CHAPTER-10 - Differentiation

Exercise-10A

Q1

f(x) = sin 4x

Let sin 4x = y and yx = t

Now,

y = sin t

differentiate both sides w.r.t. t

$\Rightarrow \frac{dy}{dt} = \frac{d}{dt}$(sin t)

$\Rightarrow \frac{dy}{dt}$ = cos t----------(i)

and t = 4x

differentiate both sides w.r.t. x

$\Rightarrow \frac{dt}{dx} = \frac{d}{dx}$(4x)

$\Rightarrow \frac{dt}{dx}$ = 4 ----------(ii)

Thus,

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

On using eq (i) and (ii), we get

= cos t × 4

$\therefore \frac{dy}{dx}$ = 4cos 4x (∵ t = 4x)

Q2

f(x) = cos 5x

Let cos 5x = y and 5x = t

Now,

y = cos t

differentiate both sides w.r.t. t

$\Rightarrow \frac{dy}{dt} = \frac{d}{dt}(\cos t)$

$\Rightarrow \frac{dy}{dt} = -\sin t$----------(i)

and t = 5x

differentiate both sides w.r.t. x

$\Rightarrow \frac{dt}{dx} = \frac{d}{dx}(5x)$

$\Rightarrow \frac{dt}{dx} = 5$----------(ii)

Thus,

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

On using eq (i) and (ii), we get

$= -\sin t \times 5$

$\therefore \frac{dy}{dx} = -5 \sin 5x$ $(\because t = 5x)$

Q3

f(x) = tan 3x

Let tan 3x = y and 3x = t

Now,

y = tan t

differentiate both sides w.r.t. t

$\Rightarrow \frac{dy}{dt} = \frac{d}{dt}(\tan t)$

$\Rightarrow \frac{dy}{dt} = \sec^2 t$----------(i)

and t = 3x

differentiate both sides w.r.t. x

$\Rightarrow \frac{dt}{dx} = \frac{d}{dx}(3x)$

$\Rightarrow \frac{dt}{dx} = 3$----------(ii)

Thus,

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

On using eq (i) and (ii), we get

$= \sec^2 t \times 3$

$\therefore \frac{dy}{dx} = 3 \sec^2 3x$ $(\because t = 3x)$

Q4

$f(x) = \cos x^3$

Let $\cos x^3 = y$ and $x^3 = t$

Now,

$y = \cos t$

differentiate both sides w.r.t. t

$\Rightarrow \frac{dy}{dt} = \frac{d}{dt}(\cos t)$

$\Rightarrow \frac{dy}{dt} = -\sin t$----------(i)

and $t = x^3$

differentiate both sides w.r.t. x

$\Rightarrow \frac{dt}{dx} = \frac{d}{dx}(x^3)$

$\Rightarrow \frac{dt}{dx} = 3x^2$----------(ii)

Thus,

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

On using eq (i) and (ii), we get

$= -\sin t \times 3x^2$

$\therefore \frac{dy}{dx} = -3x^2 \sin x^3$ $(\because t = x^3)$

Q5

$f(x) = \cot^2 x$

Let $\cot^2 x = y$ and $\cot x = t$

Now,

$y = t^2$

differentiate both sides w.r.t. t

$\Rightarrow \frac{dy}{dt} = \frac{d}{dt}(t^2)$

$\Rightarrow \frac{dy}{dt} = 2t$----------(i)

and $t = \cot x$

differentiate both sides w.r.t. x

$\Rightarrow \frac{dt}{dx} = \frac{d}{dx}(\cot x)$

$\Rightarrow \frac{dt}{dx} = -\operatorname{cosec}^2 x$ ----------(ii)

Thus,

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

On using eq (i) and (ii), we get

$= 2t \times (-\operatorname{cosec}^2 x)$

$\therefore \frac{dy}{dx} = -2\cot x \operatorname{cosec}^2 x$ $(\because t = \cot x)$

Q6

$f(x) = \tan^3 x$

Let $\cot^3 x = y$ and $\tan x = t$

Now,

$y = t^3$

differentiate both sides w.r.t. t

$\Rightarrow \frac{dy}{dt} = \frac{d}{dt}(t^3)$

$\Rightarrow \frac{dy}{dt} = 3t^2$----------(i)

and $t = \tan x$

differentiate both sides w.r.t. x

$\Rightarrow \frac{dt}{dx} = \frac{d}{dx}(\tan x)$

$\Rightarrow \frac{dt}{dx} = \sec^2 x$ ----------(ii)

Thus,

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

On using eq (i) and (ii), we get

$= 3t^2 \times \sec^2 x$

$\therefore \frac{dy}{dx} = 3\tan^2 x \sec^2 x$ ($\because t = \tan x$)

Q7

f(x) = cot $\sqrt{x}$

Let cot $\sqrt{x}$ = y and $\sqrt{x}$ = t

Now,

y = cot t

differentiate both sides w.r.t. t

$\Rightarrow \frac{dy}{dt} = \frac{d}{dt}$(cot t)

$\Rightarrow \frac{dy}{dt}$ = – cosec^2 t----------(i)

and t = $\sqrt{x}$

differentiate both sides w.r.t. x

$\Rightarrow \frac{dt}{dx} = \frac{d}{dx}(\sqrt{x})$

$\Rightarrow \frac{dt}{dx} = \frac{1}{2\sqrt{x}}$ ----------(ii)

Thus,

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

On using eq (i) and (ii), we get

$= -\text{cosec}^2 t \times \frac{1}{2\sqrt{x}}$

$\therefore \frac{dy}{dx} = -\frac{1}{2\sqrt{x}} \text{cosec}^2 \sqrt{x}$ ($\because$ t = $\sqrt{x}$)

Q8

f(x) = $\sqrt{tan\ x}$

Let $\sqrt{tan\ x}$ = y and tan x = t

Now,

y = $\sqrt{t}$

differentiate both sides w.r.t. t

$\Rightarrow \frac{dy}{dt} = \frac{d}{dt}(\sqrt{t})$

$\Rightarrow \frac{dy}{dt} = \frac{1}{2\sqrt{t}}$----------(i)

and t = tan x

differentiate both sides w.r.t. x

$\Rightarrow \frac{dt}{dx} = \frac{d}{dx}$(tan x)

$\Rightarrow \frac{dt}{dx}$ = sec^2 x ----------(ii)

Thus,

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

On using eq (i) and (ii), we get

$= \frac{1}{2\sqrt{t}} \times sec^2$ x

$\therefore \frac{dy}{dx} = \frac{sec^2\ x}{2\sqrt{tanx}}$ ($\because$ t = tan x)

Q9

$f(x) = (5 + 7x)^6$

Let $(5 + 7x)^6 = y$ and $5 + 7x = t$

Now,

$y = t^6$

differentiate both sides w.r.t. t

$\Rightarrow \frac{dy}{dt} = \frac{d}{dt}(t^6)$

$\Rightarrow \frac{dy}{dt} = 6t^5$----------(i)

and $t = 5 + 7x$

differentiate both sides w.r.t. x

$\Rightarrow \frac{dt}{dx} = \frac{d}{dx}(5 + 7x)$

$\Rightarrow \frac{dt}{dx} = \frac{d}{dx}(5) + \frac{d}{dx}(7x)$

$\Rightarrow \frac{dt}{dx} = 7$ ----------(ii)

Thus,

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

On using eq (i) and (ii), we get

$= 6t^5 \times 7$

$\therefore \frac{dy}{dx} = 42(5 + 7x)^5$ $(\because t = 5 + 7x)$

Q10

$f(x) = (3 - 4x)^5$

Let $(3 - 4x)^5 = y$ and $3 - 4x = t$

Now,

$y = t^5$

differentiate both sides w.r.t. t

$\Rightarrow \frac{dy}{dt} = \frac{d}{dt}(t^5)$

$\Rightarrow \frac{dy}{dt} = 5t^4$----------(i)

and $t = 3 - 4x$

differentiate both sides w.r.t. x

$\Rightarrow \frac{dt}{dx} = \frac{d}{dx}(3 - 4x)$

$\Rightarrow \frac{dt}{dx} = \frac{d}{dx}(3) + \frac{d}{dx}(-4x)$

$\Rightarrow \frac{dt}{dx} = -4$ ----------(ii)

