad}{dx}(5x - 6 - x^2) + b$

=-2ax+5a+b

Hence -2a=-1 and 5a+b=2.

Solving these equations,

we get and .

$a = \frac{1}{2}$

$b = -\frac{1}{2}$

We get,

$I = \frac{1}{2}\int_2^3 \frac{-2x+5}{\sqrt{5x-6-x^2}}\,dx - \frac{1}{2}\int_2^3 \frac{1}{\sqrt{5x-6-x^2}}\,dx$

Let

$I' = \int_2^3 \frac{-2x+5}{\sqrt{5x-6-x^2}}\,dx$

Let $5x-6-x^2=t$

$\Rightarrow$ (5-2x) dx=dt.

When x=2, t=0 and when x=3, y=0.

Hence

$I' = \int_0^0 \frac{1}{\sqrt{t}}\,dt = 0$

$(\ Since \int_a^a f(x)dx = 0\ )$

Let,

$I'' = \int_2^3 \frac{1}{\sqrt{5x-6-x^2}}dx$

$= \int_2^3 \frac{1}{\sqrt{\frac{1}{4}-\left(x-\frac{5}{2}\right)^2}}$

$= sin^{-1}\left(\frac{x-\frac{5}{2}}{\frac{1}{2}}\right)$

$= sin^{-1}(2x-5)|_2^3$

$=\pi$

Hence,

$I = \frac{1}{2} \times 0 - \frac{1}{2} \times \pi$

$= -\frac{\pi}{2}$

Q47

Let

$I = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{cosx}{\left(cos\left(\frac{x}{2}\right)+sin\left(\frac{x}{2}\right)\right)^3}dx$

$cosx = cos^2\left(\frac{x}{2}\right) - sin^2\left(\frac{x}{2}\right)$

$I = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{cos\left(\frac{x}{2}\right)-sin\left(\frac{x}{2}\right)}{\left(cos\left(\frac{x}{2}\right)+sin\left(\frac{x}{2}\right)\right)^2}dx$

Let

$cos\left(\frac{x}{2}\right)+sin\left(\frac{x}{2}\right)=t$

$\Rightarrow \frac{1}{2}\left(cos\left(\frac{x}{2}\right)-sin\left(\frac{x}{2}\right)\right)dx=dt$

Also, when , $x=\frac{\pi}{4}$

$t=cos\left(\frac{\pi}{8}\right)+sin\left(\frac{\pi}{8}\right)=a(let)$

and when , $t=\sqrt{2}, x=\frac{\pi}{2}$

$I=\int_a^{\sqrt{2}}\frac{2}{t^2}dt$

$=-2\times\frac{1}{t}\Big|_a^{\sqrt{2}}$

$=\frac{2}{cos\left(\frac{\pi}{8}\right)+sin\left(\frac{\pi}{8}\right)}-\sqrt{2}$

Q48

Let

$I=\int_0^{\left(\frac{\pi}{2}\right)^{\frac{1}{3}}}x^2sin(x^3)dx$

Let $x^3=t$

$\Rightarrow 3x^2=dt.$

Also, when x=0, t=0 and when , $t=\frac{\pi}{2}, x=\left(\frac{\pi}{2}\right)^{\frac{1}{3}}$

Hence,

$I = \frac{1}{3}\int_0^{\frac{\pi}{2}} sin(t)dt$

$= \frac{-1}{3} cost\Big|_0^{\frac{\pi}{2}}$

$= -\frac{1}{3}(0-1) = \frac{1}{3}$

Q49

Let

$I = \int_1^2 \frac{1}{x(1+log_e x)^2} dx$

Let

$1 + log_e x = t$

$\Rightarrow \frac{1}{x} dx = dt$

Also, when x=1, t=1 and when x=2,

$t = 1 + log_e 2$

Hence

$I = \int_1^{1+log_e2} \frac{1}{t2} dt$

$= -\frac{1}{t}\Big|_1^{1+log_e 2}$

$= 1 - \frac{1}{1+log_e 2}$

$= \frac{log_e 2}{1+log_e 2}$

Q50

Let

$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{cosecxcotx}{1+cosec^2 x}\,dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{cosx}{1+sin^2 x}\,dx$

Let sinx=t

$\Rightarrow$ cos x dx=dt.

Also, when , $t = \frac{1}{2}, x = \frac{\pi}{6}$

and when , t=1, $x = \frac{\pi}{2}$

$I = \int_{\frac{1}{2}}^{1} \frac{1}{1+t^2}\,dt$

$= tan^{-1}t|_{\frac{1}{2}}^{1}$

$= tan^{-1}1 - tan^{-1}\left(\frac{1}{2}\right)$

$= tan^{-1}\left(\frac{1-\frac{1}{2}}{1+\frac{1}{2}}\right)$

$= tan^{-1}\left(\frac{1}{3}\right)$

(Using)

$tan^{-1}a - tan^{-1}b = tan^{-1}\left(\frac{a-b}{1+ab}\right)$

Exercise 16 C

Q1

$$y = \frac{1}{2}\int_0^{\frac{\pi}{2}} \frac{2cosx}{sinx+cosx}\,dx$$

$$= \frac{1}{2}\int_0^{\frac{\pi}{2}} \frac{cosx+cosx-sin\,x+sin\,x}{sin\,x+cos\,x}\,dx$$

$$= \frac{1}{2}\int_0^{\frac{\pi}{2}} 1 + \frac{cos\,x-sin\,x}{sin\,x+cos\,x}\,dx$$

$$= \frac{1}{2}\left((x)_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} \frac{cos\,x-sin\,x}{sin\,x+cos\,x}\,dx \right)$$

Let, sin x + cos x = t

⇒ (cos x – sin x) dx = dt

At x = 0, t = 1

At x = π/2, t = 1

$$y = \frac{1}{2}\left(\frac{\pi}{2} + \int_1^1 \frac{1}{t}\,dt\right)$$

$$y = \frac{1}{2}(\frac{\pi}{2} + (Int)_1^1$$

$$y = \frac{\pi}{4}$$

Q2

$$y = \int_0^{\pi/2} \frac{\sqrt{sinx}}{(\sqrt{sin\,x}+\sqrt{cosx})}\,dx \ldots(1)$$

Use King theorem of definite integral

$$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$$

$$y = \int_0^{\pi/2} \frac{\sqrt{\sin(\frac{\pi}{2}-x)}}{(\sqrt{\sin(\frac{\pi}{2}-x)}+\sqrt{\cos(\frac{\pi}{2}-)x})} dx$$

$$y = \int_0^{\pi/2} \frac{\sqrt{\cos x}}{(\sqrt{\cos x}+\sqrt{\sin x})} dx \ \ldots(2)$$

Adding eq.(1) and eq.(2)

$$2y = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{(\sqrt{\sin x}+\sqrt{\cos x})} dx + \int_0^{\pi/2} \frac{\sqrt{\cos x}}{(\sqrt{\cos x}+\sqrt{\sin x})} dx$$

$$= \int_0^{\pi/2} \frac{\sqrt{\sin x}+\sqrt{\cos x}}{(\sqrt{\sin x}+\sqrt{\cos x})} dx$$

$$= \int_0^{\pi/2} 1dx$$

$$= (x)_{\cap}^{\frac{\pi}{2}}$$

$$y = \frac{\pi}{4}$$

Q3A

$$y = \int_0^{\pi/2} \frac{\sin^3 x}{\sin^3 x+\cos^3 x} dx \ \ldots(1)$$

Use King theorem of definite integral

$$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$$

$$y = \int_0^{\pi/2} \frac{sin^3\left(\frac{\pi}{2}-x\right)}{sin^3\left(\frac{\pi}{2}-x\right)+cos^3\left(\frac{\pi}{2}-x\right)} dx$$

$$y = \int_0^{\pi/2} \frac{cos^3 x}{sin^3 x+cos^3 x} dx \ \ldots(2)$$

Adding eq.(1) and eq.(2)

$$2y = \int_0^{\pi/2} \frac{sin^3 x}{sin^3 x+cos^3 x} dx + \int_0^{\pi/2} \frac{cos^3 x}{sin^3 x+cos^3 x} dx$$

$$= \int_0^{\pi/2} \frac{sin^3 x+cos^3 x}{sin^3 x+cos^3 x} dx$$

$$= \int_0^{\pi/2} 1 dx$$

$$2y = (x)_{\cap}^{\frac{\pi}{2}}$$

$$y = \frac{\pi}{4}$$

Q3B

$$y = \int_0^{\pi/2} \frac{cos^3 x}{sin^3 x+cos^3 x} dx \ldots(1)$$

Use King theorem of definite integral

$$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$$

$$y = \int_0^{\pi/2} \frac{cos^3\left(\frac{\pi}{2}-x\right)}{sin^3\left(\frac{\pi}{2}-x\right)+cos^3\left(\frac{\pi}{2}-x\right)} dx \ \ldots(2)$$

$$y = \int_0^{\pi/2} \frac{sin^3x}{sin^3x+cos^3x}dx$$

Adding eq.(1) and eq.(2)

$$2y = \int_0^{\pi/2} \frac{cos^3x}{sin^3x+cos^3x}dx + \int_0^{\pi/2} \frac{sin^3x}{sin^3x+cos^3x}dx$$

$$2y = \int_0^{\pi/2} \frac{cos^3x+sin^3x}{sin^3x+cos^3x}dx$$

$$2y = \int_0^{\pi/2} 1dx$$

$$2y = (x)_{\cap}^{\frac{\pi}{2}}$$

$$y = \frac{\pi}{4}$$

Q4A

$$y = \int_0^{\pi/2} \frac{sin^7x}{sin^7x+cos^7x}dx \ldots(1)$$

Use King theorem of definite integral

$$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$$

$$y = \int_0^{\pi/2} \frac{sin^7(\frac{\pi}{2}-x)}{sin^7(\frac{\pi}{2}-x)+cos^7(\frac{\pi}{2}-x)}dx$$

$$y = \int_0^{\pi/2} \frac{cos^7x}{sin^7x+cos^7x}dx \ldots(2)$$

Adding eq.(1) and eq.(2)

$2y = \int_0^{\pi/2} \frac{sin^7x}{sin^7x+cos^7x}dx + \int_0^{\pi/2} \frac{cos^7x}{sin^7x+cos^7x}dx$ Type equation here.

$2y = \int_0^{\pi/2} \frac{sin^7x+cos^7x}{sin^7x+cos^7x}dx$

$2y = \int_0^{\pi/2} 1dx$

$2y = (x)_{\cap}^{\frac{\pi}{2}}$

$y = \frac{\pi}{4}$

Q4B

$y = \int_0^{\pi/2} \frac{cos^4x}{sin^4x+cos^4x}dx \ldots(1)$

Use King theorem of definite integral

$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

$y = \int_0^{\pi/2} \frac{cos^4\left(\frac{\pi}{4}-x\right)}{sin^4\left(\frac{\pi}{2}-x\right)+cos^4\left(\frac{\pi}{2}-x\right)}dx$

$y = \int_0^{\pi/2} \frac{sin^4x}{sin^4x+cos^4x}dx \ldots(2)$

Adding eq.(1) and eq.(2)

$2y = \int_0^{\pi/2} \frac{cos^4x}{sin^4x+cos^4x}dx + \int_0^{\pi/2} \frac{sin^4x}{sin^4x+cos^4x}dx$

$2y = \int_0^{\pi/2} \frac{cos^4x+sin^4x}{sin^4x+cos^4x}dx$

$2y = \int_0^{\pi/2} 1dx$

$$2y = (x)_{\cap}^{\frac{\pi}{2}}$$

$$y = \frac{\pi}{4}$$

Q5

$$y = \int_0^{\pi/2} \frac{\cos^4 x}{\sin^4 x + \cos^4 x}\, dx \ldots\ldots\ldots (1)$$

Use King theorem of definite integral

$$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$$

$$y = \int_0^{\pi/2} \frac{cos^4\left(\frac{\pi}{2}-x\right)}{sin^4\left(\frac{\pi}{2}-x\right)+cos^4\left(\frac{\pi}{2}-x\right)}\, dx$$

$$y = \int_0^{\pi/2} \frac{sin^4 x}{sin^4+cos^4 x}\, dx \ldots\ldots (2)$$

Adding eq.(1) and eq.(2)

$$2y = \int_0^{\pi/2} \frac{cos^4 x}{sin^4 x+cos^4 x}\, dx + \int_0^{\pi/2} \frac{sin^4 x}{sin^4 x+cos^4 x}\, dx$$

$$2y = \int_0^{\pi/2} \frac{cos^4 x+sin^4 x}{sin^4 x+cos^4 x}\, dx$$

$$2y = \int_0^{\pi/2} 1 dx$$

$$2y = (x)_0^{\frac{\pi}{2}}$$

$$y = \frac{\pi}{4}$$

Q6

$$y = \int_0^{\pi/2} \frac{\cos^{\frac{1}{4}} x}{\sin^{\frac{1}{4}} x + \cos^{\frac{1}{4}} x} dx \ldots\ldots\ldots (1)$$

Use King theorem of definite integral

$$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$$

$$y = \int_0^{\pi/2} \frac{\cos^{\frac{1}{4}}\left(\frac{\pi}{2}-x\right)}{\sin^{\frac{1}{4}}\left(\frac{\pi}{2}-x\right)+\cos^{\frac{1}{4}}\left(\frac{\pi}{2}-x\right)} dx$$

$$y = \int_0^{\pi/2} \frac{\sin^{\frac{1}{4}} x}{\sin^{\frac{1}{4}} x + \cos^{\frac{1}{4}}} dx \ldots\ldots\ldots (2)$$

Adding eq.(1) and eq.(2)

$$2y = \int_0^{\pi/2} \frac{\cos^{\frac{1}{4}} x}{\sin^{\frac{1}{4}} x + \cos^{\frac{1}{4}} x} dx + \int_0^{\pi/2} \frac{\sin^{\frac{1}{4}} x}{\sin^{\frac{1}{4}} x + \cos^{\frac{1}{4}}} dx$$

$$2y = \int_0^{\pi/2} \frac{\cos^{\frac{1}{4}} x + \sin^{\frac{1}{4}} x}{\sin^{\frac{1}{4}} x + \cos^{\frac{1}{4}} x} dx$$

$$2y = \int_0^{\pi/2} 1 \, dx$$

$$2y = (x)_0^{\frac{\pi}{2}}$$

$$y = \frac{\pi}{4}$$

Q7

$$y = \int_0^{\pi/2} \frac{\sin^{\frac{3}{2}}x}{\sin^{\frac{3}{2}}x+\cos^{\frac{3}{2}}x}\,dx \ldots\ldots\ldots (1)$$

Use King theorem of definite integral

$$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$$

$$y = \int_0^{\pi/2} \frac{\sin^{\frac{3}{2}}\left(\frac{\pi}{2}-x\right)}{\sin^{\frac{3}{2}}\left(\frac{\pi}{2}-x\right)+\cos^{\frac{3}{2}}\left(\frac{\pi}{2}-x\right)}\,dx$$

$$y = \int_0^{\pi/2} \frac{\cos^{\frac{3}{2}}x}{\sin^{\frac{3}{2}}x+\cos^{\frac{3}{2}}x}\,dx \ldots\ldots\ldots (2)$$

Adding eq.(1) and eq.(2)

$$2y = \int_0^{\pi/2} \frac{\sin^{\frac{3}{2}}x}{\sin^{\frac{3}{2}}x+\cos^{\frac{3}{2}}x}\,dx + \int_0^{\pi/2} \frac{\cos^{\frac{3}{2}}x}{\sin^{\frac{3}{2}}x+\cos^{\frac{3}{2}}x}\,dx$$

$$2y = \int_0^{\pi/2} \frac{\sin^{\frac{3}{2}}x+\cos^{\frac{3}{2}}x}{\sin^{\frac{3}{2}}x+\cos^{\frac{3}{2}}x}\,dx$$

$$2y = \int_0^{\pi/2} 1\,dx$$

$$2y = (x)_0^{\frac{\pi}{2}}$$

$$y = \frac{\pi}{4}$$

Q8

$$y = \int_0^{\pi/2} \frac{\sin^n x}{\sin^n x+\cos^n x}\,dx \ldots\ldots (1)$$

Use King theorem of definite integral

$$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$$

$$y = \int_0^{\pi/2} \frac{sin^n\left(\frac{\pi}{2}-x\right)}{sin^n\left(\frac{\pi}{2}-x\right)+cos^n\left(\frac{\pi}{2}-x\right)} dx$$

$$y = \int_0^{\pi/2} \frac{cos^n x}{sin^n x+cos^n x} dx \ldots\ldots\ldots (2)$$

Adding eq.(1) and eq.(2)

$$2y = \int_0^{\pi/2} \frac{sin^n x}{sin^n x+cos^n x} dx + \int_0^{\pi/2} \frac{cos^n x}{sin^n x+cos^n x} dx$$

$$2y = \int_0^{\pi/2} \frac{sin^n x+cos^n x}{sin^n x+cos^n x} dx$$

$$2y = \int_0^{\pi/2} 1 dx$$

$$2y = (x)_0^{\frac{\pi}{2}}$$

$$y = \frac{\pi}{4}$$

Q9

$$y = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\frac{sinx}{cosxx}}}{\sqrt{\frac{sinx}{cosxx}}+\sqrt{\frac{cosx}{sinx}}} dx$$

$$y = \int_0^{\frac{\pi}{2}} \frac{sinx}{sinx+cosx} dx \ldots\ldots\ldots (1)$$

Use King theorem of definite integral

$$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$$

$$y = \int_0^{\frac{\pi}{2}} \frac{sin\left(\frac{\pi}{2}-x\right)}{sin\left(\frac{\pi}{2}-x\right)+cos\left(\frac{\pi}{2}-x\right)} dx$$

$$y = \int_0^{\frac{\pi}{2}} \frac{cosx}{sinx+cosx} dx \ldots\ldots (2)$$

Adding eq.(1) and eq.(2)

$$2y = \int_0^{\pi/2} \frac{sinx}{sinx+cosx} dx + \int_0^{\pi/2} \frac{cosx}{sinx+cosx} dx$$

$$2y = \int_0^{\pi/2} \frac{sinx+cosx}{sinx+cosx} dx$$

$$2y = \int_0^{\pi/2} 1dx$$

$$2y = (x)_0^{\frac{\pi}{2}}$$

$$y = \frac{\pi}{4}$$

Q10

$$y = \int_0^{\pi/2} \frac{\sqrt{\frac{cosx}{sinx}}}{\sqrt{\frac{sinx}{cosx}}+\sqrt{\frac{cosx}{sinx}}} dx$$

$$y = \int_0^{\pi/2} \frac{cosx}{sinx+cosx} dx \ldots\ldots (1)$$

Use King theorem of definite integral

$$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$$

$$y = \int_0^{\pi/2} \frac{\cos\left(\frac{\pi}{2}-x\right)}{\sin\left(\frac{\pi}{2}-x\right)+\cos\left(\frac{\pi}{2}-x\right)}\,dx$$

$$y = \int_0^{\pi/2} \frac{\sin x}{\sin x+\cos x}\,dx \ldots\ldots(2)$$

Adding eq.(1) and eq.(2)

$$2y = \int_0^{\pi/2} \frac{\cos x}{\sin x+\cos x}\,dx + \int_0^{\pi/2} \frac{\sin x}{\sin x+\cos x}\,dx$$

$$2y = \int_0^{\pi/2} \frac{\sin x+\cos x}{\sin x+\cos x}\,dx$$

$$2y = \int_0^{\pi/2} 1\,dx$$

$$2y = (x)_0^{\frac{\pi}{2}}$$

$$y = \frac{\pi}{4}$$

Q11

$$y = \int_0^{\frac{\pi}{2}} \frac{1}{1+\frac{\sin x}{\cos x}}\,dx$$

$$y = \int_0^{\frac{\pi}{2}} \frac{\cos x}{\sin x+\cos x}\,dx \ldots\ldots\ldots(1)$$

Use King theorem of definite integral

$$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$$

$$y = \int_0^{\pi/2} \frac{\cos\left(\frac{\pi}{2}-x\right)}{\sin\left(\frac{\pi}{2}-x\right)+\cos\left(\frac{\pi}{2}-x\right)}\,dx$$

$y - \int_0^{\pi/2} \frac{sinx}{sinx+cosx} dx \ldots\ldots\ldots (2)$

Adding eq.(1) and eq.(2)

$2y = \int_0^{\pi/2} \frac{cosx}{sinx+cosx} dx + \int_0^{\pi/2} \frac{sinx}{sinx+cosx} dx$

$2y = \int_0^{\pi/2} \frac{sinx+cosx}{sinx+cosx} dx$

$2y = \int_0^{\pi/2} 1 \, dx$

$2y = (x)_0^{\frac{\pi}{2}}$

$y = \frac{\pi}{4}$

Q12

$y = \int_0^{\frac{\pi}{2}} \frac{1}{1+\frac{cosx}{sinx}} dx$

$y = \int_0^{\frac{\pi}{2}} \frac{sinx}{sinx+cosx} dx \ldots\ldots\ldots (1)$

Use King theorem of definite integral

$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

$y = \int_0^{\pi/2} \frac{sin\left(\frac{\pi}{2}-x\right)}{sin\left(\frac{\pi}{2}-x\right)+cos\left(\frac{\pi}{2}-x\right)} dx$

$y = \int_0^{\pi/2} \frac{cosx}{sinx+cosx} dx \ldots\ldots (2)$

Adding eq.(1) and eq.(2)

$$2y = \int_0^{\pi/2} \frac{sinx}{sinx+cosx}\,dx + \int_0^{\pi/2} \frac{cosx}{sinx+cosx}\,dx$$

$$2y = \int_0^{\pi/2} \frac{sinx+cosx}{sinx+cosx}\,dx$$

$$2y = \int_0^{\pi/2} 1\,dx$$

$$2y = (x)_0^{\frac{\pi}{2}}$$

$$y = \frac{\pi}{4}$$

Q13

$$y = \int_0^{\frac{\pi}{2}} \frac{1}{1+\frac{sin^3x}{cos^3x}}\,dx$$

$$y = \int_0^{\pi/2} \frac{cos^3x}{sin^3x+cos^3x}\,dx \ldots\ldots(1)$$

Use King theorem of definite integral

$$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$$

$$y = \int_0^{\pi/2} \frac{cos^3\left(\frac{\pi}{2}-x\right)}{sin^3\left(\frac{\pi}{2}-x\right)+cos^3\left(\frac{\pi}{2}-x\right)}\,dx$$

$$y = \int_0^{\pi/2} \frac{sin^3x}{sin^3x+cos^3x}\,dx \ldots\ldots(2)$$

Adding eq.(1) and eq.(2)

$$2y = \int_0^{\pi/2} \frac{cos^3x}{sin^3x+cos^3x}\,dx + \int_0^{\pi/2} \frac{sin^3x}{sin^3x+cos^3x}\,dx$$

$$2y = \int_0^{\pi/2} \frac{cos^3x+sin^3x}{sin^3x+cos^3x}\,dx$$

$$2y = \int_0^{\pi/2} 1\, dx$$

$$2y = (x)_0^{\frac{\pi}{2}}$$

$$y = \frac{\pi}{4}$$

Q14

$$y = \int_0^{\frac{\pi}{2}} \frac{1}{1+\frac{cos^3x}{sin^3x}} dx$$

$$y = \int_0^{\pi/2} \frac{sin^3x}{sin^3x+cos^3x} dx \ldots\ldots (1)$$

Use King theorem of definite integral

$$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$$

$$y = \int_0^{\pi/2} \frac{sin^3\left(\frac{\pi}{2}-x\right)}{sin^3\left(\frac{\pi}{2}-x\right)+cos^3\left(\frac{\pi}{2}-x\right)} dx$$

$$y = \int_0^{\pi/2} \frac{cos^3x}{sin^3x+cos^3x} dx \ldots\ldots (2)$$

Adding eq.(1) and eq.(2)

$$2y = \int_0^{\pi/2} \frac{sin^3x}{sin^3x+cos^3x} dx + \int_0^{\pi/2} \frac{cos^3x}{sin^3x+cos^3x} dx$$

$$2y = \int_0^{\pi/2} \frac{sin^3x+cos^3x}{sin^3x+cos^3x} dx$$

$$2y = \int_0^{\pi/2} 1\, dx$$

$$2y = (x)_0^{\frac{\pi}{2}}$$

$y = \frac{\pi}{4}$

Q15

$y = \int_0^{\frac{\pi}{2}} \frac{1}{1+\sqrt{\frac{sinx}{cosx}}} dx$

$y = \int_0^{\pi/2} \frac{\sqrt{cosx}}{(\sqrt{sinx}+\sqrt{cosx})} dx \ldots\ldots\ldots (1)$

Use King theorem of definite integral

$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

$y = \int_0^{\pi/2} \frac{\sqrt{cos\left(\frac{\pi}{2}-x\right)}}{(\sqrt{sin\left(\frac{\pi}{2}-x\right)}+\sqrt{cos\left(\frac{\pi}{2}-x\right)}} dx$

$y = \int_0^{\pi/2} \frac{\sqrt{sinx}}{(\sqrt{cosx}+\sqrt{sinx})} dx \ldots\ldots (2)$

Adding eq.(1) and eq.(2)

$2y = \int_0^{\pi/2} \frac{\sqrt{cosx}}{(\sqrt{sinx}+\sqrt{cosx})} dx + \int_0^{\pi/2} \frac{\sqrt{sinx}}{(\sqrt{cosx}+\sqrt{sinx})} dx$

$2y = \int_0^{\pi/2} \frac{\sqrt{sinx}+\sqrt{cosx}}{(\sqrt{sinx}+\sqrt{cosx})} dx$

$2y = \int_0^{\pi/2} 1\, dx$

$2y = (x)_0^{\frac{\pi}{2}}$

$y = \frac{\pi}{4}$

Q16

$$y = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\frac{cosx}{sinx}}}{1+\sqrt{\frac{cosx}{sinx}}} dx$$

$$y = \int_0^{\pi/2} \frac{\sqrt{cosx}}{(\sqrt{sinx}+\sqrt{cosx})} dx \ldots\ldots\ldots (1)$$

Use King theorem of definite integral

$$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$$

$$y = \int_0^{\pi/2} \frac{\sqrt{cos\left(\frac{\pi}{2}-x\right)}}{\left(\sqrt{sin\left(\frac{\pi}{2}-x\right)}+\sqrt{cos\left(\frac{\pi}{2}-x\right)}\right)} dx$$

$$y = \int_0^{\pi/2} \frac{\sqrt{sinx}}{(\sqrt{cosx}+\sqrt{sinx})} dx \ldots\ldots (2)$$

Adding eq.(1) and eq.(2)

$$2y = \int_0^{\pi/2} \frac{\sqrt{cosx}}{(\sqrt{sinx}+\sqrt{cosx})} dx + \int_0^{\pi/2} \frac{\sqrt{sinx}}{(\sqrt{cosx}+\sqrt{sinx})} dx$$

$$2y = \int_0^{\pi/2} \frac{\sqrt{sinx}+\sqrt{cosx}}{(\sqrt{sinx}+\sqrt{cosx})} dx$$

$$2y = \int_0^{\pi/2} 1\, dx$$

$$2y = (x)_0^{\frac{\pi}{2}}$$

$$y = \frac{\pi}{4}$$

Q17

$$y = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\frac{sinx}{cosx}}}{1+\sqrt{\frac{sinx}{cosx}}} dx$$

$$y = \int_0^{\pi/2} \frac{\sqrt{sinx}}{(\sqrt{sinx}+\sqrt{cosx})} dx \ldots\ldots (1)$$

Use King theorem of definite integral

$$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$$

$$y = \int_0^{\pi/2} \frac{\sqrt{sin\left(\frac{\pi}{2}-x\right)}}{\left(\sqrt{sin\left(\frac{\pi}{2}-x\right)}+\sqrt{cos\left(\frac{\pi}{2}-x\right)}\right)} dx$$

$$y = \int_0^{\pi/2} \frac{\sqrt{cosx}}{(\sqrt{cosx}+\sqrt{sinx})} dx \ldots . (2)$$

Adding eq.(1) and eq.(2)

$$2y = \int_0^{\pi/2} \frac{\sqrt{sinx}}{(\sqrt{sinx}+\sqrt{cosx})} dx + \int_0^{\pi/2} \frac{\sqrt{cosx}}{(\sqrt{cosx}+\sqrt{sinx})} dx$$

$$2y = \int_0^{\pi/2} \frac{\sqrt{sinx}+\sqrt{cosx}}{(\sqrt{sinx}+\sqrt{cosx})} dx$$

$$2y = \int_0^{\pi/2} 1\, dx$$

$$2y = (x)_0^{\frac{\pi}{2}}$$

$$y = \frac{\pi}{4}$$

Q18

$y = \int_0^{\frac{\pi}{2}} \frac{sinx-cosx}{1+sinxcosx} dx \ldots\ldots (1)$

Use King theorem of definite integral

$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

$y = \int_0^{\frac{\pi}{2}} \frac{sin\left(\frac{\pi}{2}-x\right)-cos\left(\frac{\pi}{2}-x\right)}{1+sin\left(\frac{\pi}{2}-x\right)cos\left(\frac{\pi}{2}-x\right)} dx$

$y = \int_0^{\frac{\pi}{2}} \frac{cos-sinx}{1+cosxsinx} dx \ldots\ldots (2)$

Adding eq.(1) and eq.(2)

$2y = \int_0^{\frac{\pi}{2}} \frac{sinx-cosx}{1+sinxcosx} dx + \int_0^{\frac{\pi}{2}} \frac{cosx-sinx}{1+cosxsinx} dx$

$2y = \int_0^{\frac{\pi}{2}} \frac{sinx-cosx+cosx-sinx}{1+cosxsinx} dx$

$2y = \int_0^{\frac{\pi}{2}} 0 \, dx$

y = 0

Q19

$y = \int_0^1 x(1-x)^5 dx$

Use King theorem of definite integral

$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

$y = \int_0^1 (1-x)x^5 dx$

$y = \int_0^1 x^5 - x^6 dx$

$$y = \left(\frac{x^6}{6} - \frac{x^7}{7}\right)_0^1$$

$$y = \frac{1}{6} - \frac{1}{7} = \frac{1}{42}$$

Q20

$$y = \int_0^2 x\sqrt{2-x}dx$$

Use King theorem of definite integral

$$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$$

$$y = \int_0^2 (2-x)\sqrt{x}dx$$

$$y = \int_0^2 2x^{\frac{1}{2}} - x^{\frac{3}{2}}dx$$

$$y = \left(2\frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{x^{\frac{5}{2}}}{\frac{5}{2}}\right)_0^2$$

$$y = \frac{8\sqrt{2}}{3} - \frac{8\sqrt{2}}{5} = \frac{16\sqrt{2}}{15}$$

Q21

$$y = \int_0^{\pi} x\, cos^2 x\, dx \ldots . (1)$$

Use King theorem of definite integral

$$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$$

$y = \int_0^{\pi} (\pi - x)cos^2(\pi - x)dx$

$y = \int_0^{\pi} \pi cos^2 x - xcos^2 x dx \ldots\ldots (2)$

Adding eq.(1) and eq.(2)

$2y = \int_0^{\pi} xcos^2 s dx + \int_0^{\pi} \pi cos^2 x - xcos^2 x dx$

$2y = \int_0^{\pi} \pi cos^2 x dx$

$y = \frac{\pi}{2}\int_0^{\pi} \frac{1+cos2x}{2} dx$

$y = \frac{\pi}{2}\left(\frac{x}{2} + \frac{sin2x}{4}\right)_0^{\pi}$

$y = \frac{\pi}{2}\left(\frac{\pi}{2} + \frac{sin2\pi}{4}\right) = \frac{\pi^2}{4}$

Q22

$y = \int_0^{\pi} \frac{xtanx}{secxcosecx} dx \ldots . (1)$

Use King theorem of definite integral

$\int_a^b f(x)dx = \int_a^b f(a + b - x)dx$

$y = \int_0^{\pi} \frac{(\pi - x)tan(\pi - x)}{sec(\pi - x)\, cosec(\pi - x)} dx$

$y = \int_0^{\pi} \frac{-(\pi - x)tanx}{-sec\, x\, cosec\, x} dx$

$y = \int_0^{\pi} \frac{\pi tanx - xtanx}{secx\, cosecx} dx \ldots . (2)$

Adding eq.(1) and eq.(2)

$$2y = \int_0^{\pi} \frac{xtanx}{secx\ cosecx}dx + \int_0^{\pi} \frac{\pi tanx - xtanx}{secx\ cosecx}dx$$

$$2y = \int_0^{\pi} \frac{\pi tanx}{sec\ x\ cosec\ x}dx$$

$$y = \frac{\pi}{2}\int_0^{\pi} \frac{\frac{sinx}{cosx}}{\frac{1}{cosx}\times\frac{1}{sinx}}dx$$

$$y = \frac{\pi}{2}\int_0^{\pi} \frac{1-cos2x}{2}dx$$

$$y = \frac{\pi}{2}\left(\frac{x}{2} - \frac{sin2x}{4}\right)_0^{\pi}$$

$$y = \frac{\pi}{2}\left(\frac{\pi}{2} - \frac{sin2\pi}{4}\right) = \frac{\pi^2}{4}$$

Q23

$$y = \int_0^{\frac{\pi}{2}} \frac{cos^2x}{sinx+cosx}dx \ldots..(1)$$

Use King theorem of definite integral

$$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$$

$$y = \int_0^{\frac{\pi}{2}} \frac{cos^2\left(\frac{\pi}{2}-x\right)}{sin\left(\frac{\pi}{2}-x\right)+cos\left(\frac{\pi}{2}-x\right)}dx$$

$$y = \int_0^{\frac{\pi}{2}} \frac{sin^2x}{sinx+cosx}dx \ldots..(2)$$

Adding eq.(1) and eq.(2)

$$2y = \int_0^{\frac{\pi}{2}} \frac{cos^2x}{sinx+cosx}dx + \int_0^{\frac{\pi}{2}} \frac{sin^2x}{sinx+cosx}dx$$

$$2y = \int_0^{\frac{\pi}{2}} \frac{1}{sinx+cosx}\,dx$$

$$2y = \frac{1}{\sqrt{2}}\int_0^{\frac{\pi}{2}} \frac{1}{\frac{1}{\sqrt{2}}sinx+\frac{1}{\sqrt{2}}cosx}\,dx$$

$$2y = \frac{1}{\sqrt{2}}\int_0^{\frac{\pi}{2}} \frac{1}{sin\left(x+\frac{\pi}{4}\right)}\,dx$$

$$y = \frac{1}{2\sqrt{2}}\int_0^{\frac{\pi}{2}} cosec\left(x+\frac{\pi}{4}\right)dx$$

$$y = \frac{1}{2\sqrt{2}}\left(ln\left(cosec\left(x+\frac{\pi}{4}\right)-cot\left(x+\frac{\pi}{4}\right)\right)\right)_0^{\frac{\pi}{2}}$$

$$y = \frac{1}{2\sqrt{2}}\left(ln\left(cosec\frac{3\pi}{4}-cot\frac{3\pi}{4}\right)-ln\left(cosec\frac{\pi}{4}-cot\frac{\pi}{4}\right)\right)$$

$$y = \frac{1}{2\sqrt{2}}ln\frac{\sqrt{2}+1}{\sqrt{2}-1}$$

$$y = \frac{1}{2\sqrt{2}}ln\left(\sqrt{2}+1\right)^2 = \frac{1}{\sqrt{2}}ln(\sqrt{2}+1)$$

Q24

$$t = \int_0^{\pi} \frac{x\frac{sinx}{cosx}}{\frac{1}{cosx}+cosx}\,dx$$

$$y = \int_0^{\pi} \frac{xsinx}{1+cos^2x}\,dx \ldots(1)$$

Use King theorem of definite integral

$$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$$

$y = \int_0^{\pi} \frac{(\pi - x)sin(\pi - x)}{1 + cos^2(\pi - x)} dx$

$y = \int_0^{\pi} \frac{\pi sinx - xsinx}{1 + cos^2 x} dx \ldots\ldots (2)$

Adding eq.(1) and eq.(2)

$2y = \int_0^{\pi} \frac{xsinx}{1 + cos^2 x} dx + \int_0^{\pi} \frac{\pi sinx - xsinx}{1 + cos^2 x} dx$