Thus,

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

On using eq (i) and (ii), we get

$= 5t^4 \times (-4)$

$\therefore \frac{dy}{dx} = -20(3 - 4x)^4$ $(\because t = 3 - 4x)$

Q11

$f(x) = (2x^2 - 3x + 4)^5$

Let $(2x^2 - 3x + 4)^5 = y$ and $(2x^2 - 3x + 4) = t$

Now,

$y = t^5$

differentiate both sides w.r.t. t

$\Rightarrow \frac{dy}{dt} = \frac{d}{dt}(t^5)$

$\Rightarrow \frac{dy}{dt} = 5t^4$----------(i)

and $t = 2x^2 - 3x + 4$

differentiate both sides w.r.t. x

$\Rightarrow \frac{dt}{dx} = \frac{d}{dx}(2x^2 - 3x + 4)$

$\Rightarrow \frac{dt}{dx} = \frac{d}{dx}(2x^2) + \frac{d}{dx}(-3x) + \frac{d}{dx}(4)$

$\Rightarrow \frac{dt}{dx} = 4x - 3$ ----------(ii)

Thus,

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

On using eq (i) and (ii), we get

$= 5t^4 \times (4x - 3)$

$\therefore \frac{dy}{dx} = 5(4x - 3)(2x^2 - 3x + 4)^4$ $(\because t = 2x^2 - 3x + 4)$

Q12

$f(x) = (ax^2 + bx + c)^6$

Let $(ax^2 + bx + c)^6 = y$ and $(ax^2 + bx + c) = t$

Now,

$y = t^6$

differentiate both sides w.r.t. t

$\Rightarrow \frac{dy}{dt} = \frac{d}{dy}(t^6)$

$\Rightarrow \frac{dy}{dt} = 6t^5$----------(i)

and $t = ax^2 + bx + c$

differentiate both sides w.r.t. x

$\Rightarrow \frac{dt}{dx} = \frac{d}{dx}(ax^2 + bx + c)$

$\Rightarrow \frac{dt}{dx} = \frac{d}{dx}(ax^2) + \frac{d}{dx}(bx) + \frac{d}{dx}(c)$

$\Rightarrow \frac{dt}{dx} = 2ax + b$ ----------(ii)

Thus,

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

On using eq (i) and (ii), we get

$= 6t^5 \times (2ax + b)$

$\therefore \frac{dy}{dx} = 6(2ax + b)(ax^2 + bx + c)^5 \quad (\because t = ax^2 + bx + c)$

Q13

f(x) = $\frac{1}{(x^2-3x+5)^3}$

Let $\frac{1}{(x^2-3x+5)^3}$ = y and $(x^2 - 3x + 5)$ = t

Now,

y = $\frac{1}{t^3}$

differentiate both sides w.r.t. t

$\Rightarrow \frac{dy}{dt} = \frac{d}{dt}(t^{-3})$

$\Rightarrow \frac{dy}{dt} = -3t^{-4}$----------(i)

and t = $(x^2 - 3x + 5)$

differentiate both sides w.r.t. x

$\Rightarrow \frac{dt}{dx} = \frac{d}{dx}(x^2 - 3x + 5)$

$\Rightarrow \frac{dt}{dx} = \frac{d}{dx}(x^2) + \frac{d}{dx}(-3x) + \frac{d}{dx}(5)$

$\Rightarrow \frac{dt}{dx} = 2x - 3$ ----------(ii)

Thus,

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

On using eq (i) and (ii), we get

$= -3t^{-4} \times (2x - 3)$

$\therefore \frac{dy}{dx} = \frac{-3(2x-3)}{(x^2-3x+5)^4}$ $(\because t = x^2 - 3x + 5)$

Q14

$f(x) = \sqrt{\frac{a^2-x^2}{a^2+x^2}}$

Let $\sqrt{\frac{a^2-x^2}{a^2+x^2}} = y$ and $\frac{a^2-x^2}{a^2+x^2} = t$

Now,

$y = t^{1/2}$

differentiate both sides w.r.t. t

$\Rightarrow \frac{dy}{dt} = \frac{d}{dt}(t^{1/2})$

$\Rightarrow \frac{dy}{dt} = \frac{1}{2}t^{-1/2}$----------(i)

and $t = \frac{a^2-x^2}{a^2+x^2}$

differentiate both sides w.r.t. x

$\Rightarrow \frac{dt}{dx} = \frac{d}{dx}(\frac{a^2-x^2}{a^2+x^2})$

$\Rightarrow \frac{dt}{dx} = \frac{(a^2+x^2)\frac{d}{dx}(a^2-x^2)-(a^2-x^2)\frac{d}{dx}(a^2+x^2)}{(a^2+x^2)^2}$

$\Rightarrow \frac{dt}{dx} = \frac{(a^2+x^2)(0-2x)-(a^2-x^2)(0+2x)}{(a^2+x^2)^2}$

$\Rightarrow \frac{dt}{dx} = \frac{-2a^2x-2x^3-2a^2x+2x^3}{(a^2+x^2)^2}$

$\Rightarrow \frac{dt}{dx} = \frac{-4a^2x}{(a^2+x^2)^2}$ ----------(ii)

Thus,

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

On using eq (i) and (ii), we get

$= \frac{1}{2}t^{-1/2} \times \frac{-4a^2x}{(a^2+x^2)^2}$

$= \frac{1}{2} \times (\frac{a^2+x^2}{a^2-x^2})^{1/2} \times \frac{-4a^2x}{(a^2+x^2)^2}$ $(\because t = \frac{a^2-x^2}{a^2+x^2})$

$\therefore \frac{dy}{dx} = \frac{-2a^2x}{(a^2-x^2)^{1/2}.(a^2+x^2)^{3/2}}$

Q15

$f(x) = \sqrt{\frac{1+sinx}{1-sinx}}$

Let $y = \sqrt{\frac{1+sinx}{1-sinx}}$

Multiply numerator and denominator by (1 + sinx)

$\Rightarrow y = \sqrt{\frac{1+sinx}{1-sinx} \times \frac{1+sinx}{1+sinx}}$

$\Rightarrow y = \sqrt{\frac{(1+sinx)^2}{1-sin^2x}}$

$\Rightarrow y = \sqrt{\frac{(1+sinx)^2}{cos^2x}}$

$\Rightarrow y = \frac{1+sinx}{cosx}$

$\Rightarrow y = \frac{1}{cosx} + \frac{sinx}{cosx}$

$\Rightarrow y = secx + tanx$

differentiate both sides w.r.t. x

$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}(secx + tanx)$

$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}(\sec x) + \frac{d}{dx}(tanx)$

$\therefore \frac{dy}{dx} = secx.tanx + \sec^2 x$

Q16

$f(x) = \cos^2 x^3$

Let $y = \cos^2 x^3$, and $t = x^3$

Now,

$y = \cos^2 t$

Let $\cos t = u$

$\Rightarrow y = u^2$

differentiate both sides w.r.t. u

$\Rightarrow \frac{dy}{du} = \frac{d}{du}(u^2)$

$\Rightarrow \frac{dy}{du} = 2u$----------(i)

Now, $u = \cos t$

differentiate both sides w.r.t. t

$\Rightarrow \frac{du}{dt} = \frac{d}{dt}(\cos t)$

$\Rightarrow \frac{du}{dt} = -\sin t$ ----------(ii)

and $t = x^3$

differentiate both sides w.r.t. x

$\Rightarrow \frac{dt}{dx} = \frac{d}{dx}(x^3)$

$\Rightarrow \frac{dt}{dx} = 3x^2$----------(iii)

Thus,

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dt} \times \frac{dt}{dx}$

On using eq (i), (ii) and (iii), we get

$= 2u \times (-\sin t) \times 3x^2$

$= -3x^2.2\cos x^3.\sin x^3$ ($\because$ u = cost and $t = x^3$)

$\therefore \frac{dy}{dx} = -3x^2.\sin^2 x^3$ ($\because$ 2sinx.cosx = sin 2x)

Q17

$f(x) = \sec^3(x^2 + 1)$

Let $y = \sec^3(x^2 + 1)$, and $t = x^2 + 1$

Now,

$y = \sec^3 t$

Let sec t = u

$\Rightarrow y = u^3$

differentiate both sides w.r.t. u

$\Rightarrow \frac{dy}{du} = \frac{d}{du}(u^3)$

$\Rightarrow \frac{dy}{du} = 3u^2$----------(i)