$2y = \int_0^{\pi} \frac{\pi sinx}{1 + cos^2 x} dx$

Let, cos x = t

$\Rightarrow$ -sin x dx = dt

At x = 0, t = 1

At x = π, t = -1

$y = -\frac{\pi}{2} \int_1^{-1} \frac{1}{1 + t^2} dt$

$y = -\frac{\pi}{2} (tan^{-1} t)_0^{-1}$

$y = -\frac{\pi}{2} (tan^{-1}(-1) - tan^{-1} 1)$

$y = \frac{\pi^2}{4}$

Q25

$y = \int_0^{\pi} \frac{xsinx}{1 + sinx} dx \ldots\ldots (1)$

Use King theorem of definite integral

$\int_a^b f(x)dx = \int_a^b f(a + b - x)dx$

$$y = \int_0^{\pi} \frac{(\pi - x)sin(\pi - x)}{1 + sin(\pi - x)} dx$$

$$y = \int_0^{\pi} \frac{\pi sinx}{1 + sinx} - \frac{xsinx}{1 + sinx} dx \ldots \ldots (2)$$

Adding eq.(1) and eq.(2)

$$2y = \int_0^{\pi} \frac{xsinx}{1 + sinx} dx + \int_0^{\pi} \frac{\pi sinx}{1 + sinx} - \frac{xsinx}{1 + sinx} dx$$

$$2y = \int_0^{\pi} \frac{\pi(sinx + 1 - 1)}{1 + sinx} dx$$

$$y = \frac{\pi}{2} \int_0^{\pi} 1 - \frac{1}{1 + sinx} dx$$

$$y = \frac{\pi}{2} \int_0^{\pi} 1 - \frac{1 - sinx}{cos^2 x} dx$$

$$y = \frac{\pi}{2} \int_0^{\pi} 1 - sec^2 x + \frac{sinx}{cos^2 x} dx$$

Let, cos x = t

$\Rightarrow$ -sin x dx = dt

At x = 0, t = 1

At x = π, t = -1

$$y = \frac{\pi}{2} \left((x - tanx)_0^{\pi} - \int_1^{-1} \frac{1}{t^2} dt \right)$$

$$y = \frac{\pi}{2} \left(\pi - tan\pi - \left(-\frac{1}{t} \right)_0^{-1} \right)$$

$$y = \frac{\pi}{2} (\pi - 2) = \pi \left(\frac{\pi}{2} - 1 \right)$$

Q26

$y = \int_0^{\pi} \frac{x}{1+sin^2x} dx \ldots\ldots (1)$

Use King theorem of definite integral

$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

$y = \int_0^{\pi} \frac{(\pi-x)}{1+sin^2(\pi-x)} dx$

$y = \int_0^{\pi} \frac{\pi}{1+sin^2x} - \frac{\pi}{1+sin^2x} dx \ldots . (2)$

Adding eq.(1) and eq.(2)

$2y = \int_0^{\pi} \frac{x}{1+sin^2x} dx + \int_0^{\pi} \frac{\pi}{1+sin^2x} - \frac{x}{1+sin^2x} dx$

$y = \frac{\pi}{2} \int_0^{\pi} \frac{1}{1+sin^2x} dx$

$y = \frac{\pi}{2} \int_0^{\pi} \frac{\frac{1}{cos^2x}}{\frac{1+sin^2x}{cos^2x}} dx$

$y = \frac{\pi}{2} \int_0^{\pi} \frac{sec^2x}{sec^2x+tan^2x} dx$

We break it in two parts

$y = \frac{\pi}{2} \int_0^{\pi} \frac{sec^2x}{sec^2x+tan^2x} dx$

Let, tan x = t

$\Rightarrow sec^2x\ dx = dt$

At x = 0, t = 0

At x = π, t = 0

$y = \frac{\pi}{2} \int_0^{0} \frac{1}{1+2t^2} dt$

We know that when upper and lower limit is same in definite integral then value of integration is 0.

So, y = 0

Q27

$$y = \int_0^{\frac{\pi}{2}} log \frac{cos^2 x}{sin2x} dx$$

$$y = \int_0^{\frac{\pi}{2}} log \frac{cos^2 x}{2sinx\ cosx} dx$$

$$y = \int_0^{\frac{\pi}{2}} log \left(\frac{1}{2} cotx\right) dx \ldots\ldots (1)$$

Use King theorem of definite integral

$$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$$

$$y = \int_0^{\frac{\pi}{2}} log \left(\frac{1}{2} cot\left(\frac{\pi}{2} - x\right)\right) dx$$

$$y = \int_0^{\frac{\pi}{2}} log \left(\frac{1}{2} tanx\right) dx \ldots\ldots (2)$$

Adding eq.(1) and eq.(2)

$$2y = \int_0^{\frac{\pi}{2}} log \left(\frac{1}{2} cotx\right) dx + \int_0^{\frac{\pi}{2}} log \left(\frac{1}{2} tanx\right) dx$$

$$y = \frac{1}{2}\int_0^{\frac{\pi}{2}} log \left(\frac{1}{4} cotx\ tanx\right) dx \quad [\text{Use cot x tan x } = \text{ 1}]$$

$$y = \frac{1}{2}\int_0^{\frac{\pi}{2}} log \left(\frac{1}{4}\right) dx$$

$$y = \frac{1}{2} log\left(\frac{1}{4}\right)(x)_0^{\frac{\pi}{2}}$$

$$y = -\frac{\pi}{4} log4$$

Q28

$$y = \int_0^{\infty} \frac{x}{(1+x)(1+x^2)} dx$$

Let, x = tan t

$\Rightarrow$ dx = sec^2t dt

At x = 0, t = 0

At x = ∞, t = $\pi/2$

$$y = \int_0^{\frac{\pi}{2}} \frac{tant}{(1+tant)(1+tan^2t)} sec^2t\, dt$$

$$y = \int_0^{\frac{\pi}{2}} \frac{tant}{(1+tant)} dt$$

$$y = \int_0^{\frac{\pi}{2}} \frac{sint}{(cost+sint)} dt \ldots\ldots (1)$$

Use King theorem of definite integral

$$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$$

$$y = \int_0^{\pi/2} \frac{sin\left(\frac{\pi}{2}-t\right)}{sin\left(\frac{\pi}{2}-t\right)+cos\left(\frac{\pi}{2}-t\right)} dt$$

$$y = \int_0^{\pi/2} \frac{cost}{sint+cost} dt \ldots\ldots (2)$$

Adding eq.(1) and eq.(2)

$$2y = \int_0^{\pi/2} \frac{sint}{sint+cost}\,dx + \int_0^{\pi/2} \frac{cost}{sint+cost}\,dx$$

$$2y = \int_0^{\pi/2} \frac{sint+cost}{sint+cost}\,dx$$

$$2y = \int_0^{\pi/2} 1\,dx$$

$$2y(x)_0^{\frac{\pi}{2}}$$

$$y = \frac{\pi}{4}$$

Q29

Let, x = a sin t

⇒ dx = a cos t dt

At x = 0, t = 0

At x = a, t = π/2

$$y = \int_0^{\frac{\pi}{2}} \frac{a\,cost}{a\,sint+\sqrt{a^2-a^2sin^2t}}\,dt$$

$$y = \int_0^{\frac{\pi}{2}} \frac{cost}{sint+cost}\,dt$$

$$y = \frac{1}{2}\int_0^{\frac{\pi}{2}} \frac{cost+cost-sint+sint}{sint+cost}\,dt$$

$$y = \frac{1}{2}\int_0^{\frac{\pi}{2}} 1 + \frac{cost-sint}{sint+cost}\,dt$$

$$y = \frac{1}{2}\left((t)_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} \frac{cost-sint}{sint+cost}\,dt\right)$$

Again, sin t + cos t = z

$\Rightarrow (\cos t - \sin t)\, dt = dz$

At $t = 0$, $z = 1$

At $t = \pi/2$, $z = 1$

$$y = \frac{1}{2}\left(\frac{\pi}{2} + \int_1^1 \frac{1}{z} dz\right)$$

$$y = \frac{1}{2}\left(\frac{\pi}{2} + (ln\ z)_1^1\right)$$

$$y = \frac{\pi}{4}$$

Q30

$$y = \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}} dx \ldots\ldots (1)$$

Use King theorem of definite integral

$$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$$

$$y = \int_0^a \frac{\sqrt{a-x}}{\sqrt{a-x}+\sqrt{x}} dx \ldots\ldots (2)$$

Adding eq.(1) and eq.(2)

$$2y = \int_0^a \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}} dx + \int_0^a \frac{\sqrt{a-x}}{\sqrt{a-x}+\sqrt{x}} dx$$

$$2y = \int_0^a \frac{\sqrt{x}+\sqrt{a-x}}{\sqrt{a-x}+\sqrt{x}} dx$$

$$y = \frac{1}{2}\int_0^a dx$$

$$y = \frac{1}{2}(x)_o^a$$

$$y = \frac{a}{2}$$

Q31

$y = \int_0^{\pi} sin^2 x cos^3 x dx \ldots\ldots (1)$

Use King theorem of definite integral

$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

$y = \int_0^{\pi} sin^2(\pi - x) cos^3(\pi - x) dx$

$y = -\int_0^{\pi} sin2\, x cos^3 x dx \ldots\ldots (2)$

Adding eq.(1) and eq.(2)

$2y = \int_0^{\pi} sin^2 x cos^3 x dx + \left(-\int_0^{\pi} sin^2 x cos^3 x dx\right)$

y = 0

Q32

$y = \int_0^{\pi} sin^{2m} x cos^{2m+1}\, x dx \ldots\ldots (1)$

Use King theorem of definite integral

$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

$y = \int_0^{\pi} sin^{2m}(\pi - x) cos^{2m+1}(\pi - x) dx$

$y = -\int_0^{\pi} sin^{2m} x cos^{2m+1} x dx \ldots. (2)$

Adding eq.(1) and eq.(2)

$2y = \int_0^{\pi} sin^{2m} x cos^{2m+1} x dx + \left(-\int_0^{\pi} sin^{2m} x cos^{2m+1} x dx\right)$

y = 0

Q33

Let, sin x + cos x = t

⇒ cos x – sin x dx = dt

At x = 0, t = 1

At x = π/2, t = 1

$y = \int_1^1 -logt\ dt$

We know that when upper and lower limit in definite integral is equal then value of integration is zero.

So, y = 0

Q34

$y = \int_0^{\frac{\pi}{2}} log(2sinx\ cosx)dx$

$y = \int_0^{\frac{\pi}{2}} log2 + log\ sinx + log\ cosx\ dx$

Let,

$I = \int_0^{\frac{\pi}{2}} logsinx\ dx \ldots.(1)$

Use King theorem of definite integral

$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

$I = \int_0^{\pi} logsin\left(\frac{\pi}{2} - x\right)dx$

$I = \int_0^{\frac{\pi}{2}} logcosx\ dx \ldots\ldots (2)$

Adding eq.(1) and eq.(2)

$2I = \int_0^{\pi} logsinx\ dx + \int_0^{\pi} logcosx\ dx$

$2I = \int_0^{\frac{\pi}{2}} log\frac{2sinxcosx}{2} dx$

$2I = \int_0^{\frac{\pi}{2}} logsin2x - log2\ dx$

Let, 2x = t

$\Rightarrow$ 2 dx = dt

At x = 0, t = 0

At x = π/2, t = π

$2I = \frac{1}{2}\int_0^{\frac{\pi}{2}} logsint\ dt - \frac{\pi}{2}log2$

$2I = \frac{2}{2}\int_0^{\frac{\pi}{2}} logsinx\ dx - \frac{\pi}{2}log2$

$2I = I = \frac{\pi}{2}log2$

$I = \int_0^{\frac{\pi}{2}} logsinx\ dx = -\frac{\pi}{2}log2$

Similarly,

$\int_0^{\frac{\pi}{2}} logcosx\ dx = -\frac{\pi}{2}log2$

$y = \int_0^{\frac{\pi}{2}} log2dx + \int_0^{\frac{\pi}{2}} logsinx\ dx + \int_0^{\frac{\pi}{2}} logcosx\ dx$

$y = -\frac{\pi}{2}log2$

$y = -\frac{\pi}{2} log2$

Q35

$y = \int_0^{\pi} xlogsinx\ dx \ldots..(1)$

Use King theorem of definite integral

$\int_a^b f(x)dx = \int_0^b f(a+b-x)dx$

$y = \int_0^{\pi} (\pi - x)logsin(\pi - x)dx$

$y = \int_0^{\pi} \pi logsinx - xlogsinxdx \ldots\ldots(2)$

Adding eq.(1) and eq.(2)

$2y = \int_0^{\pi} xlogsinx\ dx + \int_0^{\pi} \pi logsinx - xlogsinx\ dx$

$y = \frac{\pi}{2} \int_0^{\pi} logsinx\ dx$

$y = \frac{2\pi}{2} \int_0^{\frac{\pi}{2}} logsinx\ dx \ldots..(3)$

Use King theorem of definite integral

$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

$y = \pi \int_0^{\frac{\pi}{2}} logsin\left(\frac{\pi}{2} - x\right) dx$

$y = \pi \int_0^{\frac{\pi}{2}} logcosx\ dx \ldots..(4)$

Adding eq.(3) and eq.(4)

$$2y = \pi\left(\int_0^{\frac{\pi}{2}} logsinx\ dx + \int_0^{\frac{\pi}{2}} logcosx\ dx\right)$$

$$2y = \pi\left(\int_0^{\frac{\pi}{2}} log\frac{2sinx\ cosx}{2}dx\right)$$

$$2y = \pi\left(\int_0^{\frac{\pi}{2}} logsin2x - log2\ dx\right)$$

Let, 2x = t

$\Rightarrow$ 2 dx = dt

At x = 0, t = 0

At x = $\pi/2$, t = π

$$2y = \frac{\pi}{2}\int_0^{\frac{\pi}{2}} logsint\ dt - \frac{\pi^2}{2}log2$$

$$2y = \frac{2\pi}{2}\int_0^{\pi} logsinx\ dx - \frac{\pi^2}{2}log2$$

$$2y = y - \frac{\pi^2}{2}log2$$

$$y = -\frac{\pi^2}{2}log2$$

Q36

$$y = \int_0^{\pi} log(1 + cosx)dx \ldots..(1)$$

Use King theorem of definite integral

$$\int_a^b f(x)dx = \int_a^b f(a + b - x)dx$$

$$y = \int_0^{\pi} log\big(1 + cos(\pi - x)\big)dx$$

$y = \int_0^{\pi} log(1 - cosx)dx$ ... ... (2)

Adding eq.(1) and eq.(2)

$2y = \int_0^{\pi} log(1 + cosx)dx + \int_0^{\pi} log(1 - cosx)dx$

$2y = \int_0^{\pi} logsin^2 x\, dx$

$y = 2\int_0^{\frac{\pi}{2}} logsinx\, dx$ ... ... (3)

Use King theorem of definite integral

$\int_a^b f(x)dx = \int_a^b f(a + b - x)dx$

$y = 2\int_0^{\frac{\pi}{2}} logsin\left(\frac{\pi}{2} - x\right)dx$

$y = 2\int_0^{\frac{\pi}{2}} logcosx\, dx$ ... ... (4)

Adding eq.(3) and eq.(4)

$2y = 2\left(\int_0^{\frac{\pi}{2}} logsinx\, dx + \int_0^{\frac{\pi}{2}} logcosx\, dx\right)$

$2y = 2\left(\int_0^{\frac{\pi}{2}} log\frac{2sinxcosx}{2}dx\right)$

$2y = 2\left(\int_0^{\frac{\pi}{2}} logsin2x - log2\, dx\right)$

Let, 2x = t

$\Rightarrow$ 2 dx = dt

At x = 0, t = 0

At x = π/2, t = π

$$2y = \frac{2}{2}\int_0^{\frac{\pi}{2}} logsint\ dt - \frac{2\pi}{2}log2$$

$$2y = \frac{4}{2}\int_0^{\frac{\pi}{2}} logsinx\ dx - \frac{2\pi}{2}log2$$

$$2y = y - \pi log2$$

$$y = -\pi log2$$

Q37

$$y = \int_0^{\frac{\pi}{2}} log\left(\frac{sinx}{cosx} + \frac{cosx}{sinx}\right) dx$$

$$y = \int_0^{\frac{\pi}{2}} log\frac{1}{sinxcosx}\, dx$$

$$y = -\left(\int_0^{\frac{\pi}{2}} logsinx\ dx + \int_0^{\frac{\pi}{2}} logcosx\ dx\right)$$

Let,

$$I = \int_0^{\frac{\pi}{2}} logsinx\ dx \ldots\ldots (1)$$

Use King theorem of definite integral

$$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$$

$$I = \int_0^{\frac{\pi}{2}} logsin\left(\frac{\pi}{2} - x\right) dx$$

$$I = \int_0^{\frac{\pi}{2}} logcosx\ dx \ldots\ldots (2)$$

Adding eq.(1) and eq.(2)

$2I = \int_0^{\frac{\pi}{2}} logsinx\ dx + \int_0^{\frac{\pi}{2}} logcosx\ dx$

$2I = \int_0^{\frac{\pi}{2}} log\frac{2sinxcosx}{2} dx$

$2I = \int_0^{\frac{\pi}{2}} logsin\ 2x - log2\ dx$

Let, 2x = t

$\Rightarrow$ 2 dx = dt

At x = 0, t = 0

At x = π/2, t = π

$2I = \frac{1}{2}\int_0^{\frac{\pi}{2}} logsint\ dt - \frac{\pi}{2}log2$

$2I = \frac{2}{2}\int_0^{\frac{\pi}{2}} logsinx\ dx - log2$

$2I = I - \frac{\pi}{2}log2$

$I = \int_0^{\frac{\pi}{2}} logsinx\ dx = -\frac{\pi}{2}log2$

Similarly,

$\int_0^{\frac{\pi}{2}} logcosx\ dx = -\frac{\pi}{2}log2$

$y = -\left(\int_0^{\frac{\pi}{2}} logsix\ dx + \int_0^{\frac{\pi}{2}} logcosxdx\right)$

$y = \frac{\pi}{2}log2 + \frac{\pi}{2}log2$

$y = \pi log2$

Q38

$$y = \int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} \frac{cosx}{cosx+sinx} dx \ldots\ldots (1)$$

Use King theorem of definite integral

$$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$$

$$y = \int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} \frac{cos\left(\frac{3\pi}{8}+\frac{\pi}{8}-x\right)}{sin\left(\frac{3\pi}{8}+\frac{\pi}{8}-x\right)+cos\left(\frac{3\pi}{8}+\frac{\pi}{8}-x\right)} dx$$

$$y = \int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} \frac{sinx}{sinx+cosx} dx \ldots\ldots (2)$$

Adding eq.(1) and eq.(2)

$$2y = \int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} \frac{cosx}{sinx+cosx} dx + \int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} \frac{sinx}{sinx+cosx} dx$$

$$\int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} \frac{sinx+cosx}{sinx+cosx} dx$$

$$2y = \int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} 1\, dx$$

$$2y = (x)_{\frac{\pi}{8}}^{\frac{3\pi}{8}}$$

$$2y = \frac{3\pi}{8} - \frac{\pi}{8} = \frac{\pi}{8}$$

Q39

$$y = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{cosx}}{\sqrt{sinx}+\sqrt{cosx}}\,dx$$

Use King theorem of definite integral

$$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$$

$$y = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{cos\left(\frac{\pi}{3}+\frac{\pi}{6}-x\right)}}{\left(\sqrt{sin\left(\frac{\pi}{3}+\frac{\pi}{6}-x\right)}+\sqrt{cos\left(\frac{\pi}{3}+\frac{\pi}{6}-x\right)}\right)}\,dx$$

$$y = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{sinx}}{(\sqrt{cosx}+\sqrt{sinx})}\,dx \ldots\ldots(2)$$

Adding eq.(1) and eq.(2)

$$2y = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{cosx}}{(\sqrt{sinx}+\sqrt{cosx})}\,dx + \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{sinx}}{(\sqrt{cosx}+\sqrt{sinx})}\,dx$$

$$2y = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{sinx}+\sqrt{cox}}{(\sqrt{sinx}+\sqrt{cosx})}\,dx$$

$$2y = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 1\,dx$$

$$2y = (x)_{\frac{\pi}{6}}^{\frac{\pi}{3}}$$

$$y = \frac{\pi}{12}$$

Q40

$$y = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{1}{2cos^2\frac{x}{2}}\,dx$$

$$y = \frac{1}{2}\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} sec^2\frac{x}{2}\,dx$$

$$y = \frac{1}{2}\left(\frac{tanx}{\frac{2}{\frac{1}{2}}}\right)_{\frac{\pi}{4}}^{\frac{3\pi}{4}}$$

$$y = tan\frac{3\pi}{8} - tan\frac{\pi}{8}$$

$$y = \left(\sqrt{2}+1\right) - \left(\sqrt{2}-1\right) = 2$$

Q 41

$$y = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{x}{1+sinx}\,dx \ldots\ldots(1)$$

Use King theorem of definite integral

$$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$$

$$y = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{\left(\frac{3\pi}{4}+\frac{\pi}{4}-x\right)}{1+sin\left(\frac{3\pi}{4}+\frac{\pi}{4}-x\right)}\,dx$$

$$y = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{\pi-x}{1+sinx}\,dx \ldots\ldots(2)$$

Adding eq.(1) and eq.(2)

$$2y = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{x}{1+sinx}\,dx + \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{\pi-x}{1+sinx}\,dx$$

$$y = \frac{\pi}{2}\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{1}{1+sinx}\,dx$$

$$y = \frac{\pi}{2}\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{1}{1+sinx} \times \frac{1-sinx}{1-sinx} dx$$

$$y = \frac{\pi}{2}\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{(1-sinx)}{cos^2 x} dx$$

$$y = \frac{\pi}{2}\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} sec^2 x - \frac{sinx}{cos^2 x} dx$$

Let, cos x = t

$\Rightarrow$ -sin x dx = dt

At x = π/4, t = $\frac{1}{\sqrt{2}}$

At x = 3π/4, t = $\frac{-1}{\sqrt{2}}$

$$y = \frac{\pi}{2}\left((tanx)_{\frac{\pi}{4}}^{\frac{3\pi}{4}} + \int_{\frac{1}{\sqrt{2}}}^{-\frac{1}{\sqrt{2}}} \frac{1}{t^2} dt\right)$$

$$y = \frac{\pi}{2}\left(tan\frac{3\pi}{4} - tan\frac{\pi}{4} + \left(\frac{-1}{t}\right)_{\frac{1}{\sqrt{2}}}^{-\frac{1}{\sqrt{2}}}\right.$$

$$y = \frac{\pi}{2}\left(-1 - 1 + \sqrt{2} + \sqrt{2}\right) = \pi\left(\sqrt{2} - 1\right)$$

Q42

$$y = \int_{\frac{a}{4}}^{\frac{3a}{4}} \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}} dx \ldots . (1)$$

Use King theorem of definite integral

$$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$$

$$y = \int_{\frac{a}{4}}^{\frac{3a}{4}} \frac{\sqrt{\frac{3a}{4}+\frac{a}{4}-x}}{\sqrt{\frac{3a}{4}+\frac{a}{4}-x}+\sqrt{x}}dx$$

$$y = \int_{\frac{a}{4}}^{\frac{3a}{4}} \frac{\sqrt{a-x}}{\sqrt{a-x}+\sqrt{x}}dx$$

Adding eq.(1) and eq.(2)

$$2y = \int_{\frac{a}{4}}^{\frac{3a}{4}} \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}}dx + \int_{\frac{a}{4}}^{\frac{3a}{4}} \frac{\sqrt{a-x}}{\sqrt{a-x}+\sqrt{x}}dx$$

$$2y = \int_{\frac{a}{4}}^{\frac{3a}{4}} \frac{\sqrt{x}+\sqrt{a-x}}{\sqrt{a-x}+\sqrt{x}}dx$$

$$y = \frac{1}{2}\int_{\frac{a}{4}}^{\frac{3a}{4}} 1\,dx$$

$$y = \frac{1}{2}(x)_{\frac{a}{4}}^{\frac{3a}{4}}$$

$$y = \frac{a}{4}$$

Q43

$$y = \int_1^4 \frac{\sqrt{x}}{\sqrt{5-x}+\sqrt{x}}dx$$

Use King theorem of definite integral

$$\int_a^b f(x) = \int_a^b f(a+b-x)dx$$

$$y = \int_1^4 \frac{\sqrt{4+1-x}}{\sqrt{4+1-x}+\sqrt{x}}dx$$

$$y = \int_1^4 \frac{\sqrt{5-x}}{\sqrt{5-x}+\sqrt{x}}dx$$

Adding eq.(1) and eq.(2)

$$2y = \int_1^4 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{5-x}}dx + \int_1^4 \frac{\sqrt{5-x}}{\sqrt{5-x}+\sqrt{x}}dx$$

$$2y = \int_1^4 \frac{\sqrt{x}+\sqrt{5-x}}{\sqrt{5-x}+\sqrt{x}}dx$$

$$y = \frac{1}{2}\int_1^4 1\,dx$$

$$y = \frac{1}{2}(x)_1^4$$

$$y = \frac{3}{2}$$

Q44

Use integration by parts

$$\int I \times II dx = I\int II\,dx - \int \frac{d}{dx}I\left(\int II\,dx\right)dx$$

$$y = x\int cotx\,dx - \int \frac{d}{dx}x\left(\int cotx\,dx\right)dx$$

$$y = (xlogsinx)_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} logsinx\,dx$$

Let,

$$I = \int_0^{\frac{\pi}{2}} logsinx\,dx \ldots\ldots(1)$$

Use King theorem of definite integral

$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

$I = \int_0^{\frac{\pi}{2}} logsin\left(\frac{\pi}{2} - x\right) dx$

$I = \int_0^{\frac{\pi}{2}} logcosx\, dx$ ..... (2)

Adding eq.(1) and eq.(2)

$2I = \int_0^{\frac{\pi}{2}} logsinx\, dx + \int_0^{\frac{\pi}{2}} logcosx\, dx$

$2I = \int_0^{\frac{\pi}{2}} log\frac{2sinxcosx}{2} dx$

$2I = \int_0^{\frac{\pi}{2}} logsin2x - log2\, dx$

Let, 2x = t

$\Rightarrow$ 2 dx = dt

At x = 0, t = 0

At x = π/2, t = π

$2I = \frac{1}{2}\int_0^{\frac{\pi}{2}} logsint\, dt - \frac{\pi}{2}log2$

$2I = \frac{2}{2}\int_0^{\frac{\pi}{2}} logsinx\, dx - \frac{\pi}{2}log2$

$2I = I - \frac{\pi}{2}log2$

$I = \int_0^{\frac{\pi}{2}} logsinx\, dx = -\frac{\pi}{2}log2$

$y = (xlogsinx)_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} logsinx\, dx$

$y = \frac{\pi}{2} log sin \frac{\pi}{2} - \left(-\frac{\pi}{2} log 2\right)$

$y = \frac{\pi}{2} log 2$

Q45

Let, x = sin t

⇒ dx = cos t dt

At x = 0, t = 0

At x = 1, t = π/2

$y = \int_0^{\frac{\pi}{2}} \frac{sin^{-1} sint}{sint} cost\ dt$

$y = \int_0^{\frac{\pi}{2}} \frac{tcost}{sint} dt$

$y = \int_0^{\frac{\pi}{2}} 1 cott\ dt$

Use integration by parts

$\int I \times II\ dt = I \int II\ dt - \int \frac{d}{dt} I \left(\int II\ dt\right) dt$

$y = t \int cott\ dt - \int \frac{d}{dt} t \left(\int cott\ dt\ \right) dt$

$y = (tlogsint)_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} logsint\ dt$

Let,

$I = \int_0^{\frac{\pi}{2}} logsint\ dt \ldots \ldots (1)$

Use King theorem of definite integral

$\int_a^b f(t)dt = \int_a^b f(a+b-t)dt$

$I = \int_0^{\pi} logsin\left(\frac{\pi}{2} - t\right) dt$

$I = \int_0^{\frac{\pi}{2}} logcost\ dt \ldots\ldots(2)$

Adding eq.(1) and eq.(2)

$2I = \int_0^{\frac{\pi}{2}} logsint\ dt + \int_0^{\frac{\pi}{2}} logcost\ dt$

$2I = \int_0^{\frac{\pi}{2}} log\frac{2sintcost}{2} dt$

$2I = \int_0^{\frac{\pi}{2}} logsin2t - log2dt$

Let, 2t = z

$\Rightarrow$ 2 dt = dz

At t = 0, z = 0

At t = π/2, z = π

$2I = \frac{1}{2}\int_0^{\pi} logsinz\ dz - \frac{\pi}{2}log2$

$2I = \frac{2}{2}\int_0^{\frac{\pi}{2}} logsinz\ dz = \frac{\pi}{2}log2$

$2I = I - \frac{\pi}{2}log2$

$I = \int_0^{\frac{\pi}{2}} logsinx\ dz = -\frac{\pi}{2}log2$

$y = (tlogsint)_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} logt\ dt$

$y = \frac{\pi}{2} log sin \frac{\pi}{2} - \left(-\frac{\pi}{2} log 2\right)$

$y = \frac{\pi}{2} log 2$

Q46

Use integration by parts

$\int I \times II \, dt = I \int II \, dt - \int \frac{d}{dt} I \left(\int II \, dt\right) dt$

$y = logx \int \frac{\sqrt{1}}{\sqrt{1-x^2}} dx - \int \frac{d}{dx} logx \left(\int \frac{1}{\sqrt{1-x^2}} dx\right) dx$

$y = (logx sin^{-1}x)_0^1 - \int_0^1 \frac{sin^{-1}x}{x} dx$

$y = -\int_0^1 \frac{sin^{-1}x}{x} dx$

Let, x = sin t

⇒ dx = cos t dt

At x = 0, t = 0

At x = 1, t = π/2

$y = -\int_0^{\frac{\pi}{2}} \frac{sin^{-1}sint}{sint} cost \, dt$

$y = -\int_0^{\frac{\pi}{2}} \frac{tcost}{sint} dt$

$y = -\int_0^{\frac{\pi}{2}} tcott \, dt$

Use integration by parts

$$\int I \times II\, dt = I \int II\, dt - \int \frac{d}{dt} I \left(\int II\, dt\right) dt$$

$$y = -(t \int cott\, dt - (\int cott\, dt)\, dt)$$

$$y = -\left((tlogsint)_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} logsint\, dt\right)$$

Let,

$$I = \int_0^{\frac{\pi}{2}} logsint\, dt \ldots\ldots (1)$$

Use King theorem of definite integral

$$\int_a^b f(t)dt = \int_a^b f(a+b-x)dt$$

$$I = \int_0^{\frac{\pi}{2}} logsin\left(\frac{\pi}{2} - t\right) dt$$

$$I = \int_0^{\frac{\pi}{2}} logcost\, dt \ldots\ldots (2)$$

Adding eq.(1) and eq.(2)

$$2I = \int_0^{\frac{\pi}{2}} logsint\, dt + \int_0^{\frac{\pi}{2}} logcost\, dt$$

$$2I = \int_0^{\frac{\pi}{2}} log\, \frac{2sintcost}{2} dt$$

$$2I = \int_0^{\frac{\pi}{2}} logsin2t - log2dt$$

Let, 2t = z

$\Rightarrow$ 2 dt = dz

At t = 0, z = 0

At t = π/2, z = π

$$2I = \frac{1}{2}\int_0^{\frac{\pi}{2}} logsinzdz - \frac{\pi}{2}log2$$

$$2I = \frac{2}{2}\int_0^{\frac{\pi}{2}} logsinz\, dz - \frac{\pi}{2}log2$$

$$2I = I - \frac{\pi}{2}log2$$

$$I = \int_0^{\frac{\pi}{2}} logsinz\, dz = -\frac{\pi}{2}log2$$

$$y = -\left((tlogsint)_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} logt\, dt\right)$$

$$y = -\frac{\pi}{2}logsin\frac{\pi}{2} + \left(-\frac{\pi}{2}log2\right)$$

$$y = \frac{-\pi}{2}log2$$

Q47

Let x = tan t

⇒ dx = sec^2t dt

At x = 0, t = 0

At x = 1, t = π/4

$$y = \int_0^{\frac{\pi}{4}} \frac{log(1+tant)}{1+tan^2t} sec^2tdt$$

$$y = \int_0^{\frac{\pi}{4}} log(1+tant)dt \ldots\ldots(1)$$

Use King theorem of definite integral

$\int_a^b f(t)dt = \int_a^b f(a+b-t)dt$

$y = \int_0^{\frac{\pi}{4}} log\left(1 + tan\left(\frac{\pi}{4} - t\right)\right) dt$

$y = \int_0^{\frac{\pi}{4}} log\left(1 + \frac{1-tant}{1+tant}\right) dt$

$y = \int_0^{\frac{\pi}{4}} log\left(\frac{2}{1+tant}\right) dt \ldots\ldots (2)$

Adding eq.(1) and eq.(2)

$2y = \int_0^{\frac{\pi}{4}} log(1+tant)dt + \int_0^{\frac{\pi}{4}} log\left(\frac{2}{1+tant}\right) dt$

$2y = \int_0^{\frac{\pi}{4}} log(1+tant)\left(\frac{2}{1+tant}\right) dt$

$2y = \int_0^{\frac{\pi}{4}} log2\, dt$

$y = \frac{\pi}{8} log2$

Q 48

$y = \int_{-a}^{a} x^3\sqrt{a^2 - x^2}dx \ldots\ldots (1)$

Use King theorem of definite integral

$\int_a^b f(t)dt = \int_a^b f(a+b-t)dt$

$y = \int_{-a}^{a} (a-a-x)^3\sqrt{a^2 - (a-a-x)^2}dx$

$y = \int_{-a}^{a} -x^3\sqrt{a^2 - x^2}dx \ldots (2)$

Adding eq.(1) and eq.(2)

$2y = \int_{-a}^{a} x^3\sqrt{a^2 - x^2}dx + \left(-\int_{-a}^{a} x^3\sqrt{a^2 - x^2}dx\right)$

y = 0

Q49

$y = \int_{-\pi}^{\pi} sin^{75}x + x^{125}dx \ldots\ldots(1)$

Use King theorem of definite integral

$\int_a^b f(t)dt = \int_a^b f(a + b - t)dt$

$y = \int_{-\pi}^{\pi} sin^{75}(\pi - \pi - x) + (\pi - \pi - x)^{125}dx$

$y = \int_{-\pi}^{\pi} -sin^{75}x - x^{125}dx \ldots..(2)$

Adding eq.(1) and eq.(2)

$2y = \int_{-\pi}^{\pi} sin^{75}x + x^{125}dx + \left(-\int_{\pi}^{\pi} sin^{75}x + x^{125}dx\right)$

y = 0

Q50

$y = \int_{-\pi}^{\pi} x^{12}sin^9 xdx \ldots\ldots(1)$

Use King theorem of definite integral

$\int_a^b f(t)dt = \int_a^b f(a + b - t)dt$

$y = \int_{-\pi}^{\pi} (\pi - \pi - x)^{12}sin^9(\pi - \pi - x)dx$

$y = \int_{-\pi}^{\pi} -x^{12}sin9x\,dx \ldots\ldots(2)$

Adding eq.(1) and eq.(2)

$2y = \int_{-\pi}^{\pi} x^{12} sin^9 dx + \left(-\int_{-\pi}^{\pi} x^{12} sin^9 x dx\right)$

y = 0

Q51

We know that

|x| = -x in [-1, 0)

|x| = x in [0, 1]

$y = \int_{-1}^{0} e^{|x|} dx + \int_{0}^{1} e^{|x|} dx$

$y = \int_{-1}^{0} e^{-x} dx + \int_{0}^{1} e^{x} dx$

$y = (-e^{-x})_{-1}^{0} + (e^{x})_{0}^{1}$

y= -(1-e)+(e-1)

y = 2(e – 1)

Q52

We know that

|x+1| = -(x+1) in [-2, -1)

|x+1| = (x+1) in [-1, 2]

$y = \int_{-2}^{-1} |x+1| dx + \int_{-1}^{2} |x+1| dx$

$= -\int_{-2}^{-1} (x+1) dx + \int_{-1}^{2} (x+1) dx$

$$= -\left(\frac{x^2}{2} + x\right)_{-2}^{-1} + \left(\frac{x^2}{2} + x\right)_{-1}^{2}$$

$$= -\left(\frac{1}{2} - 1 - 2 + 2\right) + \left(2 + 2 - \frac{1}{2} + 1\right)$$