Now, u = sec t

differentiate both sides w.r.t. t

$\Rightarrow \frac{du}{dt} = \frac{d}{dt}(\sec t)$

$\Rightarrow \frac{du}{dt} = \sec t.\tan t$ ----------(ii)

and $t = x^2 + 1$

differentiate both sides w.r.t. x

$\Rightarrow \frac{dt}{dx} = \frac{d}{dx}(x^2) + \frac{d}{dx}(1)$

$\Rightarrow \frac{dt}{dx} = 2x$----------(iii)

Thus,

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dt} \times \frac{dt}{dx}$

On using eq (i), (ii) and (iii), we get

$= 3u^2 \times (\sec t.\tan t) \times 2x$

$= -3\sec^2(x^2 + 1).(\sec(x^2 + 1).\tan(x^2 + 1)).2x$

($\because$ u = sec t and $t = x^2 + 1$)

$\therefore \frac{dy}{dx}$ $= -6x.\sec^3(x^2 + 1).\tan(x^2 + 1)$

Q18

f(x) = $\sqrt{cos3x}$

Let y = $\sqrt{cos3x}$, and t = 3x

Now,

y = $\sqrt{cos\ t}$

Let cos t = u

$\Rightarrow$ y = $\sqrt{u}$

differentiate both sides w.r.t. u

$\Rightarrow \frac{dy}{du} = \frac{d}{du}(\sqrt{u})$

$\Rightarrow \frac{dy}{du} = \frac{1}{2\sqrt{u}}$----------(i)

Now, u = cos t

differentiate both sides w.r.t. t

$\Rightarrow \frac{du}{dt} = \frac{d}{dt}(\cos t)$

$\Rightarrow \frac{du}{dt} = -\sin t$ ----------(ii)

and t = 3x

differentiate both sides w.r.t. x

$\Rightarrow \frac{dt}{dx} = \frac{d}{dx}(3x)$

$\Rightarrow \frac{dt}{dx} = 3$----------(iii)

Thus,

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dt} \times \frac{dt}{dx}$

On using eq (i), (ii) and (iii), we get

$= \frac{1}{2\sqrt{u}} \times (-\sin t) \times 3$

$= \frac{3}{2\sqrt{\cos 3x}}.(-\sin 3x)$

($\because$ u = cos t and t = 3x)

$\therefore \frac{dy}{dx} = -\frac{3}{2}.\frac{\sin 3x}{\sqrt{\cos 3x}}$

Q19

f(x) = $\sqrt[3]{sin2x}$

Let y = $\sqrt[3]{sin2x}$, and t = 2x

Now,

y = $\sqrt[3]{sint}$

Let sin t = u

$\Rightarrow$ y = $u^{1/3}$

differentiate both sides w.r.t. u

$\Rightarrow \frac{dy}{du} = \frac{d}{du}(u^{1/3})$

$\Rightarrow \frac{dy}{du} = \frac{1}{3}u^{-2/3}$----------(i)

Now, u = sin t

differentiate both sides w.r.t. t

$\Rightarrow \frac{du}{dt} = \frac{d}{dt}(\sin t)$

$\Rightarrow \frac{du}{dt} = \cos t$ ----------(ii)

and t = 2x

differentiate both sides w.r.t. x

$\Rightarrow \frac{dt}{dx} = \frac{d}{dx}(2x)$

$\Rightarrow \frac{dt}{dx} = 2$----------(iii)

Thus,

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dt} \times \frac{dt}{dx}$

On using eq (i), (ii) and (iii), we get

$= \frac{1}{3}u^{-2/3} \times (\cos t) \times 2$

$= \frac{2}{3}\sin^{-2/3}2x.\cos 2x$

($\because$ u = sin t and t = 2x)

$\therefore \frac{dy}{dx} = \frac{2}{3}.\frac{\cos 2x}{\sin^{2/3}2x}$

Q20

f(x) = $\sqrt{1 + cot\ x}$

Let y =$\sqrt{1 + cot\ x}$, and t = 1 + cot x

Now,

y = $\sqrt{t}$

differentiate both sides w.r.t. t

$\Rightarrow \frac{dy}{dt} = \frac{d}{dt}(t^{1/2})$

$\Rightarrow \frac{dy}{dt} = \frac{1}{2}t^{-1/2}$----------(i)

Now, t = 1 + cot x

differentiate both sides w.r.t. x

$\Rightarrow \frac{dt}{dx} = \frac{d}{dx}(1 + \cot x)$

$\Rightarrow \frac{dt}{dx} = \frac{d}{dx}(1) + \frac{d}{dx}(\cot x)$

$\Rightarrow \frac{dt}{dx} = 0 - \text{cosec}^2\ x$----------(ii)

Thus,

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

On using eq (i) and (ii), we get

$= \frac{1}{2}u^{-1/2} \times (-\text{cosec}^2\ x)$

$= \frac{1}{2}(1 + \cot x)^{-1/2}(-\text{cosec}^2 x)$ $(\because u = 1 + \cot x)$

$\therefore \frac{dy}{dx} = -\frac{1}{2}.\frac{cosec^2 x}{\sqrt{1+cot\, x}}$

Q21

$f(x) = \text{cosec}^3 \frac{1}{x^2}$

Let $y = \text{cosec}^3 \frac{1}{x^2}$, and $t = \frac{1}{x^2}$

Now,

$y = \text{cosec}^3 t$

Let cosec t = u

$\Rightarrow y = u^3$

differentiate both sides w.r.t. u

$\Rightarrow \frac{dy}{du} = \frac{d}{du}(u^3)$

$\Rightarrow \frac{dy}{du} = 3u^2$----------(i)

Now, u = cosec t

differentiate both sides w.r.t. t

$\Rightarrow \frac{du}{dt} = \frac{d}{dt}(\text{cosec } t)$

$\Rightarrow \frac{du}{dt} = -\text{cosec t.cot t}$----------(ii)

and $t = \frac{1}{x^2}$

differentiate both sides w.r.t. x

$\Rightarrow \frac{dt}{dx} = \frac{d}{dx}(x^{-2})$

$\Rightarrow \frac{dt}{dx} = -2x^{-3}$----------(iii)

Thus,

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dt} \times \frac{dt}{dx}$

On using eq (i), (ii) and (iii), we get

$= 3u^2 \times (-\text{ cosec t.cot t}) \times (-2x^{-3})$

$= -6x^{-3}.\text{cosec}^2 \frac{1}{x^2}.(-\text{ cosec } \frac{1}{x^2}.\cot \frac{1}{x^2})$

($\because$ u = cosec t and t = $\frac{1}{x^2}$)

$\therefore \frac{dy}{dx} = 6x^{-3}.\text{cosec}^3 \frac{1}{x^2}.\cot \frac{1}{x^2}$

Q22

$f(x) = \sqrt{sinx^3}$

Let y = $\sqrt{sinx^3}$, and t = x^3

Now,

$y = \sqrt{sin\ t}$

Let sin t = u

$\Rightarrow y = \sqrt{u}$

differentiate both sides w.r.t. u

$\Rightarrow \frac{dy}{du} = \frac{d}{du}(\sqrt{u})$

$\Rightarrow \frac{dy}{du} = \frac{1}{2\sqrt{u}}$----------(i)

Now, u = sin t

differentiate both sides w.r.t. t

$\Rightarrow \frac{du}{dt} = \frac{d}{dt}(\sin t)$

$\Rightarrow \frac{du}{dt} = \cos t$----------(ii)

and $t = x^3$

differentiate both sides w.r.t. x

$\Rightarrow \frac{dt}{dx} = \frac{d}{dx}(x^3)$

$\Rightarrow \frac{dt}{dx} = 3x^2$----------(iii)

Thus,

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dt} \times \frac{dt}{dx}$

On using eq (i), (ii) and (iii), we get

$= \frac{1}{2\sqrt{u}} \times (\cos t) \times 3x^2$

$= \frac{3x^2}{2\sqrt{\sin x^3}}.(\cos x^3)$

($\because$ u = sin t and $t = x^3$)

$\therefore \frac{dy}{dx} = -\frac{3}{2}x^2.\frac{\cos x^3}{\sqrt{\sin x^3}}$

Q23

f(x) = $\sqrt{xsinx}$

Let y =$\sqrt{xsinx}$, and t = x.sinx

Now,

y = $\sqrt{t}$

differentiate both sides w.r.t. t

$\Rightarrow \frac{dy}{dt} = \frac{d}{du}(\sqrt{t})$

$\Rightarrow \frac{dy}{dt} = \frac{1}{2\sqrt{t}}$----------(i)

and t = x.sinx

differentiate both sides w.r.t. x

$\Rightarrow \frac{dt}{dx} = \frac{d}{dx}$(x.sinx)