=5

Q53

We know that

|x – 5| = -(x – 5) in [0, 5)

|x – 5| = (x – 5) in [5, 8]

$$y = \int_0^5 |x - 5| dx + \int_5^8 |x - 5| dx$$

$$y = -\int_0^5 (x - 5) dx + \int_5^8 (x - 5) dx$$

$$y = -\left(\frac{x^2}{2} - 5x\right)_0^5 + \left(\frac{x^2}{2} - 5x\right)_5^8$$

$$y = -\left(\frac{25}{2} - 25\right) + \left(32 - 40 - \frac{25}{2} + 25\right)$$

=17

Q54

We know that

|cos x| = cos x in [0, π/2)

|cos x| = -cos x in [π/2, 3π/2)

|cos x| = cos x in [3π/2, 2π]

$$y = \int_0^{\frac{\pi}{2}} |cosx| dx + \int_0^{\frac{3\pi}{2}} |cosx| dx + \int_0^{2\pi} |cosx| dx$$

$$y = \int_0^{\frac{\pi}{2}} cosx\, dx - \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} cosx\, dx + \int_{\frac{3\pi}{2}}^{2\pi} cosx\, dx$$

$$y = (sinx)_0^{\frac{\pi}{2}} - (sinx)_{\pi}^{\frac{3\pi}{2}} + (sinx)_{\frac{3\pi}{2}}^{2\pi}$$

y=(1-0)—1-1+(0+1) =4

Q55

We know that

|sin x| = -sin x in [-π/4, 0)

|sin x| = sin x in [0, π/4]

$$y = \int_{-\frac{\pi}{4}}^{0} |sinx| dx + \int_0^{\frac{\pi}{4}} |sinx| dx$$

$$y = -\int_{-\frac{\pi}{4}}^{0} sinx\, dx + \int_0^{\frac{\pi}{4}} sinx\, dx$$

$$y = -(-cosx)_{-\frac{\pi}{4}}^{0} + (-cosx)_0^{\frac{\pi}{4}}$$

$$y = \left(1 - \frac{1}{\sqrt{2}}\right) - \left(\frac{1}{\sqrt{2}} - 1\right) = 2 - \frac{1}{\sqrt{2}}$$

Q56

$$y = \int_1^3 f(x)dx$$

$$y = \int_1^2 f(x)dx + \int_2^3 f(x)dx$$

$$y = \int_1^2 2x + 1\, dx + \int_2^3 x^2 + 1 dx$$

$$y = (x^2 + x)_1^2 + \left(\frac{x^3}{3} + x\right)_2^3$$

$$y = (4 + 2 - 1 - 1) + (9 + 3 - \frac{8}{3} - 2) = \frac{34}{3}$$

Q57

$$y = \int_0^4 f(x)dx$$

$$y = \int_0^2 f(x)dx + \int_2^4 f(x)dx$$

$$y = \int_0^2 3x^2 + 4dx + \int_2^4 9x - 2dx$$

$$y = (x^3 + 4x)_0^2 + \left(\frac{9x^2}{2} - 2x\right)_2^4$$

y=(8+8)+(72-8-18+4) =66

Q58

$$y = \int_0^4 |x| + |x - 2| + |x - 4| dx$$

$$y = \int_0^2 |x| + |x - 2| + |x - 4| dx + \int_2^4 |x| + |x - 2| + |x - 4| dx$$

$$y = \int_0^2 x - (x - 2) - (x - 4) dx + \int_2^4 x + (x - 2) - (x - 4) dx$$

$$y = \left(-\frac{x^2}{2} + 6x\right)_0^2 + \left(\frac{x^2}{2} + 2x\right)_0^4$$

y=(-2+12)+(8+8-2-4) =20

Exercise 16D

Q1

f(x) is continuous in [0,2]

$\int_a^b f(x)dx = \lim_{n\to\infty} h\sum_{r=0}^{n-1} f(a+rh), where\ h = (b-a)/n$

here h=2/n

$\int_0^2 (x+4)dx = \lim_{n\to\infty}\left(\frac{2}{n}\right)\sum_{r=0}^{n-1} f(2r/n)$

$= \lim_{n\to\infty}\left(\frac{2}{n}\right)\sum_{r=0}^{n-1}\left(\frac{2r}{n}\right)+4$

$= \lim_{n\to\infty}\left(\frac{2}{n}\right)\left(\frac{(n-1)(n)}{n}+4(n-1)\right)$

$= \lim_{n\to\infty}\frac{2}{n}\frac{n^2-n+4n^2-4n}{n}$

$= \lim_{n\to\infty}\frac{2}{n}\frac{5n^2-5n}{n}$

$= \lim_{n\to\infty}\frac{10n^2-10n}{n^2}$

$= \lim_{n\to\infty} 10-(10/n)$

=10

Q2

f(x) is continuous in [1,2]

$\int_a^b f(x)dx = \lim_{n\to\infty} h\sum_{r=0}^{n-1} f(a+rh), where\ h = (b-a)/n$

here h=1/n

$\int_1^2 (3x-2)dx = \lim_{n\to\infty} \left(\frac{1}{n}\right)\sum_{r=0}^{n-1}(3+3\frac{r}{n}-2)$

$\lim_{n\to\infty} \left(\frac{1}{n}\right)\sum_{r=0}^{n-1}(\ 3+3\frac{r}{n}-2)$

$\lim_{n\to\infty} \left(\frac{1}{n}\right)\left(n+\frac{3(n-1)(n)}{2n}\right)$

$= \lim_{n\to\infty} \left(\frac{1}{n}\right)\left(\frac{2n^2+3n^2-3n}{2n}\right)$

$= \lim_{n\to\infty} \left(\frac{5n^2-3n}{2n^2}\right)$

$= \lim_{n\to\infty} \left(\frac{5}{2}\right) - \left(\frac{3}{2n}\right)$ =5/2

Q3

f(x) is continuous in [1,3]

$\int_a^b f(x)dx = \lim_{n\to\infty} h\ \sum_{r=0}^{n-1} f(a+rh), where\ h = (b-a)/n$

here h=2/n

$\int_1^3 (x^2)dx = \lim_{n\to\infty} \left(\frac{2}{n}\right)\sum_{r=0}^{n-1} f\left(1+\left(\frac{2r}{n}\right)\right)$

$= \lim_{n\to\infty} \left(\frac{2}{n}\right)\sum_{r=0}^{n-1}\left(1+\left(\frac{2r}{n}\right)\right)^2$

$= \lim_{n\to\infty} \left(\frac{2}{n}\right)\sum_{r=0}^{n-1}\left(\frac{4r^2}{n^2}+1+\frac{4r}{n}\right)$

$$= \lim_{n\to\infty} \frac{2}{n}\left(\frac{4(n-1)(n)(2n-1)}{6n^2} + n + \frac{4(n-1)(n)}{2n}\right)$$

$$= \lim_{n\to\infty} \frac{2}{n}\left(\frac{4(2n^3-2n^2-n^2+n)}{6n^2} + n + \frac{2(n^2-n)}{n}\right)$$

$$= \lim_{n\to\infty} \frac{2}{n}\left(\frac{(8n^3-12n^2+4n)+(6n^3)+(12n^3-12n^2)}{6n^2}\right)$$

$$= \lim_{n\to\infty} \frac{2}{n}\left(\frac{26n^3-24n^2+4n}{6n^2}\right)$$

$$= \lim_{n\to\infty} \left(\frac{52n^3-48n^2+8n}{6n^3}\right)$$

$$= \lim_{n\to\infty} \left(\frac{52}{5}\right) - \left(\frac{26}{6n}\right) + \left(\frac{8}{6n^2}\right) = 26/3$$

Q4

f(x) is continuous in [0,3]

$$\int_a^b f(x)dx = \lim_{n\to\infty} h\sum_{r=0}^{n-1} f(a+rh), where\ h = b-a/n$$

here h=3/n

$$\int_0^3 (x^2+1)dx = \lim_{n\to\infty} \left(\frac{3}{n}\right)\sum_{r=0}^{n-1} f\left(\left(\frac{3r}{n}\right)\right)$$

$$= \lim_{n\to\infty} \left(\frac{3}{n}\right)\sum_{r=0}^{n-1} \left(\left(\frac{3r}{n}\right)^2 + 1\right)$$

$$= \lim_{n\to\infty} \left(\frac{3}{n}\right)\sum_{r=0}^{n-1} \left(\frac{9r^2}{n^2} + 1\right)$$

$$= \lim_{n\to\infty} \frac{3}{n}\left(\frac{9(n-1)(n)(2n-1)}{6n^2} + n\right)$$

$$= \lim_{n\to\infty} \frac{3}{n}\left(\frac{9(n^2-n)(2n-1)}{6n^2} + n\right)$$

$$= \lim_{n\to\infty} \frac{3}{n}\left(\frac{9(2n^3-2n^2-n^2+n)}{6n^2} + n\right)$$

$$= \lim_{n\to\infty} \frac{3}{n}\left(\frac{(18n^3-27n^2+9n)+(6n^3)}{6n^2}\right)$$

$$= \lim_{n\to\infty} \frac{3}{n}\left(\frac{24n^3-27n^2+9n}{6n^2}\right)$$

$$= \lim_{n\to\infty} \left(\frac{72^3-81n^2+27}{6n^3}\right)$$

$$= \lim_{n\to\infty} \left(\frac{72}{6}\right) - \left(\frac{81}{6n}\right) + \left(\frac{27}{6n^2}\right)$$

=12

Q5

f(x) is continuous in [2,5]

$$\int_a^b f(x)dx = \lim_{n\to\infty} h\sum_{r=0}^{n-1} f(a+rh), where\ h = (b-a)/n$$

here h=3/n

$$\int_2^5 (3x^2-5)dx = \lim_{n\to\infty} \left(\frac{3}{n}\right)\sum_{r=0}^{n-1} f\left(\left(2+\frac{3r}{n}\right)\right)$$

$$= \lim_{n\to\infty} \left(\frac{3}{n}\right)\sum_{r=0}^{n-1} \left(3\left(2+\frac{3r}{n}\right)^2 - 5\right)$$

$$= \lim_{n\to\infty} \left(\frac{3}{n}\right)\sum_{r=0}^{n-1} 3\left(\frac{9r^2}{n^2} + 4 + \frac{12r}{n}\right) - 5$$

$$= \lim_{n\to\infty} \frac{3}{n}\left(\frac{27(n-1)(n)(2n-1)}{6n^2} + 12n + \frac{18n(n-1)}{n} - 5n\right)$$

$$= \lim_{n\to\infty} \frac{3}{n}\left(\frac{27(n^2-n)(2n-1)}{6n^2} + 12n + \frac{18n(n-1)}{n} - 5n\right)$$

$$= \lim_{n\to\infty} \frac{3}{n}\left(\frac{27(2n^3-2n^2-n^2+n)}{6n^2} + 12n + \frac{18n(n-1)}{n} - 5n\right)$$

$$= \lim_{n\to\infty} \frac{3}{n}\left(\frac{(54n^3-81n^2+27n)+(42n^3)(108n^3-108n^2)}{6n^2}\right)$$

$$= \lim_{n\to\infty} \frac{3}{n}\left(\frac{204n^3-189n^2+27n}{6n^2}\right)$$

$$= \lim_{n\to\infty} \left(\frac{612n^3-567n^2+27n}{6n^3}\right)$$

$$= \lim_{n\to\infty} \left(\frac{612}{6}\right) - \left(\frac{567}{6n}\right) + \left(\frac{27}{6n^2}\right)$$

=102

Q6

f(x) is continuous in [2,5]

$$\int_a^b f(x)dx = \lim_{n\to\infty} h\sum_{r=0}^{n-1} f(a+rh), where\ h = (b-a)/n$$

here h=3/n

$$\int_0^3 (x^2+2x)dx = \lim_{n\to\infty} \left(\frac{3}{n}\right)\sum_{r=0}^{n-1} f\left(\left(\frac{3r}{n}\right)\right)$$

$$= \lim_{n\to\infty} \left(\frac{3}{n}\right)\sum_{r=0}^{n-1} \left(\left(\frac{3r}{n}\right)^2 + \frac{6r}{n}\right)$$

$$= \lim_{n\to\infty}\left(\frac{3}{n}\right)\sum_{r=0}^{n-1}\left(\frac{9r^2}{n^2}+\frac{6r}{n}\right)$$

$$= \lim_{n\to\infty}\frac{3}{n}\left(\frac{9(n-1)(n)(2n-1)}{6n^2}+\frac{3n(n-1)}{n}\right)$$

$$= \lim_{n\to\infty}\frac{3}{n}\left(\frac{9(n^2-n)(2n-1)}{6n^2}+\frac{3n(n-1)}{n}\right)$$

$$= \lim_{n\to\infty}\frac{3}{n}\left(\frac{9(2n^3-2n^2-n^2+n)}{6n^2}+\frac{3n(n-1)}{n}\right)$$

$$= \lim_{n\to\infty}\frac{3}{n}\left(\frac{(18n^3-27n^2+9n)+(18n^3-18n^2)}{6n^2}\right)$$

$$= \lim_{n\to\infty}\frac{3}{n}\left(\frac{36n^3-45n^2+9n}{6n^2}\right)$$

$$= \lim_{n\to\infty}\left(\frac{108n^3-135n^2+27n}{6n^3}\right)$$

$$= \lim_{n\to\infty}\left(\frac{108}{6}\right)-\left(\frac{135}{6n}\right)+\left(\frac{27}{6n^2}\right)$$

=18

Q7

f(x) is continuous in [1,4]

$$\int_a^b f(x)dx = \lim_{n\to\infty} h\sum_{r=0}^{n-1} f(a+rh), where\ h=(b-a)/n$$

here h=3/n

$$\int_1^4 (3x^2+2x)dx = \lim_{n\to\infty}\left(\frac{3}{n}\right)\sum_{r=0}^{n-1} f\left(\left(1+\frac{3r}{n}\right)\right)$$

$$= \lim_{n\to\infty}\left(\frac{3}{n}\right)\sum_{r=0}^{n-1}\left(3\left(1+\frac{3r}{n}\right)^2+2\left(1+\frac{3r}{n}\right)\right)$$

$$= \lim_{n\to\infty}\left(\frac{3}{n}\right)\sum_{r=0}^{n-1}3\left(\frac{9r^2}{n^2}+1+\frac{6r}{n}\right)+2\left(1+\frac{3r}{n}\right)$$

$$= \lim_{n\to\infty}\frac{3}{n}\left(\frac{27(n-1)(n)(2n-1)}{6n^2}+3n+\frac{9n(n-1)}{n}+2n+\frac{3n(n-1)}{n}\right)$$

$$= \lim_{n\to\infty}\frac{3}{n}\left(\frac{27(n^2-n)(2n-1)}{6n^2}+5n+\frac{12n(n-1)}{n}\right)$$

$$= \lim_{n\to\infty}\frac{3}{n}\left(\frac{27(2n^3-2n^2-n^2+n)}{6n^2}+5n+\frac{12n(n-1)}{n}\right)$$

$$= \lim_{n\to\infty}\frac{3}{n}\left(\frac{(54n^3-81n^2+27n)+(30n^3)+(72n^3-72n^2)}{6n^2}\right)$$

$$= \lim_{n\to\infty}\frac{3}{n}\left(\frac{156n^3-153n^2+27n}{6n^2}\right)$$

$$= \lim_{n\to\infty}\left(\frac{468n^3-459n^2+81n}{6n^3}\right)$$

$$= \lim_{n\to\infty}\left(\frac{468}{6}\right)-\left(\frac{459}{6n}\right)+\left(\frac{81}{6n^2}\right)$$

=78

Q8

f(x) is continuous in [1,3]

$$\int_a^b f(x)dx = \lim_{n\to\infty} h\sum_{r=0}^{n-1} f(a+rh), where\ h=(b-a)/n$$

here h=3/n

$$\int_1^3 (x^2+5x)dx = \lim_{n\to\infty}\left(\frac{2}{n}\right)\sum_{r=0}^{n-1} f\left(\left(1+\frac{2r}{n}\right)\right)$$

$$= \lim_{n\to\infty}\left(\frac{2}{n}\right)\sum_{r=0}^{n-1}\left(\left(1+\frac{2r}{n}\right)^2+5\left(1+\frac{2r}{n}\right)\right)$$

$$= \lim_{n\to\infty}\left(\frac{2}{n}\right)\sum_{r=0}^{n-1}\left(1+\frac{4r^2}{n^2}+\frac{4r}{n}+5+\frac{10r}{n}\right)$$

$$= \lim_{n\to\infty}\left(\frac{2}{n}\right)\sum_{r=0}^{n-1}\left(1+\frac{4r^2}{n^2}+\frac{4r}{n}+5+\frac{10r}{n}\right)$$

$$= \lim_{n\to\infty}\frac{2}{n}\left(\frac{4(n-1)(n)(2n-1)}{6n^2}+6n+\frac{7n(n-1)}{n}\right)$$

$$= \lim_{n\to\infty}\frac{2}{n}\left(\frac{4(n^2-n)(2n-1)}{6n^2}+6n+\frac{7n(n-1)}{n}\right)$$

$$= \lim_{n\to\infty}\frac{2}{n}\left(\frac{4(2n^3-2n^2-n^2+n)}{6n^2}+6n+\frac{7n(n-1)}{n}\right)$$

$$= \lim_{n\to\infty}\frac{2}{n}\left(\frac{(8n^3-12n^2+4n)+(42n^3-42n^2)+(36n^3)}{6n^2}\right)$$

$$= \lim_{n\to\infty}\frac{2}{n}\left(\frac{86n^3-54n^2+4n}{6n^2}\right)$$

$$= \lim_{n\to\infty}\left(\frac{172n^3-108n^2+8n}{6n^3}\right)$$

$$= \lim_{n\to\infty}\left(\frac{172}{6}\right)-\left(\frac{108}{6n}\right)+\left(\frac{8}{6n^2}\right) = 86/3$$

Q9

f(x) is continuous in [1,3]

$$\int_a^b f(x)dx = \lim_{n\to\infty} h\sum_{r=0}^{n-1} f(a+rh), where\ h=(b-a)/n$$

here h=2/n

$$\int_1^3 (2x^2+5x)dx = \lim_{n\to\infty}\left(\frac{2}{n}\right)\sum_{r=0}^{n-1} f\left(\left(1+\frac{2r}{n}\right)\right)$$

$$= \lim_{n\to\infty}\left(\frac{2}{n}\right)\sum_{r=0}^{n-1}\left(2\left(1+\frac{2r}{n}\right)^2+5\left(1+\frac{2r}{n}\right)\right)$$

$$= \lim_{n\to\infty}\left(\frac{2}{n}\right)\sum_{r=0}^{n-1}\left(2+\frac{8r^2}{n^2}+\frac{8r}{n}+5+\frac{10r}{n}\right)$$

$$= \lim_{n\to\infty}\left(\frac{2}{n}\right)\sum_{r=0}^{n-1}\left(7+\frac{8r^2}{n^2}+\frac{18r}{n}\right)$$

$$= \lim_{n\to\infty}\frac{2}{n}\left(\frac{8(n-1)(n)(2n-1)}{6n^2}+7n+\frac{9n(n-1)}{n}\right)$$

$$= \lim_{n\to\infty}\frac{2}{n}\left(\frac{8(n^2-n)(2n-1)}{6n^2}+7n+\frac{9n(n-1)}{n}\right)$$

$$= \lim_{n\to\infty}\frac{2}{n}\left(\frac{8(2n^3-2n^2-n^2+n)}{6n^2}+7n+\frac{9n(n-1)}{n}\right)$$

$$= \lim_{n\to\infty}\frac{2}{n}\left(\frac{(16n^3-24n^2+8n)+(54n^3-54^2)+(42n^3)}{6n^2}\right)$$

$$= \lim_{n\to\infty}\frac{2}{n}\left(\frac{112n^3-78n^2+8n}{6n^2}\right)$$

$$= \lim_{n\to\infty}\left(\frac{224n^3-156n^2+8n}{6n^3}\right)$$

$$= \lim_{n\to\infty}\left(\frac{224}{6}\right)-\left(\frac{156}{6n}\right)+\left(\frac{8}{6n^2}\right) = 112/3$$

Q10

f(x) is continuous in [0,2]

$$\int_a^b f(x)dx = \lim_{n\to\infty} h\sum_{r=0}^{n-1} f(a+rh), where\ h=(b-a)/n$$

here h=2/n

$$\int_0^2 (x^3)dx = \lim_{n\to\infty}\left(\frac{2}{n}\right)\sum_{r=0}^{n-1} f\left(\frac{2r}{n}\right)$$

$$= \lim_{n\to\infty}\left(\frac{2}{n}\right)\sum_{r=0}^{n-1}\left(\frac{2r}{n}\right)^3$$

$$= \lim_{n\to\infty}\left(\frac{2}{n}\right)\sum_{r=0}^{n-1}\left(\frac{8r^3}{n^3}\right)$$

$$= \lim_{n\to\infty}\frac{2}{n}\left(\frac{8(n-1)^2(n)^2}{4n^3}\right)$$

$$= \lim_{n\to\infty}\frac{2}{n}\left(\frac{8(n^2-2n+1)(n^2)}{4n^3}\right)$$

$$= \lim_{n\to\infty}\frac{2}{n}\left(\frac{8(n^4-2n^3+n^2)}{4n^3}\right)$$

$$= \lim_{n\to\infty}\left(\frac{16n^4-32n^3+16n^2}{4n^4}\right)$$

$$= \lim_{n\to\infty}\left(\frac{16}{4}\right)-\left(\frac{32}{4n}\right)+\left(\frac{16}{4n^2}\right) = 4$$

Q11

f(x) is continuous in [2,4]

$$\int_a^b f(x)dx = \lim_{n\to\infty} h\sum_{r=0}^{n-1} f(a+rh), where\ h=(b-a)/n$$

here h=3/n

$$\int_2^4 (x^2-3x+2)dx = \lim_{n\to\infty}\left(\frac{2}{n}\right)\sum_{r=0}^{n-1} f\left(\left(2+\frac{2r}{n}\right)\right)$$

$$= \lim_{n\to\infty}\left(\frac{2}{n}\right)\sum_{r=0}^{n-1}\left(\left(2+\frac{2r}{n}\right)^2-3\left(2+\frac{2r}{n}\right)+2\right)$$

$$= \lim_{n\to\infty}\left(\frac{2}{n}\right)\sum_{r=0}^{n-1}\left(\frac{4r^2}{n^2}+\frac{8r}{n}+4-6-\frac{6r}{n}+2\right)$$

$$= \lim_{n\to\infty}\frac{2}{n}\left(\frac{4(n-1)(n)(2n-1)}{6n^2}+\frac{n(n-1)}{n}\right)$$

$$= \lim_{n\to\infty}\frac{2}{n}\left(\frac{4(n^2-n)(2n-1)}{6n^2}+\frac{n(n-1)}{n}\right)$$

$$= \lim_{n\to\infty}\frac{2}{n}\left(\frac{4(2n^3-2n^2-n^2+n)}{6n^2}+\frac{n(n-1)}{n}\right)$$

$$= \lim_{n\to\infty}\frac{2}{n}\left(\frac{(8n^3-12n^2+4n)+(6n^3-6n^2)}{6n^2}\right)$$

$$= \lim_{n\to\infty}\frac{2}{n}\left(\frac{14n^3-18n^2+4n}{6n^2}\right)$$

$$= \lim_{n\to\infty}\left(\frac{28n^3-36n^2+8n}{6n^3}\right)$$

$$= \lim_{n\to\infty}\left(\frac{28}{6}\right)-\left(\frac{36}{6n}\right)+\left(\frac{8}{6n^2}\right) = 14/3$$

Q12

f(x) is continuous in [0,2]

$$\int_a^b f(x)dx = \lim_{n\to\infty} h\sum_{r=0}^{n-1} f(a+rh), where\ h=(b-a)/n$$

here h=2/n

$$\int_0^2 (x^2+x)dx = \lim_{n\to\infty}\left(\frac{2}{n}\right)\sum_{r=0}^{n-1} f\left(\frac{2r}{n}\right)$$

$$= \lim_{n\to\infty}\left(\frac{2}{n}\right)\sum_{r=0}^{n-1}\left(\left(\frac{2r}{n}\right)^2+\left(\frac{2r}{n}\right)\right)$$

$$= \lim_{n\to\infty}\left(\frac{2}{n}\right)\sum_{r=0}^{n-1}\left(\frac{4r^2}{n^2}+\frac{2r}{n}\right)$$

$$= \lim_{n\to\infty}\frac{2}{n}\left(\frac{4(n-1)(n)(2n-1)}{6n^2}+\frac{n(n-1)}{n}\right)$$

$$= \lim_{n\to\infty}\frac{2}{n}\left(\frac{4\left(n^2-n\right)(2n-1)}{6n^2}+\frac{n(n-1)}{n}\right)$$

$$= \lim_{n\to\infty}\frac{2}{n}\left(\frac{4\left(2n^3-2n^2-n^2+n\right)}{6n^2}+\frac{n(n-1)}{n}\right)$$

$$= \lim_{n\to\infty}\frac{2}{n}\left(\frac{\left(8n^3-12n^2+4n\right)+\left(6n^3-6n^2\right)}{6n^2}\right.$$

$$= \lim_{n\to\infty}\frac{2}{n}\left(\frac{14n^3-18n^2+4n}{6n^2}\right)$$

$$= \lim_{n\to\infty}\left(\frac{28n^3-36n^2+8n}{6n^3}\right)$$

$$= \lim_{n\to\infty}\left(\frac{28}{6}\right)-\left(\frac{36}{6n}\right)+\left(\frac{8}{6n^2}\right)$$

=14/3

Q13

f(x) is continuous in [0,3]

$$\int_a^b f(x)dx = \lim_{n\to\infty} h\sum_{r=0}^{n-1} f(a+rh), where\ h=(b-a)/n$$

here h=3/n

$$\int_0^3(2x^2+3x+5)dx = \lim_{n\to\infty}\left(\frac{3}{n}\right)\sum_{r=0}^{n-1} f\left(\frac{3r}{n}\right)$$

$$= \lim_{n\to\infty}\left(\frac{3}{n}\right)\sum_{r=0}^{n-1}\left(2\left(\frac{3r}{n}\right)^2+3\left(\frac{3r}{n}\right)+5\right)$$

$$= \lim_{n\to\infty}\left(\frac{3}{n}\right)\sum_{r=0}^{n-1}\left(\frac{18r^2}{n^2}+\frac{9r}{n}+5\right)$$

$$= \lim_{n\to\infty}\frac{3}{n}\left(\frac{18(n-1)(n)(2n-1)}{6n^2}+\frac{9n(n-1)}{2n}+5n\right)$$

$$= \lim_{n\to\infty}\frac{3}{n}\left(\frac{18(n^2-n)(2n-1)}{6n^2}+\frac{9n(n-1)}{2n}+5n\right)$$

$$= \lim_{n\to\infty}\frac{3}{n}\left(\frac{18(2n^3-2n^2-n^2+n)}{6n^2}+\frac{9n(n-1)}{2n}+5n\right)$$

$$= \lim_{n\to\infty}\frac{3}{n}\left(\frac{(36n^3-54n^2+18n)+(27n^3-27n^2)+30n^3}{6n^2}\right)$$

$$= \lim_{n\to\infty}\frac{3}{n}\left(\frac{93n^3-81n^2+18n}{6n^2}\right)$$

$$= \lim_{n\to\infty}\left(\frac{279n^3-243n^2+54n}{6n^3}\right)$$

$$= \lim_{n\to\infty}\left(\frac{279}{6}\right)-\left(\frac{243}{6n}\right)+\left(\frac{54}{6n^2}\right) = 93/2$$

Q14

Since it is modulus function so we need to break the function and then solve it

$$f(x) = \int_0^{\frac{1}{3}}(1-3x)dx + \int_{\frac{1}{3}}^{1}(3x-1)dx$$

it is continuous in [0,1]

let $g(x) = \int_0^{\frac{1}{3}}(1-3x)dx$ and $h(x) = \int_{\frac{1}{3}}^{1}(3x-1)dx$

$$g(x) = \int_0^{\frac{1}{3}}(1-3x)dx$$

here h=1/3n

$$\int_0^{\frac{1}{3}}(1-3x)dx = \lim_{n\to\infty}\left(\frac{1}{3n}\right)\sum_{r=0}^{n-1} f(r/3n)$$

$$= \lim_{n\to\infty}\left(\frac{1}{3n}\right)\sum_{r=0}^{n-1}\left(1-3\left(\frac{r}{3n}\right)\right)$$

$$= \lim_{n\to\infty}\left(\frac{1}{3n}\right)\left(n-\frac{3(n-1)(n)}{6n}\right)$$

$$= \lim_{n\to\infty}\frac{1}{3n}\frac{6n^2-3n^2+3n}{3n}$$

$$= \lim_{n\to\infty}\frac{1}{3n}\frac{3n^2+3n}{3n}$$

$$= \lim_{n\to\infty}\frac{3n^2+3n}{9n^2}$$

$$= \lim_{n\to\infty}\frac{1}{3}+\left(\frac{3}{9n}\right)$$

=1/3

$$h(x) = \int_{\frac{1}{3}}^{1}(3x-1)dx$$

here h=2/3n

$$\int_{\frac{1}{3}}^{1}(3x-1)dx = \lim_{n\to\infty}\left(\frac{2n}{3n}\right)\sum_{r=0}^{n-1} f\left(\left(\frac{1}{3}\right)+\left(\frac{2r}{3n}\right)\right)$$

$$= \lim_{n\to\infty}\left(\frac{2}{3n}\right)\sum_{r=0}^{n-1}\left(3\left(\frac{1}{3}+\frac{2r}{3n}\right)-1\right)$$

$$= \lim_{n\to\infty}\left(\frac{2}{3n}\right)\left(\frac{(n-1)(n)}{n}\right)$$

$$= \lim_{n\to\infty}\frac{2}{3n}.\frac{n^2-n}{n}$$

$= \lim_{n\to\infty} \frac{2}{3n} . \frac{n^2-n}{n}$

$= \lim_{n\to\infty} \frac{2n^2-2n}{3n^2}$

$= \lim_{n\to\infty} \frac{2}{3} - \left(\frac{2}{3n}\right)$

=2/3

f(x)=g(x)+h(x)

=(1/3)+(2/3)

=3/3

=1

Q15

f(x) is continuous in [0,2]

$\int_a^b f(x)dx = \lim_{n\to\infty} h \sum_{r=0}^{n-1} f(a+rh), where\ h = (b-a)/n$

here h=2/n

$\int_0^2 (e^x)dx = \lim_{n\to\infty} \left(\frac{2}{n}\right) \sum_{r=0}^{n-1} f\left(\frac{2r}{n}\right)$

$= \lim_{n\to\infty} \left(\frac{2}{n}\right) \sum_{r=0}^{n-1} e^{\frac{2r}{n}}$

$= \lim_{n\to\infty} \left(\frac{2}{n}\right) (e^0 + e^h + e^{2h} + \cdots \ldots \ldots \ldots . . + e^{nh}$

$sum\ of\ e^0 + e^h + e^{2h} + \cdots \ldots \ldots \ldots \ldots + e^{nh}$

Which is g.p with common ratio $e^{1/n}$

Whose sum is $= \frac{e^h(1-e^{nh})}{1-e^h}$

$= \lim_{n\to\infty}\left(\frac{2}{n}\right)\left(\frac{e^h(1-e^{nh})}{1-e^h}\right)$

$= \lim_{n\to\infty}\left(\frac{2}{n}\right)\left(\frac{e^h(1-e^{nh})}{\frac{1-e^h.h}{h}}\right)$

$\lim_{n\to\infty}\frac{1-e^h}{h} = -1$

$= \lim_{n\to\infty}\left(\frac{2}{n}\right).\frac{e^h(1-e^{nh})}{-h}$

As h=2/n

$= \lim_{n\to\infty}\left(\frac{2}{n}\right).\frac{e^{\left(\frac{2}{n}\right)}(1-e^{n*((2/n)})}{-2/n}$

$=e^2-1$

Q16

f(x) is continuous in [1,3]

$\int_a^b f(x)dx = \lim_{n\to\infty} h\sum_{r=0}^{n-1} f(a+rh), where\ h = (b-a)/n$

here h=2/n

$\int_1^3 (e^{-x})dx = \lim_{n\to\infty}\left(\frac{2}{n}\right)\sum_{r=0}^{n-1} f\left(1+\left(\frac{2r}{n}\right)\right)$

$= \lim_{n\to\infty}\left(\frac{2}{n}\right)\sum_{r=0}^{n-1} e^{-\left(1+\frac{2r}{n}\right)}$

$= \lim_{n\to\infty} \left(\frac{2}{n}\right) \sum_{r=0}^{n-1} e^{-1}.e^{-\frac{2r}{n}}$

Common ratio is

$h = -2/n$

$sum = e^{-1}(e^0 + e^h + e^{2h} + \cdots \ldots \ldots \ldots + e^{nh})$

$= \lim_{n\to\infty} \left(\frac{2e^{-1}}{n}\right) (e^0 + e^h + e^{2h} + \cdots \ldots \ldots \ldots + e^{nh}$

$sum\ of = e^0 + e^h + e^{2h} + \cdots \ldots \ldots \ldots + e^{nh}$

Which is g.p. with common ratio $e^{1/n}$

Whose sum is $= \frac{e^h(1-e^{nh})}{1-e^h}$

$= \lim_{n\to\infty} \left(\frac{2e^{-1}}{n}\right) \left(\frac{e^h(1-e^{nh})}{1-e^h}\right)$

$= \lim_{n\to\infty} \left(\frac{2e^{-1}}{n}\right) (\frac{e^h(1-e^{nh})}{\frac{1-e^h.h}{h}}$

$\lim_{n\to\infty} \frac{1-e^h}{h} = -1$

$= \lim_{n\to\infty} \left(\frac{2e^{-1}}{n}\right) (\frac{e^h(1-e^{nh})}{-h}$

As h=-2/n

$= \lim_{n\to\infty} \left(\frac{2e^{-1}}{n}\right) (\frac{e^{\left(-\frac{2}{n}\right)}\left(1-e^{n*(-\frac{2}{n}}\right)}{2/n} = \frac{1-e^{-2}}{e}$

$= \frac{(e^2-1)}{e^3}$

Q17

f(x) is continuous in [a,b]

$$\int_a^b f(x)dx = \lim_{n\to\infty} h\sum_{r=0}^{n-1} f(a+rh), where\ h = (b-a)/n$$

here h=(b-a)/n

$$\int_a^b (cosx)dx = \lim_{n\to\infty}\left(\frac{b-a}{n}\right)\sum_{r=0}^{n-1} f(a+rh)$$

$$= \lim_{n\to\infty}\left(\frac{b-a}{n}\right)\sum_{r=0}^{n-1} cos(a+rh)$$

S=cos(a)+ cos(a+h)+cos(a+2h)+cos(a+3h)+……..+ cos(a+(n-1)h)

$$= \frac{sin\left(\frac{nh}{2}\right)cos\left(a+\frac{(n-1)h}{2}\right)}{sin\left(\frac{h}{2}\right)}$$

Putting h=(b-a)/n

$$\lim_{n\to\infty}\left(\frac{b-a}{n}\right)\frac{sin\left(\frac{n(b-a)}{2n}\right)cos\left(a+\frac{(n-1)(b-a)}{2n}\right)}{\frac{sin\left(\frac{b-a}{2n}\right)}{\frac{b-a}{2n}}.\frac{b-a}{2n}}$$

As we know

$$\lim_{h\to 0}\left(\frac{sinh}{h}\right) = 1$$

$$= \lim_{n\to\infty} 2sin\left(\frac{(b-a)}{a}\right)cos\left(a+\left(\frac{1}{2}-\frac{1}{2n}\right)(b-a)\right)$$

$$= 2sin\left(\frac{b-a}{2}\right)cos\left(\frac{b+a}{2}\right)$$

Which is trigonometry formula of sin(b)-sin(a)

Final answer is sin(b)-sin(a)

Objective Exercise

Q1

$y = \int_1^4 x\sqrt{x}dx$

$= \int_1^4 x^{\frac{3}{2}} dx$

$= \left(\frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1}\right)$

$= \frac{2}{5}(4^{\frac{5}{2}} - 1^{\frac{5}{2}})$

$= \frac{2}{5}(32 - 1) = \frac{62}{5}$ =12.4

Q2

$y = \int_0^2 \sqrt{6x + 4}dx$

$= \left(\frac{(6x+4)^{\frac{1}{2}+1}}{6\left(\frac{1}{2}+1\right)}\right)_0^2$

$= \frac{2}{6\times3}\left(16^{\frac{3}{2}} - 4^{\frac{3}{2}}\right)$

$= \frac{2}{6\times3}(64 - 6) = \frac{56}{9}$

Q3

$y = \int_0^1 \frac{dx}{\sqrt{5+3}}$

$$= \left(\frac{(5x+3)^{-\frac{1}{2}+1}}{5\left(-\frac{1}{2}+1\right)} \right)_0^1$$

$$= \frac{2}{5}(8^{\frac{1}{2}} - 3^{\frac{1}{2}}) = \frac{2}{5}\left(\sqrt{8} - \sqrt{3}\right)$$