$\Rightarrow \frac{dt}{dx}$ = x$\frac{d}{dx}$(sinx) + sinx$\frac{d}{dx}$(x)

$\Rightarrow \frac{dt}{dx}$ = x.cosx + sinx----------(ii)

Thus,

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

On using eq (i) and (ii), we get

$= \frac{1}{2\sqrt{t}} \times$ (x.cosx + sinx)

$\therefore \frac{dy}{dx} = \frac{x.cosx+sinx}{2\sqrt{x.sinx}}$ ($\because$ t = x.sinx)

Q24

f(x) = $\sqrt{cot\sqrt{x}}$

Let y =$\sqrt{cot\sqrt{x}}$, and u = cot $\sqrt{x}$

Now,

y = $\sqrt{u}$

$\Rightarrow$ y = $\sqrt{u}$

differentiate both sides w.r.t. u

$\Rightarrow \frac{dy}{du} = \frac{d}{du}(\sqrt{u})$

$\Rightarrow \frac{dy}{du} = \frac{1}{2\sqrt{u}}$----------(i)

Now, u = cot $\sqrt{x}$

Let t = $\sqrt{x}$

$\Rightarrow$ u = cot t

differentiate both sides w.r.t. t

$\Rightarrow \frac{du}{dt} = \frac{d}{dt}(\cot t)$

$\Rightarrow \frac{du}{dt}$ = – cosec^2 t----------(ii)

and t = $\sqrt{x}$

differentiate both sides w.r.t. u

$\Rightarrow \frac{dy}{du} = \frac{d}{du}(\sqrt{x})$

$\Rightarrow \frac{dy}{du} = \frac{1}{2\sqrt{x}}$----------(i)

Thus,

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dt} \times \frac{dt}{dx}$

On using eq (i), (ii) and (iii), we get

$= \frac{1}{2\sqrt{u}} \times (-\text{ cosec}^2 \text{ t}) \times \frac{1}{2\sqrt{x}}$

$= \frac{1}{2\sqrt{\cot\sqrt{x}}} \times (-\text{ cosec}^2 \sqrt{x}) \times \frac{1}{2\sqrt{x}}$

($\because$ u = cot t and t = $\sqrt{x}$)

$\therefore \frac{dy}{dx} = \frac{-cosec^2 \sqrt{x}}{4\sqrt{x}\sqrt{\cot\sqrt{x}}}$)

Q25

$f(x) = \cot^3 x^2$

Let $y = co^3 x^2$, and $t = x^2$

Now,

$y = \cot^3 t$

Let cot t = u

$\Rightarrow y = u^3$

differentiate both sides w.r.t. u

$\Rightarrow \frac{dy}{du} = \frac{d}{du}(u^3)$

$\Rightarrow \frac{dy}{du} = 3u^2$----------(i)

Now, u = cot t

differentiate both sides w.r.t. t

$\Rightarrow \frac{du}{dt} = \frac{d}{dt}(\cot t)$

$\Rightarrow \frac{du}{dt} = -\operatorname{cosec}^2 t$----------(ii)

and $t = x^2$

differentiate both sides w.r.t. x

$\Rightarrow \frac{dt}{dx} = \frac{d}{dx}(x^2)$

$\Rightarrow \frac{dt}{dx} = 2x$----------(iii)

Thus,

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dt} \times \frac{dt}{dx}$

On using eq (i), (ii) and (iii), we get

$= 3u^2 \times (-\operatorname{cosec}^2 t) \times 2x$

$\therefore \frac{dy}{dx} = -6x.\cot^2 x^2.\operatorname{cosec}^2 x^2$ ($\because$ u = cot t and $t = x^2$)

Q26

$f(x) = \cos(\sin\sqrt{ax + b})$

Let y = cos(sin$\sqrt{ax+b}$), t = sin$\sqrt{ax+b}$, u = $\sqrt{ax+b}$)

and w = ax + b

Now,

y = cos t

differentiate both sides w.r.t. t

$\Rightarrow \frac{dy}{dt} = \frac{d}{dt}$(cos t)

$\Rightarrow \frac{dy}{dt}$ = –sin t----------(i)

Now, t = sin u

differentiate both sides w.r.t. u

$\Rightarrow \frac{dt}{du} = \frac{d}{du}$(sin u)

$\Rightarrow \frac{du}{dt}$ = cos u----------(ii)

Now, u = $\sqrt{w}$

differentiate both sides w.r.t. w

$\Rightarrow \frac{du}{dw} = \frac{d}{dw}(\sqrt{w})$

$\Rightarrow \frac{du}{dw} = \frac{1}{2\sqrt{w}}$----------(iii)

And w = ax + b

differentiate both sides w.r.t. x

$\Rightarrow \frac{dw}{dx} = \frac{d}{dx}$(ax + b)

$\Rightarrow \frac{dw}{dx}$ = a----------(iv)

Thus,

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{du} \times \frac{du}{dw} \times \frac{dw}{dx}$

On using eq (i), (ii), (iii) and (iv), we get

$= (-\sin t) \times (\cos u) \times (\frac{1}{2\sqrt{w}}) \times (a)$

$= (-\sin.(\sin\sqrt{ax+b})) \times (\cos \sqrt{ax+b}) \times (\frac{1}{2\sqrt{ax+b}}) \times (a)$

$(\because t = \sin\sqrt{ax+b}$, $u = \sqrt{ax+b}$ and $w = ax + b)$

$\therefore \frac{dy}{dx} = \frac{-a}{2\sqrt{ax+b}}.(\sin(\sin\sqrt{ax+b})).(\cos \sqrt{ax+b})$

Q27

$f(x) = \sqrt{cosec(x^3+1)}$

Let $y = \sqrt{cosec(x^3+1)}$, $t = \text{cosec}(x^3 + 1)$ and $u = x^3 + 1$

Now,

$y = \sqrt{t}$

$\Rightarrow y = \sqrt{t}$

differentiate both sides w.r.t. t

$\Rightarrow \frac{dy}{dt} = \frac{d}{du}(\sqrt{t})$

$\Rightarrow \frac{dy}{dt} = \frac{1}{2\sqrt{t}}$----------(i)

Now, t = cosec u

differentiate both sides w.r.t. u

$\Rightarrow \frac{dt}{du} = \frac{d}{du}(\text{cosec}\, u)$

$\Rightarrow \frac{dt}{du}$ = – cosec u.cot u----------(ii)

Now, u = $x^3 + 1$

differentiate both sides w.r.t. x

$\Rightarrow \frac{du}{dx} = \frac{d}{dx}(x^3 + 1)$

$\Rightarrow \frac{du}{dx} = 3x^2$----------(iii)

Thus,

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{du} \times \frac{du}{dx}$

On using eq (i), (ii) and (iii), we get

$= \frac{1}{2\sqrt{t}} \times$ (– cosec u.cot u) × $3x^2$

$= \frac{-3x^2}{2\sqrt{\text{cosec}(x^3+1)}}$.(cosec ($x^3 + 1$).cot ($x^3$+ 1))

(∵ t = cosec($x^3 + 1$) and u = $x^3 + 1$)

$\therefore \frac{dy}{dx} = \frac{-3x^2}{2\sqrt{\text{cosec}(x^3+1)}}.\sqrt{cosec\ (x^3+1)}$.cot($x^3$+ 1)

Q28

f(x) = sin 5x.cos 3x

Let y = sin 5x.cos 3x

Multiplying Numerator and denominator by 2

$\Rightarrow y = \frac{1}{2} \times 2\sin 5x.\cos 3x$

$\Rightarrow y = \frac{1}{2} \times \{\sin(5x+3x) + \sin(5x - 3x)\}$

$(\because 2\sin a.\cos b = \sin(a+b) + \sin(a - b))$

$\Rightarrow y = \frac{1}{2} \times [\sin(8x) + \sin(2x)]$

differentiating both sides w.r.t. x

$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}[\frac{1}{2} \times (\sin 8x + \sin 2x)]$

$\Rightarrow \frac{dy}{dx} = \frac{1}{2}[\frac{d}{dx}(\sin 8x) + \frac{d}{dx}(\sin 2x)]$

$\Rightarrow \frac{dy}{dx} = \frac{1}{2}[8\cos 8x + 2\cos 2x]$

$\therefore \frac{dy}{dx} = 4\cos 8x + \cos 2x$

Q29

f(x) = sin 2x.sin x

Let y = sin 2x.sin x

Multiplying Numerator and denominator by 2

$\Rightarrow y = \frac{1}{2} \times 2\sin 2x.\sin x$

$\Rightarrow y = \frac{1}{2} \times \{\cos(2x - x) - \cos(2x + x)\}$

$(\because 2\sin a.\sin b = \cos(a - b) - \cos(a + b))$

$\Rightarrow y = \frac{1}{2} \times [\cos(x) - \cos(3x)]$

differentiating both sides w.r.t. x

$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}[\frac{1}{2} \times (\cos x - \cos 3x)]$

$\Rightarrow \frac{dy}{dx} = \frac{1}{2}[\frac{d}{dx}(\cos x) - \frac{d}{dx}(\cos 3x)]$

$\Rightarrow \frac{dy}{dx} = \frac{1}{2}[(-\sin x) - (-3\sin 3x)]$

$\therefore \frac{dy}{dx} = \frac{3}{2}\sin 3x - \frac{1}{2}\sin x$

Q30

$f(x) = \cos 4x.\cos 2x$

Let $y = \cos 4x.\cos 2x$

Multiplying Numerator and denominator by 2

$\Rightarrow y = \frac{1}{2} \times 2\cos 4x.\cos 2x$

$\Rightarrow y = \frac{1}{2} \times \{\cos(4x + 2x) + \cos(4x - 2x)\}$

$(\because 2\cos a.\cos b = \cos(a + b) + \cos(a - b))$

$\Rightarrow y = \frac{1}{2} \times [\cos(6x) + \cos(2x)]$

differentiating both sides w.r.t. x

$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}[\frac{1}{2}\times(\cos 6x + \cos 2x)]$

$\Rightarrow \frac{dy}{dx} = \frac{1}{2}[\frac{d}{dx}(\cos 6x) + \frac{d}{dx}(\cos 2x)]$

$\Rightarrow \frac{dy}{dx} = \frac{1}{2}[(-6\sin 6x) + (-2\sin 2x)]$

$\therefore \frac{dy}{dx} = -3\sin 6x - \sin 2x$

Q31

$y = \sin(\frac{1+x^2}{1-x^2})$

Let $t = \frac{1+x^2}{1-x^2}$

$\Rightarrow y = \sin t$

differentiating both sides w.r.t. t

$\Rightarrow \frac{dy}{dt} = \frac{d}{dt}(\sin t)$

$\Rightarrow \frac{dy}{dt} = \cos t$----------(i)

Now, $t = \frac{1+x^2}{1-x^2}$

differentiating both sides w.r.t. x

$\Rightarrow \frac{dt}{dx} = \frac{d}{dx}(\frac{1+x^2}{1-x^2})$

$$\Rightarrow \frac{dt}{dx} = \frac{(1-x^2)\frac{d}{dx}(1+x^2)-(1+x^2)\frac{d}{dx}(1-x^2)}{(1-x^2)^2}$$

$$\Rightarrow \frac{dt}{dx} = \frac{(1-x^2)2x-(1+x^2)(-2x)}{(1-x^2)^2}$$

$$\Rightarrow \frac{dt}{dx} = \frac{(1-x^2)(0+2x)-(1+x^2)(0-2x)}{(1-x^2)^2}$$

$$\Rightarrow \frac{dt}{dx} = \frac{(2x-2x^3)+(2x+2x^3)}{(1-x^2)^2}$$

$$\Rightarrow \frac{dt}{dx} = \frac{4x}{(1-x^2)^2}$$----------(ii)

Thus,

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

On using eq (i) and (ii), we get

$$\frac{dy}{dx} = \cos t \times \frac{4x}{(1-x^2)^2}$$

$$\therefore \frac{dy}{dx} = \frac{4x}{(1-x^2)^2}.\cos\frac{1+x^2}{1-x^2} \qquad (\because t = \frac{1+x^2}{1-x^2})$$

Q32

$$y = \frac{sinx + x^2}{\cot 2x}$$

differentiating both sides w.r.t. x

$$\Rightarrow \frac{dt}{dx} = \frac{d}{dx}\left(\frac{sinx + x^2}{\cot 2x}\right)$$

$$\Rightarrow \frac{dt}{dx} = \frac{(\cot 2x)\frac{d}{dx}(sinx + x^2) - (sinx + x^2)\frac{d}{dx}(cot2x)}{(cot2x)^2}$$

$$\Rightarrow \frac{dt}{dx} = \frac{(\cot 2x)(cosx + 2x) - (sinx + x^2)(-2cosec^2 2x)}{cot^2 2x}$$

$$\Rightarrow \frac{dt}{dx} = \frac{(cosx.cot2x + 2x.cot2x)}{cot^2 2x} + \frac{(sinx + x^2)(2cosec^2 2x)}{cot^2 2x}$$

$$\Rightarrow \frac{dt}{dx} = \frac{(cosx.cot2x + 2x.cot2x)}{cot^2 2x} + \frac{(sinx + x^2)(\frac{2}{sin^2 2x})}{\frac{cos^2 2x}{sin^2 2x}}$$

$$\Rightarrow \frac{dt}{dx} = \frac{(2x + cosx)}{cot\ 2x} + \frac{2(sinx + x^2)}{cos^2 2x}$$

$\therefore \frac{dt}{dx} = \tan 2x.(2x + \cos x) + 2\sec^2 2x(\sin x + x^2)$

Q33

To prove: $\frac{dy}{dx} + y^2 + 1 = 0$

Given: $y = \frac{\cos x - \sin x}{\cos x + \sin x}$

Proof: $y = \frac{\cos x - \sin x}{\cos x + \sin x}$

dividing numerator and denominator in RHS by cos x

$$\Rightarrow y = \frac{\frac{\cos x}{\cos x} - \frac{\sin x}{\cos x}}{\frac{\cos x}{\cos x} + \frac{\sin x}{\cos x}}$$

$$\Rightarrow y = \frac{1 - tanx}{1 + tanx}$$

$$\Rightarrow y = \frac{\tan\frac{\pi}{4} - tanx}{1 + \tan\frac{\pi}{4}.tanx} \qquad (\because \tan\frac{\pi}{4} = 1)$$

$\Rightarrow y = \tan(\frac{\pi}{4} - x)$ $(\because \tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A . \tan B})$

differentiating both sides w.r.t. x

$\Rightarrow \frac{dy}{dx} = \frac{dy}{dx}[\tan(\frac{\pi}{4} - x)]$

$\Rightarrow \frac{dy}{dx} = \sec^2(\frac{\pi}{4} - x)(0 - 1)$

$\Rightarrow \frac{dy}{dx} = -\sec^2(\frac{\pi}{4} - x)$

On putting the y and $\frac{dy}{dx}$ in $\frac{dy}{dx} + y^2 + 1 = 0$, we get

$-\sec^2(\frac{\pi}{4} - x) + \tan^2(\frac{\pi}{4} - x) + 1 = 0$

$\Rightarrow -\sec^2(\frac{\pi}{4} - x) + \sec^2(\frac{\pi}{4} - x) = 0$ $(\because 1 + \tan^2 A = \sec^2 A)$

$\Rightarrow 0 = 0$

LHS = RHS

Hence Proved

Q34

To prove: $\frac{dy}{dx} = \sec^2(\frac{\pi}{4} - x)$

Given: $y = \frac{\cos x + \sin x}{\cos x - \sin x}$

Proof: $y = \frac{\cos x + \sin x}{\cos x - \sin x}$

dividing numerator and denominator in RHS by cos x

$\Rightarrow y = \frac{\frac{\cos x}{\cos x} + \frac{\sin x}{\cos x}}{\frac{\cos x}{\cos x} - \frac{\sin x}{\cos x}}$

$\Rightarrow y = \frac{1 + tanx}{1 - tanx}$

$\Rightarrow y = \frac{\tan\frac{\pi}{4} + tanx}{1 - \tan\frac{\pi}{4}.tanx}$ $(\because \tan\frac{\pi}{4} = 1)$

$\Rightarrow y = \tan(\frac{\pi}{4} + x)$ $(\because \tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A.\tan B})$

differentiating both sides w.r.t. x

$\Rightarrow \frac{dy}{dx} = \frac{dy}{dx}[\tan(\frac{\pi}{4} + x)]$

$\Rightarrow \frac{dy}{dx} = \sec^2(\frac{\pi}{4} + x)(0 + 1)$

$\Rightarrow \frac{dy}{dx} = \sec^2(\frac{\pi}{4} + x)$

Hence proved

Exercise – 10D

Q1

$f(x) = \sin^{-1}\{\sqrt{\frac{1 - cosx}{2}}\}$

Let $y = \sin^{-1}\{\sqrt{\frac{1 - cosx}{2}}\}$

$\Rightarrow y = \sin^{-1}\{\sqrt{\frac{2sin^2\frac{x}{2}}{2}}\}$ $(\because \cos A = 2\sin^2\frac{A}{2})$