Q4

$$y = \int_0^1 \frac{1}{1+x^2} dx$$

$$= (tan^{-1}x)_0^1$$

$$= tan^{-1}1 - tan^{-1}0$$

$$= \frac{\pi}{4} - 0$$

Q5

$$y = \int_0^2 \frac{dx}{\sqrt{4-x^2}}$$

Use formula $\frac{\int dx}{\sqrt{a^2-x^2}} = sin^{-1}\frac{x}{a}$

$$y = \left(\frac{sin^{-1}x}{2}\right)_0^2$$

$$= sin^{-1}1 - sin^{-1}0$$

$$= \frac{\pi}{2}$$

Q6

$y = \int_{\sqrt{3}}^{\sqrt{8}} x\sqrt{1+x^2}dx$

Let, $x^2 = t$

Differentiating both side with respect to t

$2x\frac{dx}{dt} = 1$

$\Rightarrow xdx = \frac{1}{2}dt$

At $x = \sqrt{3}, t = 3$

At $x = \sqrt{8}, t = 8$

$y = \frac{1}{2}\int_2^8 \sqrt{1+t}\,dt$

$= \frac{1}{2}\left(\frac{(1+t)^{\frac{1}{2}+1}}{\left(\frac{1}{2}+1\right)}\right)_3^8$

$= \frac{1}{3}\left(9^{\frac{3}{2}} - 4^{\frac{3}{2}}\right)$

$= \frac{1}{3}(27-8) = \frac{19}{3}$

Q7

Let, $x^4 = t$

Differentiating both side with respect to

$4x^3\frac{dx}{dt} = 1$

$\Rightarrow x^3dx = \frac{1}{4}dt$

At x = 0, t = 0

At x = 1, t = 1

$y = \frac{1}{4}\int_0^1 \frac{1}{1+t^2}dt$

$= \frac{1}{4}(tan^{-1}t)_0^1$

$= \frac{1}{4}(tan^{-1}1 - tan^{-1}0) = \frac{\pi}{16}$

Q8

Let, log x = t

Differentiating both side with respect to t

$\frac{1}{x}\frac{dx}{dt} = 1$

$\Rightarrow \frac{1}{x}dx = dt$

At x = 1, t = 0

At x = e, t = 1

$y = \int_0^1 t^2 dt$

$= \left(\frac{t^3}{3}\right)_0^1 = \frac{1}{3}$

Q9

$y = \left(ln(sinx)\right)_{\frac{\pi}{4}}^{\frac{\pi}{2}}$

$= ln\left(sin\frac{\pi}{2}\right) - ln\left(sin\frac{\pi}{4}\right)$

$= ln1 - ln\frac{1}{\sqrt{2}}$

$= \frac{1}{2}ln2$

Q10

$y = \int_0^{\frac{\pi}{4}}(sec^2x - 1)dx$

$= (tanx - x)_0^{\frac{\pi}{4}}$

$= \left(tan\frac{\pi}{4} - \frac{\pi}{4}\right) - (tan0 - 0)$

$= 1 - \frac{\pi}{4}$

Q11

$y = \int_0^{\frac{\pi}{2}}\frac{1+cos2x}{2}dx$

$= \left(\frac{x}{2} + \frac{sin2x}{4}\right)_0^{\pi/2}$

$= \left(\frac{\frac{\pi}{2}}{2} + \frac{sin\pi}{4}\right) - \left(\frac{0}{2} + \frac{sin0}{4}\right) = \frac{\pi}{4}$

Q12

$$y = \left(ln(cosecx - cotx)\right)_{\frac{\pi}{3}}^{\frac{\pi}{2}}$$

$$= ln\left(cosec\frac{\pi}{2} - cot\frac{\pi}{2}\right) - ln\left(cosec\frac{\pi}{3} - cot\frac{\pi}{3}\right)$$

$$= lm(1 - 0) - ln\left(\frac{2}{\sqrt{3}} - \frac{1}{\sqrt{3}}\right)$$

$$= \frac{1}{2}log3$$

Q13

$$y = \int_0^{\frac{\pi}{2}} cosx(1 - sin^2x)dx$$

Let, sin x = t

Differentiating both side with respect to t

$$cosx\frac{dx}{dt} = 1$$

$$\Rightarrow cosx\ dx = dt$$

At x = 0, t = 0

At x = π/2, t = 1

$$y = \int_0^1 1 - t^2\ dt$$

$$= \left(t - \frac{t^3}{3}\right)_0^1$$

$$= 1 - \frac{1}{3} = \frac{2}{3}$$

Q14

$y = \int_0^{\frac{\pi}{4}} e^{tanx} sec^2 x \, dx$

Let, tan x = t

Differentiating both side with respect to t

$sec^2 x \frac{dx}{d} = 1$

$\Rightarrow sec^2 x \, dx = dt$

At x = 0, t = 0

At x =π/4, t = 1

$y = \int_0^1 e^t \, dt$

$= e^t{}_0^1$

$= e^1 - e^0$

$= e - 1$

Q15

Let, sin x = t

Differentiating both side with respect to t

$cosx \frac{dx}{dt} = 1$

$\Rightarrow cosx \, dx = dt$

At x = 0, t = 0

At x = $\frac{\pi}{2}$, t = 1

$y = \int_0^1 \frac{1}{1+t^2} dt$v

$= (tan^{-1}t)_0^1$

= $\tan^{-1}1 - \tan^{-1}0 = \pi/4$

Q16

Let, 1/x = t

Differentiating both side with respect to t

$\frac{-1}{x^2}\frac{dx}{dt} = 1$

$\Rightarrow \frac{1}{x^2} dx = -dt$

At x = 1/π, t = π

At x = 2/π, t = π/2

$y = \int_{\pi}^{\frac{\pi}{2}} sint\ dt$

$= (-cost)_{\pi}^{\frac{\pi}{2}} = 1$

Q17

$y = \int_0^{\pi} \frac{1}{1+sinx} \times \frac{1-sinx}{1-sinx} dx$

$= \int_0^{\pi} \frac{1-sinx}{cos^2x} dx$

$= \int_0^{\pi} \frac{1}{cos^2x} - \frac{sinx}{cos^2x} dx$

$= \int_0^{\pi} sec^2x dx - \int_0^{\pi} \frac{sinx}{cos^2x} dx$

Let, cos x = t

Differentiating both side with respect to

$-sinx \frac{dx}{dt} = 1$

$\Rightarrow sinx\ dx = -dt$

At x = 0, t = 1

At x = π, t = -1

$y = (tanx)_0^{\pi} + \int_1^{-1} \frac{1}{t^2} dt$

$= (tan\pi - tan0) + \left(\frac{t^{-1}}{-1}\right)_1^{-1}$ =2

Q18

$y = \int_0^{\pi/2} sin^{\frac{3}{2}}x cos^3x dx$

$y = \int_o^{\frac{\pi}{2}} sin^{\frac{3}{2}}x cosx)(1 - sin^2x) dx$

Let, sin x = t

Differentiating both side with respect to t

$cosx \frac{dx}{dt} = 1$

$\Rightarrow \cos x\, dx = dt$

At x = 0, t = 0

At x = π/2, t = 1

$$y = \int_0^1 t^{\frac{3}{2}} - t^{\frac{7}{2}} dt$$

$$= \left(\frac{t^{\frac{5}{2}}}{\frac{5}{2}} - \frac{t^{\frac{9}{2}}}{\frac{9}{2}} \right)_0^1$$

$$= \frac{2}{5} - \frac{2}{9}$$

$$= \frac{8}{45}$$

Q19

$$y = \int_0^1 \frac{e^x(x+1-1)}{(1+x)^2} dx$$

$$= \int_0^1 e^x \left(\frac{1}{1+x} - \frac{1}{(1+x)^2} \right) dx$$

Use formula $\int e^x(f(x) + f'(x))dx = e^x f(x)$

If $f(x) = \frac{1}{1+x}$

then $f'(x) = -\frac{1}{(1+x)^2}$

$$y = \left(\frac{e^x}{1+x} \right)_0^1$$

$$y = \frac{e}{2} - 1$$

Q20

$$y = \int_0^{\frac{\pi}{2}} e^x \left(\frac{1+sinx}{2cos^2\frac{x}{2}}\right) dx$$

$$= \int_0^{\frac{\pi}{2}} e^x \left(\frac{1}{2cos^2\frac{x}{2}} + \frac{sinx}{2cos^2\frac{x}{2}}\right) dx$$

$$= \int_0^{\frac{\pi}{2}} e^x \left(\frac{1}{2cos^2\frac{x}{2}} + \frac{2sin\frac{x}{2}cos\frac{x}{2}}{2cos^2\frac{x}{2}}\right) dx$$

$$= \int_0^{\frac{\pi}{2}} e^x \left(\frac{1}{2}sec^2\frac{x}{2} + tan\frac{x}{2}\right) dx$$

Use formula $\int e^x(f(x) + f'(x))dx = e^x f(x)$

If $f(x) = tan\frac{x}{2}$ then $f'(x) = \frac{1}{2}sec^2\frac{x}{2}$

$$y = \left(e^x tan\frac{x}{2}\right)_0^{\frac{\pi}{2}}$$

$$= e^{\frac{\pi}{2}} tan\frac{\frac{\pi}{2}}{2} - e^0 tan\frac{0}{2}$$

$$= e^{\frac{\pi}{2}}$$

Q21

$$y = \int_0^{\frac{\pi}{4}} \sqrt{sin^2x + cos^2x + 2sinxcosx}dx$$

$$= \int_0^{\frac{\pi}{4}} sinx + cosxdx$$

$$= (-cosx + sinx)_0^{\frac{\pi}{4}}$$

$= \left(-\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) - (-1 + 0)$

y = 1

Q22

$y = \int_0^{\frac{\pi}{2}} \sqrt{2cos^2 x}dx$

$= \int_0^{\frac{\pi}{2}} \sqrt{2}cosxdx$

$= \sqrt{2}(sinx)_0^{\frac{\pi}{2}}$ =√2

Q23

$y = \int_0^1 \frac{1-x-1+1}{1+x}dx$

$= \int_0^1 \frac{2}{1+x} - 1\ dx$

$= (2ln(1+x) - x)_0^1$

= 2 ln 2 – 1

Q24

$y = \int_0^{\frac{\pi}{2}} \frac{1-cos2x}{2}dx$

$= \left(\frac{x}{2} - \frac{sin2x}{4}\right)_0^{\frac{\pi}{2}}$

$= \frac{\pi}{4} - \frac{sin\pi}{4} = \frac{\pi}{4}$

Q25

$y = \int_0^{\frac{\pi}{6}} cosx(1 - 2sin^2x)dx$

$= \int_0^{\frac{\pi}{6}} cosx - 2cosxsin^2x\, dx$

$= (sinx)_0^{\frac{\pi}{6}} - 2\int_0^{\frac{\pi}{6}} cosxsin^2xdx$

Let, sin x = t

Differentiating both side with respect to t

$cosx\frac{dx}{dt} = 1$

$\Rightarrow cosxdx = dt$

At x = 0, t = 0

At x = π/6, t = 1/2

$y = sin\frac{\pi}{6} - sin0 - 2\int_0^{\frac{1}{2}} t^2dt$

$= \frac{1}{2} - 2\left(\frac{t^3}{3}\right)_0^{\frac{1}{2}}$

$= \frac{1}{2} - \frac{1}{12}$

$= \frac{5}{12}$

Q26

$y = \int_0^{\frac{\pi}{2}} sinx(2\ sinxcosx)dx$

$= 2\int_0^{\frac{\pi}{2}} sin^2 xcosxdx$

Let, sin x = t

Differentiating both side with respect to t

$cosx\frac{dx}{dt} = 1$

$\Rightarrow cosxdx = dt$

At x = 0, t = 0

At x = π/2, t = 1

$y = 2\int_0^1 t^2 dt$

$= 2\left(\frac{t^3}{3}\right)_0^1 = \frac{2}{3}$

Q27

$y = \int_0^{\pi} (2sinxcosx)(4cos^3 x - 3cosx)dx$

Let, cos x = t

Differentiating both side with respect to t

$-sinx\frac{dx}{dt} = 1$

$\Rightarrow sinxdx = -dt$

At x = 0, t = 1

At x = π, t = -1

$$y = -\int_1^{-1} 8t^4 - 6t^2 dt$$

$$= -\left(8\frac{t^5}{5} - 6\frac{t^3}{3}\right)_1^{-1}$$

$$= -\left[\left(-\frac{8}{5} + 2\right) - \left(\frac{8}{5} - 2\right)\right] = -\frac{4}{5}$$

Q28

$$y = \int_0^1 \frac{e^x}{1+e^{2x}} dx$$

Let $e^x = t$

Differentiating both side with respect to t

$$e^x \frac{dx}{dt} = 1$$

$$\Rightarrow e^x dx = dt$$

At x = 0, t = 1

At x = 1, t = e

$$y = \int_1^e \frac{1}{1+t^2} dt$$

$$= (tan^{-1}t)_1^e$$

$= \tan^{-1}e - \tan^{-1}1$

$= \tan^{-1}e - \pi/4$

Q29

Let, $x = t^2$

Differentiating both side with respect to t

$\frac{dx}{dt} = 2t$

$\Rightarrow dx = 2t\,dt$

At x = 0, t = 0

At x = 9, t = 3

$y = \int_0^3 \frac{2t}{1+t} dt$

$= 2\int_0^3 \frac{t+1-1}{1+t} dt$

$= 2\int_0^3 1 - \frac{1}{1+t} dt$

$= 2\left(t - ln(1+t)\right)_0^3$

y = 2[(3 – ln 4) – (0 – ln 1)]

= 6 – 2 log 4

Q30

Use integration by parts

$$y = x\int_0^{\frac{\pi}{2}} \cos x\,dx - \int_0^{\frac{\pi}{2}} \frac{d}{dx}x\left(\int \cos x\,dx\right)dx$$

$$= (x\sin x)_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \sin x\, dx$$

$$= \frac{\pi}{2} - (-\cos x)_0^{\frac{\pi}{2}}$$

$= \pi/2 - 1$

Q31

On converting denominator into perfect square we get

$$= \left(x+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2$$

$$y = \int_0^1 \frac{1}{\left(x+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} dx$$

$$y = \left(\frac{1}{\frac{\sqrt{3}}{2}} \tan^{-1} \frac{x+\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right)_0^1$$

$$= \frac{2}{\sqrt{3}}\left(\tan^{-1}\frac{2}{\sqrt{3}}\left(\frac{3}{2}\right) - \tan^{-1}\frac{2}{\sqrt{3}}\left(\frac{1}{2}\right)\right)$$

$= \pi/3\sqrt{3}$

Q32

Let, x = sin t

Differentiating both side with respect to t

$\frac{dx}{dt} = cost$

This means $dx = cost\ dt$

At x = 0, t = 0

At x = 1, t = π/2

$$y = \int_0^{\frac{\pi}{2}} \sqrt{\frac{1-\sin t}{1+\sin t}} \cos t\, dt$$

$$= \int_0^{\frac{\pi}{2}} \frac{1-\sin t}{\cos t} \cos t\, dt$$

$$= \int_0^{\frac{\pi}{2}} 1-\sin t\, dt$$

$$= (t+\cos t)_0^{\frac{\pi}{2}}$$

$$= \frac{\pi}{2} - 1$$

Q33

$$y = \int_0^1 \frac{1-x+1-1}{1+x} dx$$

$$= \int_0^1 \frac{2}{1+x} - 1 \, dx$$

$2\ln(1+x) - x$ within the limits 0 and 1

= 2 log 2 – 1

Q34

Let, x = a sin t

Differentiating both side with respect to t

$\frac{dx}{dt} = cost$

This means $dx = cost\ dt$

At x = -a, t = - π/2

At x = a, t = π/2

$$y = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{\frac{a-a\sin t}{a+a\sin t}} a\cos t \, dt$$

$$= a \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1-\sin t}{\cos t} \cos t \, dt$$

$$= a\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 1-\sin t\,dt$$

$$= a(t+\cos t)_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

$$= a\left[\left(\frac{\pi}{2}+0\right)-\frac{\pi}{2}+0\right)$$

$= a\pi$

Q35

$$y = \int_0^{\sqrt{2}} \sqrt{(\sqrt{2})^2 - x^2}\,dx$$

$$= \left(\frac{x}{2}\sqrt{2-x^2}+\frac{2}{2}\sin^{-1}\frac{x}{\sqrt{2}}\right)_0^{\sqrt{2}}$$

$$= \left(\frac{\sqrt{2}}{2}\sqrt{2-2}+\sin^{-1}\frac{\sqrt{2}}{\sqrt{2}}\right)-(0+\sin^{-1}0)$$