$\Rightarrow y = \sin^{-1}\sin\frac{x}{2}$

$\Rightarrow y = \frac{x}{2}$

differentiate both sides w.r.t. x

$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}(\frac{x}{2})$

$\therefore \frac{dy}{dx} = \frac{1}{2}$

Q2

$f(x) = \tan^{-1}(\frac{\sin x}{1+\cos x})$

Let $y = \tan^{-1}(\frac{\sin x}{1+\cos x})$

$\Rightarrow y = \tan^{-1}\{\frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{2cos^2\frac{x}{2}}\}$

$(\because 1 + \cos A = 2\cos^2\frac{A}{2}$ and $\sin 2A = 2\sin A \cos A)$

$\Rightarrow y = \tan^{-1}\{\frac{\sin\frac{x}{2}}{cos\frac{x}{2}}\}$

$\Rightarrow y = \tan^{-1}\{\tan\frac{x}{2}\}$

$\Rightarrow y = \frac{x}{2}$

differentiate both sides w.r.t. x

$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}(\frac{x}{2})$

$\therefore \frac{dy}{dx} = \frac{1}{2}$

Q3

$f(x) = \cot^{-1}(\frac{1+\cos x}{\sin x})$

Let $y = \cot^{-1}(\frac{1+\cos x}{\sin x})$

$\Rightarrow y = \cot^{-1}\{\frac{2\cos^2\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}}\}$

$(\because 1 + \cos A = 2\cos^2\frac{A}{2}$ and $\sin 2A = 2\sin A \cos A)$

$\Rightarrow y = \cot^{-1}\{\frac{\cos\frac{x}{2}}{\sin\frac{x}{2}}\}$

$\Rightarrow y = \cot^{-1}\{\cot\frac{x}{2}\}$

$\Rightarrow y = \frac{x}{2}$

differentiate both sides w.r.t. x

$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}(\frac{x}{2})$

$\therefore \frac{dy}{dx} = \frac{1}{2}$

Q4

$f(x) = \cot^{-1}(\sqrt{\frac{1+\cos x}{1-\cos x}})$

Let $y = \cot^{-1}\left(\sqrt{\frac{1+\cos x}{1-\cos x}}\right)$

$\Rightarrow y = \cot^{-1}\left(\sqrt{\frac{1+\cos x}{1-\cos x}} \times \sqrt{\frac{1+\cos x}{1-\cos x}}\right)$

$\Rightarrow y = \cot^{-1}\left(\sqrt{\frac{(1+\cos x)^2}{1-\cos^2 x}}\right)$

$\Rightarrow y = \cot^{-1}\left(\sqrt{\frac{(1+\cos x)^2}{\sin^2 x}}\right)$

$\Rightarrow y = \cot^{-1}\left(\frac{1+cosx}{\sin x}\right)$

$\Rightarrow y = \cot^{-1}\left\{\frac{2\cos^2\frac{x}{2}}{2\sin\frac{x}{2}cos\frac{x}{2}}\right\}$

$(\because 1 + \cos A = 2\cos^2\frac{A}{2}$ and $\sin 2A = 2\sin A \cos A)$

$\Rightarrow y = \cot^{-1}\left\{\frac{\cos\frac{x}{2}}{sin\frac{x}{2}}\right\}$

$\Rightarrow y = \cot^{-1}\left\{\cot\frac{x}{2}\right\}$

$\Rightarrow y = \frac{x}{2}$

differentiate both sides w.r.t. x

$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}\left(\frac{x}{2}\right)$

$\therefore \frac{dy}{dx} = \frac{1}{2}$

Q5

$f(x) = \tan^{-1}\left(\frac{cosx+\sin x}{\cos x-sinx}\right)$

Let $y = \tan^{-1}\left(\frac{cosx+\sin x}{\cos x-sinx}\right)$

Dividing numerator and denominator by cos x

$\Rightarrow y = \tan^{-1}\left(\frac{\frac{\cos x}{\cos x}+\frac{\sin x}{\cos x}}{\frac{\cos x}{\cos x}-\frac{\sin x}{\cos x}}\right)$

$\Rightarrow y = \tan^{-1}\left(\frac{1+tanx}{1-tanx}\right)$

$\Rightarrow y = \tan^{-1}\left(\frac{\tan\frac{\pi}{4}+tanx}{1-\tan\frac{\pi}{4}.tanx}\right)$ $\quad (\because \tan\frac{\pi}{4} = 1)$

$\Rightarrow y = \tan^{-1}(\tan(\frac{\pi}{4} + x))$ $\quad (\because \tan(A + B) = \frac{\tan A + \tan B}{1-\tan A.\tan B})$

$\Rightarrow y = \frac{\pi}{4} + x$

differentiate both sides w.r.t. x

$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}(\frac{\pi}{4} + x)$

$\therefore \frac{dy}{dx} = 1$

Q6

$f(x) = \cot^{-1}\left(\frac{\cos x-\sin x}{\cos x+\sin x}\right)$

$y = \cot^{-1}\left(\frac{\cos x-\sin x}{\cos x+\sin x}\right)$

dividing numerator and denominator in RHS by cos x

$\Rightarrow y = \cot^{-1}\left(\frac{\frac{\cos x}{\cos x} - \frac{\sin x}{\cos x}}{\frac{\cos x}{\cos x} + \frac{\sin x}{\cos x}}\right)$

$\Rightarrow y = \cot^{-1}\left(\frac{1 - tanx}{1+tanx}\right)$

$\Rightarrow y = \cot^{-1}\left(\frac{\tan\frac{\pi}{4} - tanx}{1+\tan\frac{\pi}{4}.tanx}\right)$ $\quad (\because \tan\frac{\pi}{4} = 1)$

$\Rightarrow y = \cot^{-1}(\tan(\frac{\pi}{4} - x))$ $\quad (\because \tan(A - B) = \frac{\tan A - \tan B}{1+\tan A.\tan B})$

$\Rightarrow y = \cot^{-1}(\cot(\frac{\pi}{2} - (\frac{\pi}{4} - x))$

$\Rightarrow y = \cot^{-1}(\cot(\frac{\pi}{4} + x))$

$\Rightarrow y = (\frac{\pi}{4} + x)$

differentiate both sides w.r.t. x

$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}(\frac{\pi}{4} + x)$

$\therefore \frac{dy}{dx} = 1$

Q7

$f(x) = \cot^{-1}\left(\sqrt{\frac{1+\cos 3x}{1-\cos 3x}}\right)$

Let $y = \cot^{-1}\left(\sqrt{\frac{1+\cos 3x}{1-\cos 3x}}\right)$

$\Rightarrow y = \cot^{-1}\left(\sqrt{\frac{1+\cos 3x}{1-\cos 3x}} \times \sqrt{\frac{1+\cos 3x}{1-\cos 3x}}\right)$

$\Rightarrow y = \cot^{-1}\left(\sqrt{\frac{(1+\cos 3x)^2}{1-\cos^2 3x}}\right)$

$\Rightarrow y = \cot^{-1}\left(\sqrt{\frac{(1+\cos 3x)^2}{\sin^2 3x}}\right)$

$\Rightarrow y = \cot^{-1}\left(\frac{1+cos3x}{\sin 3x}\right)$

$\Rightarrow y = \cot^{-1}\left\{\frac{2\cos^2\frac{3x}{2}}{2\sin\frac{3x}{2}cos\frac{3x}{2}}\right\}$

$(\because 1 + \cos A = 2\cos^2\frac{A}{2}$ and $\sin 2A = 2\sin A \cos A)$

$\Rightarrow y = \cot^{-1}\left\{\frac{\cos\frac{3x}{2}}{sin\frac{3x}{2}}\right\}$

$\Rightarrow y = \cot^{-1}\left\{\cot\frac{3x}{2}\right\}$

$\Rightarrow y = \frac{3x}{2}$

differentiate both sides w.r.t. x

$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}\left(\frac{3x}{2}\right)$

$\therefore \frac{dy}{dx} = \frac{3}{2}$

Q8

$f(x) = \sec^{-1}\left(\frac{1+tan^2x}{1-tan^2 x}\right)$

Let $y = \sec^{-1}\left(\frac{1+tan^2x}{1-tan^2 x}\right)$

Dividing numerator and denominator by $1+\tan^2 x$

$\Rightarrow y = \sec^{-1}\left(\frac{\frac{1+\tan^2 x}{1+\tan^2 x}}{\frac{1-\tan^2 x}{1+\tan^2 x}}\right)$

$\Rightarrow y = \sec^{-1}\left(\frac{1}{\cos 2x}\right)$ $\left(\because \cos 2A = \frac{1-\tan^2 A}{1+\tan^2 A}\right)$

$\Rightarrow y = \sec^{-1}(\sec 2x)$

$\Rightarrow y = 2x$

differentiate both sides w.r.t. x

$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}(2x)$

$\therefore \frac{dy}{dx} = 2$

Q9

$f(x) = \sin^{-1}\left(\frac{1-\tan^2 x}{1+\tan^2 x}\right)$

Let $y = \sin^{-1}\left(\frac{1-\tan^2 x}{1+\tan^2 x}\right)$

$\Rightarrow y = \sin^{-1}(\cos 2x)$ $\left(\because \cos 2A = \frac{1-\tan^2 A}{1+\tan^2 A}\right)$

$\Rightarrow y = \sin^{-1}\left(\sin\left(\frac{\pi}{2} - 2x\right)\right)$

$\Rightarrow y = \frac{\pi}{2} - 2x$

differentiate both sides w.r.t. x

$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}\left(\frac{\pi}{2} - 2x\right)$

$\therefore \frac{dy}{dx} = -2$

Q10

$f(x) = \text{cosec}^{-1}\left(\frac{1+tan^2x}{2tanx}\right)$

Let $y = \text{cosec}^{-1}\left(\frac{1+tan^2x}{2tanx}\right)$

Dividing numerator and denominator by $1+\tan^2 x$

$\Rightarrow y = \text{cosec}^{-1}\left(\frac{\frac{1+tan^2x}{1+tan^2x}}{\frac{2tanx}{1+tan^2x}}\right)$

$\Rightarrow y = \text{cosec}^{-1}\left(\frac{1}{\sin 2x}\right)$ $\left(\because \sin 2A = \frac{2tan\,A}{1+tan^2A}\right)$

$\Rightarrow y = \text{cosec}^{-1}(\text{cosec } 2x)$

$\Rightarrow y = 2x$

differentiate both sides w.r.t. x

$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}(2x)$

$\therefore \frac{dy}{dx} = 2$

Q11

$f(x) = \cot^{-1}(\text{cosec } x + \cot x)$

Let $y = \cot^{-1}(\text{cosec } x + \cot x)$

$\Rightarrow y = \cot^{-1}\left(\frac{1}{\sin x} + \frac{\cos x}{\sin x}\right)$

$\Rightarrow y = \cot^{-1}\left(\frac{1+\cos x}{\sin x}\right)$

$\Rightarrow y = \cot^{-1}\left(\frac{2\cos^2\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}}\right)$

$(\because 1 + \cos A = 2\cos^2\frac{A}{2}$ and $\sin 2A = 2\sin A \cos A)$

$\Rightarrow y = \cot^{-1}\left(\frac{\cos\frac{x}{2}}{\sin\frac{x}{2}}\right)$

$\Rightarrow y = \cot^{-1}\left(\cot\frac{x}{2}\right)$

differentiate both sides w.r.t. x

$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}\left(\frac{x}{2}\right)$

$\therefore \frac{dy}{dx} = \frac{1}{2}$

Q12

$f(x) = \tan^{-1}(\cot x) + \cot^{-1}(\tan x)$

Let $y = \tan^{-1}(\cot x) + \cot^{-1}(\tan x)$

$\Rightarrow y = \tan^{-1}\left(\tan\left(\frac{\pi}{2} - x\right)\right) + \cot^{-1}\left(\cot\left(\frac{\pi}{2} - x\right)\right)$

$\Rightarrow y = \left(\frac{\pi}{2} - x\right) + \left(\frac{\pi}{2} - x\right)$

$\Rightarrow y = \pi - 2x$

differentiate both sides w.r.t. x

$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}(\pi - 2x)$

$\therefore \frac{dy}{dx} = -2$

Q13

$f(x) = \sin^{-1}\{\sqrt{1-x^2}\}$

Let $x = \cos\theta$ (i.e. $\theta = \cos^{-1}x$)

$\Rightarrow f(x) = \sin^{-1}\{\sqrt{1-\cos^2\theta}\}$

$\Rightarrow f(x) = \sin^{-1}\{\sqrt{\sin^2\theta}\}$

$\Rightarrow f(x) = \sin^{-1}\{\sin\theta\}$

$\Rightarrow f(x) = \theta$

Putting $\theta = \cos^{-1}x$

$\Rightarrow f(x) = \cos^{-1}x$

differentiating both sides w.r.t. x

$\Rightarrow \frac{df(x)}{dx} = \frac{d}{dx}(\cos^{-1}x)$

$\therefore \frac{df(x)}{dx} = -\frac{1}{\sqrt{1-x^2}}$

Q14

$f(x) = \sin^{-1}\{\sqrt{\frac{1-x}{2}}\}$

Let $x = \cos\theta$ (i.e. $\theta = \cos^{-1}x$)

$\Rightarrow f(x) = \sin^{-1}\{\sqrt{\frac{1-\cos\theta}{2}}\}$

$\Rightarrow f(x) = \sin^{-1}\{\sqrt{\frac{2\sin^2\frac{\theta}{2}}{2}}\}$

$\Rightarrow f(x) = \sin^{-1}\{\sin\frac{\theta}{2}\}$

$\Rightarrow f(x) = \frac{\theta}{2}$

Putting $\theta = \cos^{-1}x$

$\Rightarrow f(x) = \frac{1}{2}\cos^{-1}x$

differentiating both sides w.r.t. x

$\Rightarrow \frac{df(x)}{dx} = \frac{1}{2}\frac{d}{dx}(\cos^{-1}x)$

$\therefore \frac{df(x)}{dx} = -\frac{1}{2\sqrt{1-x^2}}$

Q15

$f(x) = \cos^{-1}\{\sqrt{\frac{1+x}{2}}\}$

Let $x = \cos\theta$ (i.e. $\theta = \cos^{-1}x$)

$\Rightarrow f(x) = \cos^{-1}\{\sqrt{\frac{1+\cos\theta}{2}}\}$

$\Rightarrow f(x) = \cos^{-1}\{\sqrt{\frac{2\cos^2\frac{\theta}{2}}{2}}\}$

$\Rightarrow f(x) = \cos^{-1}\{\cos\frac{\theta}{2}\}$

$\Rightarrow f(x) = \frac{\theta}{2}$

Putting $\theta = \cos^{-1}x$

$\Rightarrow f(x) = \frac{1}{2}\cos^{-1}x$

differentiating both sides w.r.t. x

$\Rightarrow \frac{df(x)}{dx} = \frac{1}{2}\frac{d}{dx}(\cos^{-1}x)$

$\therefore \frac{df(x)}{dx} = -\frac{1}{2\sqrt{1-x^2}}$

Q16

$f(x) = \cos^{-1}\{\sqrt{1-x^2}\}$

Let $x = \sin\theta$ (i.e. $\theta = \sin^{-1}x$)

$\Rightarrow f(x) = \cos^{-1}\{\sqrt{1-\sin^2\theta}\}$

$\Rightarrow f(x) = \cos^{-1}\{\sqrt{\cos^2\theta}\}$

$\Rightarrow f(x) = \cos^{-1}\{\cos\theta\}$

$\Rightarrow f(x) = \theta$

Putting $\theta = \sin^{-1}x$

$\Rightarrow f(x) = \sin^{-1}x$

differentiating both sides w.r.t. x

$\Rightarrow \frac{df(x)}{dx} = \frac{d}{dx}(\sin^{-1}x)$

$\therefore \frac{df(x)}{dx} = \frac{1}{\sqrt{1-x^2}}$

Q17

$f(x) = \sin^{-1}\{2x\sqrt{1-x^2}\}$

Let $x = \sin\theta$ (i.e. $\theta = \sin^{-1}x$)