$= \frac{\pi}{2}$

Q36

We know that

$|x| = -x$ in $[-2, 0)$

|x| = x in [0, 2]

$$y = \int_{-2}^{0} |x|\,dx + \int_{0}^{2} |x|\,dx$$

$$= \int_{-2}^{0} -x\,dx + \int_{0}^{2} x\,dx$$

$$= \left(-\frac{x^2}{2}\right)_{-2}^{0} + \left(\frac{x^2}{2}\right)_{0}^{2}$$

y = 0 – (-2) + 2 – 0

= 4

Q37

We know that

|2x – 1| = -(2x – 1) in [0, 1/2)

|2x – 1| = (2x – 1) in [1/2, 1]

$$y = \int_{0}^{\frac{1}{2}} |2x-1|\,dx + \int_{\frac{1}{2}}^{1} |2x-1|\,dx$$

$$= \int_{0}^{\frac{1}{2}} -(2x-1)\,dx + \int_{\frac{1}{2}}^{1} 2x-1\,dx$$

$$= -(x^2 - x)_0^{\frac{1}{2}} + (x^2 - x)_{\frac{1}{2}}^1$$

$$= -\left[\left(\frac{1}{4} - \frac{1}{2}\right) - (0 - 0)\right] + \left[(1 - 1) - \left(\frac{1}{4} - \frac{1}{2}\right)\right]$$

y = ½

Q38

We know that

|2x + 1| = -(2x + 1) in [-2, -1/2)

|2x + 1| = (2x + 1) in [-1/2, 1]

$$y = \int_{-2}^{-\frac{1}{2}} |2x + 1|\, dx + \int_{-\frac{1}{2}}^{1} |2x + 1|\, dx$$

$$= \int_{-2}^{-\frac{1}{2}} -(2x + 1)\, dx + \int_{-\frac{1}{2}}^{1} 2x + 1\, dx$$

$$= -(x^2 + x)_{-2}^{-\frac{1}{2}} + (x^2 + x)_{-\frac{1}{2}}^1$$

$$= -\left[\left(\frac{1}{4} - \frac{1}{2}\right) - (4 - 2)\right] + \left[(1 + 1) - \left(\frac{1}{4} - \frac{1}{2}\right)\right]$$

y = 9/2

Q39

We know that

$|x| = -x$ in $[-2, 0)$

$|x| = x$ in $[0, 1]$

$$y = \int_{-2}^{0} \frac{|x|}{x} dx + \int_{0}^{1} \frac{|x|}{x} dx$$

$$= \int_{-2}^{0} \frac{-x}{x} dx + \int_{0}^{1} \frac{x}{x} dx$$

$$= \int_{-2}^{0} -1 \, dx + \int_{0}^{1} 1 \, dx$$

$$= (-x)_{-2}^{0} + (x)_{0}^{1}$$

$= -(0 - (-2)) + (1 - 0)$

$= -1$

Q40

We know that

$|x| = -x$ in $[-a, 0)$ where $a > 0$

$|x| = x$ in $[0, a]$ where $a > 0$

$$y = \int_{-a}^{0} x|x|\,dx + \int_{0}^{a} x|x|\,dx$$

$$= \int_{-a}^{0} x(-x)\,dx + \int_{0}^{a} x(x)\,dx$$

$$= -\int_{-a}^{0} x^2\,dx + \int_{0}^{a} x^2\,dx$$

$$= -\left(\frac{x^3}{3}\right)_{-a}^{0} + \left(\frac{x^3}{3}\right)_{0}^{a}$$

$= 0$

Q41

Find the equivalent expression to $|\cos x|$ at $0 \le x \le \pi$

In $0 \le x \le \pi/2$

$=\cos x$

In $\pi/2 \le x \le \pi$

$=-\cos x$

$$\Rightarrow \int_{0}^{\frac{\pi}{2}} \cos x\,dx + \int_{\frac{\pi}{2}}^{\pi} -\cos x\,dx$$

$$\Rightarrow \sin\frac{\pi}{2} - \sin 0 - \cos\pi + \cos\frac{\pi}{2}$$

⇒1-0-(-1) +0=2

Q42

Find the equivalent expression to |sin x| at 0<x<2π

In $0 \le x \le \pi$

|sin x| = sin x

In $\pi \le x \le 2\pi$

|sin x| = -sin x

$\Rightarrow \int_0^{\pi} \sin x\,dx + \int_{\pi}^{2\pi} -\sin x dx$

=-cos π-(-cos 0)+cos 2π-cos π

=-(-1)+1+1-(-1)

=2+2

=4

Q43

∴ Here,a = π/2

$$f(x) = \frac{\sin x}{(\sin x + \cos x)}$$

$$\frac{\sin\left(\frac{\pi}{2} - x\right)}{\sin\left(\frac{\pi}{2} - x\right) + \cos\left(\frac{\pi}{2} - x\right)} = \frac{\cos x}{\cos x + \sin x}$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sin x + \cos x}{\cos x + \sin x} dx$$

$$= \int_0^{\pi/2} 1 \, dx$$

$2I = \pi/2$

$I = \frac{\pi}{4}$

Q44

$a = \frac{\pi}{2}$

$$f(x) = \frac{\sqrt{\sin x}}{\sqrt{\cos x} + \sqrt{\sin x}}$$

$$\frac{\sqrt{\sin\left(\frac{\pi}{2} - x\right)}}{\sqrt{\cos\left(\frac{\pi}{2} - x\right)} + \sqrt{\sin\left(\frac{\pi}{2} - x\right)}} = \frac{\sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}}$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x} + \sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx$$

$$= \int_0^{\pi/2} 1 \, dx$$

$2I = \frac{\pi}{2}$

$= \frac{\pi}{4}$

Q45

$a = \frac{\pi}{2}$

$$f(x) = \frac{\sin^4 x}{\sin^4 x + \cos^4 x}$$

$$\frac{\sin^4\left(\frac{\pi}{2} - x\right)}{\sin^4\left(\frac{\pi}{2} - x\right) + \cos^4\left(\frac{\pi}{2} - x\right)} = \frac{\cos^4 x}{\sin^4 x + \cos^4 x}$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sin^4 x + \cos^4 x}{\sin^4 x + \cos^4 x} dx$$

$= \int_0^{\pi/2} 1 \, dx$

$2I = \pi/2$

$= \frac{\pi}{4}$

Q46

$a = \frac{\pi}{2}$

$$f(x) = \frac{\cos^{\frac{1}{4}} x}{\sin^{\frac{1}{4}} x + \cos^{\frac{1}{4}} x}$$

$$\frac{\cos^{\frac{1}{4}}\left(\frac{\pi}{2} - x\right)}{\sin^{\frac{1}{4}}}\left(\frac{\pi}{2} - x\right)\cos^{\frac{1}{4}}\left(\frac{\pi}{2} - x\right) = \sin^{\frac{1}{4}} x \sin^{\frac{1}{4}} x + \cos^{\frac{1}{4}} x$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{1}{4}} x + \cos^{\frac{1}{4}} x}{\sin^{\frac{1}{4}} x + \cos^{\frac{1}{4}} x} dx$$

$$= \int_0^{\frac{\pi}{2}} 1 dx$$

$2I = \pi/2$

$= \frac{\pi}{4}$

Q47

$a = \frac{\pi}{2}$

$$f(x) = \frac{\sin^n x}{\cos^n x + \sin^n x}$$

$$= \frac{\cos^n x}{\cos^n x + \sin^n x}$$

$2I = \int_0^{\pi/2} 1 \, dx$

$2I = \frac{\pi}{2}$

$I = \frac{\pi}{4}$

Q48

$a = \pi/2$

$f(x) = \frac{\sqrt{Cot\ x}}{\sqrt{Cot\ x} + \sqrt{\tan x}} = \frac{\sqrt{tan\ x}}{\sqrt{Cot\ x} + \sqrt{\tan x}}$

$2\ I = \int_0^{\pi/2} 1\ dx$

$= \frac{\pi}{4}$

Q49

$$= \frac{\sqrt[3]{\tan x}}{\sqrt[3]{\cot x} + \sqrt[3]{\tan x}}$$

$$= \frac{\sqrt[3]{\frac{\sin x}{\cos x}}}{\sqrt[3]{\frac{\sin x}{\cos x}} + \sqrt[3]{\frac{\cos x}{\sin x}}}$$

$$= \frac{\sqrt[3]{\frac{\sin x}{\cos x}} * \left(\sqrt[3]{\sin x}\ \sqrt[3]{\cos x}\right)}{\sin^{\frac{2}{3}} x + \cos^{\frac{2}{3}} x}$$

$$= \frac{\sin^{\frac{2}{3}} x}{\sin^{\frac{2}{3}} x + \cos^{\frac{2}{3}} x}$$

$a = \frac{\pi}{2}$

$$f(x) = \frac{\sin^{\frac{2}{3}} x}{\sin^{\frac{2}{3}} x + \cos^{\frac{2}{3}} x}$$

$$= \frac{\cos^{\frac{2}{3}} x}{\sin^{\frac{2}{3}} x + \cos^{\frac{2}{3}} x}$$

$$= \int_0^{\pi/2} 1 \, dx$$

$$= \frac{\pi}{4}$$

Q50

$$\frac{1}{1+\tan x} = \frac{1}{1+\frac{\sin x}{\cos x}}$$

$$= \frac{1}{(\cos x + \sin x)\frac{1}{\cos x}}$$

$$= \frac{\cos x}{\cos x + \sin x}$$

So our integral becomes, $\int_0^{\frac{\pi}{2}} \frac{\cos x}{\cos x + \sin x} dx$

$$a = \frac{\pi}{2}$$

$$f(x) = \frac{\sin x}{(\sin x + \cos x)}$$

$$= \frac{\sin\left(\frac{\pi}{2} - x\right)}{\sin\left(\frac{\pi}{2} - x\right) + \cos\left(\frac{\pi}{2} - x\right)}$$

$$= \frac{\cos x}{\cos x + \sin x}$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sin x + \cos x}{\cos x + \sin x} dx$$

$$= \int_0^{\pi/2} 1 \, dx$$

$$= \frac{\pi}{4}$$

Q 51

So our integral becomes

$$\frac{1}{\sqrt{\cot x} + 1} = \frac{1}{\sqrt{\frac{\cos x}{\sin x}} + 1}$$

$$= \frac{\sqrt{\sin x}}{\sqrt{\cos x} + \sqrt{\sin x}}$$

$$a = \frac{\pi}{2}$$

$$f(x) = \frac{\sqrt{\sin x}}{\sqrt{\cos x} + \sqrt{\sin x}}$$

$$= \frac{\sqrt{\sin\left(\frac{\pi}{2} - x\right)}}{\sqrt{\cos\left(\frac{\pi}{2} - x\right)} + \sqrt{\sin\left(\frac{\pi}{2} - x\right)}}$$

$$= \frac{\sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}}$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x} + \sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx$$

$$= \int_0^{\pi/2} 1 \, dx$$

$$= \frac{\pi}{4}$$

Q52

$$\frac{1}{1 + \tan^3 x} = \frac{\cos^3 x}{\sin^3 x + \cos^3 x}$$

$$a = \frac{\pi}{2}$$

$$f(x) = \frac{\cos^3 x}{\sin^3 x + \cos^3 x}$$

We know that,

$$f(a - x) = \frac{\sin^3 x}{\sin^3 x + \cos^3 x}$$

$$2I = \int_0^{\pi/2} 1 \, dx$$

$$2I = \frac{\pi}{2}$$

$$= \frac{\pi}{4}$$

Q53

so our integral becomes,

$$\frac{\sec^5 x}{\sec^5 x + \operatorname{cosec}^5 x} = \frac{\frac{1}{\cos^5 x}}{\frac{1}{\cos^5 x} + \frac{1}{\sin^5 x}}$$

$$= \frac{\sin^5 x}{\sin^5 x + \cos^5 x}$$

Here $a = \frac{\pi}{2}$ and $f(x) = \frac{\sin^5 x}{\sin^5 x + \cos^5 x}$

$$f(a - x) = \frac{\cos^5 x}{\sin^5 x + \cos^5 x}$$

We know that,

$2I = \int_0^{\pi/2} 1 \, dx$

$= \frac{\pi}{4}$

Q54

So our integral becomes,

$$\frac{\sqrt{\cot x}}{1 + \sqrt{\cot x}} = \frac{\sqrt{\frac{\cos x}{\sin x}}}{1 + \sqrt{\frac{\cos x}{\sin x}}}$$

$$= \frac{\sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}}$$

$a = \frac{\pi}{2}$

$$f(a-x)=\frac{\sqrt{\sin x}}{\sqrt{\cos x}+\sqrt{\sin x}}$$

$$\therefore\ f(x)=\frac{\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}$$

$$\therefore\ 2I=\int_0^a f(x)+\int_0^a f(a-x)$$

$$=\int_0^{\frac{\pi}{2}}\frac{\sqrt{\sin x}+\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}dx$$

$$\int_0^{\pi/2}1\,dx=\frac{\pi}{4}$$

Q55

So our integral becomes,

$$\frac{\tan x}{1+\tan x}=\frac{\sin x}{\cos x}\left(\frac{1}{1+\frac{\sin x}{\cos x}}\right)$$

$$=\frac{\sin x}{\sin x+\cos x}$$

a = π/2

$$f(x)=\frac{\sin x}{(\sin x+\cos x)}$$

$$=\frac{\sin\left(\frac{\pi}{2}-x\right)}{\sin\left(\frac{\pi}{2}-x\right)+\cos\left(\frac{\pi}{2}-x\right)}$$

$$= \frac{\cos x}{\cos x + \sin x}$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sin x + \cos x}{\cos x + \sin x} dx$$

$\int_0^{\pi/2} 1 \, dx = 0 = \frac{\pi}{4}$

Q56

Here $f(x)=x^4 \sin x$

we will see $f(-x)=(-x)^4 \sin(-x)$

$=- x^4 \sin x$

Therefore, f(x) is a odd function,

$\int_{-\pi}^{\pi} x^4 Sinx \, dx$

Q57

Here $f(x)=x^3 \cos^3 x$

we will see $f(-x)= (-x)^3 \cos^3(-x)$

$=-x^3 \cos^3 x$

Therefore, f(x) is a odd function,

$\int_{-\pi}^{\pi} x^3 \cos^3 x \, dx = 0$

Q58

$f(x)=\sin^5 x$

$f(-x)=\sin^5(-x)$

$=-\sin^5 x$

Therefore, f(x) is a odd function,

$\int_{-\pi}^{\pi} \text{Sin}^5\, x\ \ dx = 0$

Q59

$$\int_{-1}^{-2} x^3(1-x^2)dx = \int_{-1}^{-2} (x^3-x^5)dx$$

$$= \left[\frac{x^4}{4} - \frac{x^6}{6}\right]$$

$$= \left[\frac{2^4}{4} - \frac{1^6}{4} - \frac{2^6}{6} + \frac{1^6}{6}\right]$$

$$= -\frac{27}{4}$$

Q60

$f(x) = \log\frac{a-x}{a+x}$

$f(-x) = \log\frac{a-(-x)}{a-x} = \log\frac{a+x}{a-x} = -\log\frac{a-x}{a+x}$

Hence it is a odd function

$\int_{-a}^{a} \log \frac{a-x}{a+x} dx = 0$

Q61

$\sin^{61}x$ and $x^{123}$is an odd function,

so there integral is zero.

Q62

f(x)=tan x

f(-x) =tan(-x)

=-tan x

hence the function is odd,

therefore, I=0

Q63

By by parts,

$$\int \log\left(x+\sqrt{x^2+1}\right) = x\log\left(x+\sqrt{x^2+1}\right) - \int \frac{x}{\left(x+\sqrt{x^2+1}\right)\left(1+\frac{x}{\sqrt{x^2+1}}\right)}$$

$$= {}_x\log(x+\sqrt{x^2+1})_- \int \frac{x}{\sqrt{x^2+1}} = {}_x\log(x+\sqrt{x^2+1})_-\sqrt{x^2+1}$$

Q64

cosx is an even function so,

$$\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx$$

$$\therefore \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x dx = 2\int_{0}^{\frac{\pi}{2}} \cos x dx$$

=2(1-0) =2

Q65

Here,

$$f(x) = \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}}$$

$$f(a-x) = \frac{\sqrt{a-x}}{\sqrt{x}+\sqrt{a-x}}$$

$$2I = \int_{0}^{a} \frac{\sqrt{x}+\sqrt{a-x}}{\sqrt{x}+\sqrt{a-x}} dx$$

$$= \int_{0}^{a} dx$$

$$I = \frac{a}{2}$$

Q66

let $I = \int_0^{\frac{\pi}{4}} \log(1+\tan x)dx$

$$= \log\left(1 + \frac{\left(\tan\frac{\pi}{4} - \tan x\right)}{1 + \tan\frac{\pi}{4}\tan x}\right)$$

$$= \log(1 + 1(1 - \tan x)\frac{1}{1+\tan x}$$

$$= \log\frac{2}{1+\tan x}$$

$$= \int_0^{\frac{\pi}{4}} \log\frac{2}{1+\tan x}\,dx$$

$$= \int_0^{\frac{\pi}{4}} \log 2dx - \int_0^{\frac{\pi}{4}} (1+\tan x)dx$$

$$\therefore I = \int_0^{\frac{\pi}{4}} \log 2\,dx - I$$

$$I = \frac{\pi}{8}\log 2$$

Q67

$$\int_{-a}^{a} f(x)dx = \int_{-a}^{0} f(x)dx + \int_0^a f(x)dx$$

$$\int_0^a f(-x)dx = \int_{-a}^{0} f(x)dx$$

$$\int_0^a f(-x)dx + \int_0^a f(x)dx$$

Q68

$\int_0^{1.5}[x]dx = \int_0^1[x]dx + \int_1^{1.5}[x]dx$

$= \int_0^1 0.dx + \int_1^{1.5} 1.dx$

= ½

Q69

$\int_0^2[x]dx = \int_{-1}^0[x]dx + \int_0^1[x]dx$

$= \int_{-1}^0 -1dx + \int_0^1 0dx$

=-1-0+0 = -1

Q70

$\int_0^2|x^2 - 3x + 2|dx$

∴ x^2-3x+2=0

(x-2)(x-1)=0

so, 2, and 1 itself are the limits so no breaking points for the integral,

$\int_1^2(-x^2 + 3x - 2)dx$

$[\frac{-x^3}{3} + \frac{3x^2}{2} - 2x]$ (limits 1 to 2

= 1/6

Q71

∴ sin x=0

∴ x=0,π,2π….

So π, 2π are the limits so no breaking points for the integral,

$\int_{\pi}^{2\pi} -Sinxdx = -Cosx$ (π, 2π

=2

Q72

put $\sin^{-1} x = t$

$dt = \frac{dx}{\sqrt{1-x^2}}$

x=sin t

$\sin^{-1} 1/\sqrt{2} = \pi/4$

=t = 0 and $\sin^{-1} 0=0$

Limit changes to,

$\int_0^{\frac{\pi}{4}} \frac{tdt}{1-\sin^2 t} = \int_0^{\frac{\pi}{4}} t \sec^2 t\, dt$

= t tant - $\int_0^{\frac{\pi}{4}} \tan t\, dt$

[t tan t + log cost] within the limits (0, π/4)

$= \frac{\pi}{4} - \frac{1}{2} log2$

Q73

put x=tan y

dx=sec^{2}ydy

$\int_0^{\frac{\pi}{4}} \sin^{-1}(Sin\ 2y) \sec^2 y\, dy$

$= 2\int_0^{\frac{\pi}{4}} y \sec^2 y\, dy$

$= 2[y \tan y - \int_0^{\frac{\pi}{4}} \tan y\, dy]$

= 2[y tany – log cosy] within the limite 0, π/4

$= \frac{\pi}{2} - \log 2$