$\Rightarrow f(x) = \sin^{-1}\{2\sin\theta\sqrt{1-sin^2\theta}\}$

$\Rightarrow f(x) = \sin^{-1}\{2\sin\theta\sqrt{cos^2\theta}\}$

$\Rightarrow f(x) = \sin^{-1}\{2\sin\theta.\cos\theta\}$

$\Rightarrow f(x) = \sin^{-1}\{\sin 2\theta\}$

$\Rightarrow f(x) = 2\theta$

Putting $\theta = \sin^{-1}x$

$\Rightarrow f(x) = 2\sin^{-1}x$

differentiating both sides w.r.t. x

$\Rightarrow \frac{df(x)}{dx} = 2\frac{d}{dx}(\sin^{-1}x)$

$\therefore \frac{df(x)}{dx} = \frac{2}{\sqrt{1-x^2}}$

Q18

$f(x) = \sin^{-1}(3x - 4x^3)$

Let $x = \sin\theta$ (i.e. $\theta = \sin^{-1}x$)

$\Rightarrow f(x) = \sin^{-1}(3x - 4x^3)$

$\Rightarrow f(x) = \sin^{-1}(3\sin\theta - 4\sin^3\theta)$

$\Rightarrow f(x) = \sin^{-1}(\sin 3\theta)$

$\Rightarrow f(x) = \sin^{-1}\{\sin 3\theta\}$

$\Rightarrow f(x) = 3\theta$

Putting $\theta = \sin^{-1}x$

$\Rightarrow f(x) = 3\sin^{-1}x$

differentiating both sides w.r.t. x

$\Rightarrow \frac{df(x)}{dx} = 3\frac{d}{dx}(\sin^{-1}x)$

$\therefore \frac{df(x)}{dx} = \frac{3}{\sqrt{1-x^2}}$

Q19

$f(x) = \sin^{-1}(1 - 2x^2)$

Let $x = \sin\theta$ (i.e. $\theta = \sin^{-1}x$)

$\Rightarrow f(x) = \sin^{-1}(1 - 2x)$

$\Rightarrow f(x) = \sin^{-1}(1 - 2\sin^2\theta)$

$\Rightarrow f(x) = \sin^{-1}(\cos 2\theta)$

$\Rightarrow f(x) = \sin^{-1}\{\sin(\frac{\pi}{2} - 2\theta)\}$

$\Rightarrow f(x) = \frac{\pi}{2} - 2\theta$

Putting $\theta = \sin^{-1}x$

$\Rightarrow f(x) = \frac{\pi}{2} - 2\sin^{-1}x$

differentiating both sides w.r.t. x

$\Rightarrow \frac{df(x)}{dx} = 3\frac{d}{dx}(\frac{\pi}{2} - 2\sin^{-1}x)$

$\therefore \frac{df(x)}{dx} = \frac{-2}{\sqrt{1-x^2}}$

Q20

$f(x) = \sec^{-1}(\frac{1}{\sqrt{1-x^2}})$

Let $x = \sin\theta$ (i.e. $\theta = \sin^{-1}x$)

$\Rightarrow f(x) = \sec^{-1}(\frac{1}{\sqrt{1-\sin^2\theta}})$

$\Rightarrow f(x) = \sec^{-1}(\frac{1}{\sqrt{\cos^2\theta}})$

$\Rightarrow f(x) = \sec^{-1}(\frac{1}{\cos\theta})$

$\Rightarrow f(x) = \sec^{-1}(\sec\theta)$

$\Rightarrow f(x) = \theta$

Putting $\theta = \sin^{-1}x$

$\Rightarrow f(x) = \sin^{-1}x$

differentiating both sides w.r.t. x

$\Rightarrow \frac{df(x)}{dx} = \frac{d}{dx}(\sin^{-1}x)$

$\therefore \frac{df(x)}{dx} = \frac{1}{\sqrt{1-x^2}}$

Q21

$f(x) = \tan^{-1}(\frac{x}{\sqrt{1-x^2}})$

Let $x = \sin\theta$ (i.e. $\theta = \sin^{-1}x$)

$\Rightarrow f(x) = \tan^{-1}(\frac{\sin\theta}{\sqrt{1-\sin^2\theta}})$

$\Rightarrow f(x) = \tan^{-1}(\frac{\sin\theta}{\sqrt{\cos^2\theta}})$

$\Rightarrow f(x) = \tan^{-1}(\frac{\sin\theta}{\cos\theta})$

$\Rightarrow f(x) = \tan^{-1}(\tan\theta)$

$\Rightarrow f(x) = \theta$

Putting $\theta = \sin^{-1}x$

$\Rightarrow f(x) = \sin^{-1}x$

differentiating both sides w.r.t. x

$\Rightarrow \frac{df(x)}{dx} = \frac{d}{dx}(\sin^{-1}x)$

$\therefore \frac{df(x)}{dx} = \frac{1}{\sqrt{1-x^2}}$

Q22

$f(x) = \tan^{-1}\left(\frac{x}{1+\sqrt{1-x^2}}\right)$

Let $x = \sin\theta$ (i.e. $\theta = \sin^{-1}x$)

$\Rightarrow f(x) = \tan^{-1}\left(\frac{\sin\theta}{1+\sqrt{1-\sin^2\theta}}\right)$

$\Rightarrow f(x) = \tan^{-1}\left(\frac{\sin\theta}{1+\sqrt{\cos^2\theta}}\right)$

$\Rightarrow f(x) = \tan^{-1}\left(\frac{\sin\theta}{1+\cos\theta}\right)$

$\Rightarrow f(x) = \tan^{-1}\left(\frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{\cos^2\frac{\theta}{2}}\right)$

$\Rightarrow f(x) = \tan^{-1}\left(\tan\frac{\theta}{2}\right)$

$\Rightarrow f(x) = \frac{\theta}{2}$

Putting $\theta = \sin^{-1}x$

$\Rightarrow f(x) = \frac{1}{2}\sin^{-1}x$

differentiating both sides w.r.t. x

$\Rightarrow \frac{df(x)}{dx} = \frac{1}{2}\frac{d}{dx}(\sin^{-1}x)$

$\therefore \frac{df(x)}{dx} = \frac{1}{2\sqrt{1-x^2}}$

Q23

$f(x) = \cot^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$

Let $x = \sin\theta$ (i.e. $\theta = \sin^{-1}x$)

$\Rightarrow f(x) = \cot^{-1}\left(\frac{\sqrt{1-\sin^2\theta}}{\sin\theta}\right)$

$\Rightarrow f(x) = \cot^{-1}\left(\frac{\sqrt{\cos^2\theta}}{\sin\theta}\right)$

$\Rightarrow f(x) = \cot^{-1}\left(\frac{\cos\theta}{\sin\theta}\right)$

$\Rightarrow f(x) = \cot^{-1}(\cot\theta)$

$\Rightarrow f(x) = \theta$

Putting $\theta = \sin^{-1}x$

$\Rightarrow f(x) = \sin^{-1}x$

differentiating both sides w.r.t. x

$\Rightarrow \frac{df(x)}{dx} = \frac{d}{dx}(\sin^{-1}x)$

$\therefore \frac{df(x)}{dx} = \frac{1}{\sqrt{1-x^2}}$

Q24

$f(x) = \sec^{-1}\left(\frac{1}{1-2x^2}\right)$

Let $x = \sin\theta$ (i.e. $\theta = \sin^{-1}x$)

$\Rightarrow f(x) = \sec^{-1}\left(\frac{1}{1-2\sin^2\theta}\right)$

$\Rightarrow f(x) = \sec^{-1}\left(\frac{1}{\cos 2\theta}\right)$

$\Rightarrow f(x) = \sec^{-1}(\sec 2\theta)$

$\Rightarrow f(x) = \theta$

Putting $\theta = \sin^{-1}x$

$\Rightarrow f(x) = \sin^{-1}x$

differentiating both sides w.r.t. x

$\Rightarrow \frac{df(x)}{dx} = \frac{d}{dx}(\sin^{-1}x)$

$\therefore \frac{df(x)}{dx} = \frac{1}{\sqrt{1-x^2}}$

Q25

$f(x) = \sin^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)$

Let $x = \cot\theta$ (i.e. $\theta = \cot^{-1}x$)

$\Rightarrow f(x) = \sin^{-1}\left(\frac{1}{\sqrt{1+\cot^2\theta}}\right)$

$\Rightarrow f(x) = \sin^{-1}\left(\frac{1}{\sqrt{\text{cosec}^2\theta}}\right)$

$\Rightarrow f(x) = \sin^{-1}\left(\frac{1}{\text{cosec}\,\theta}\right)$

$\Rightarrow f(x) = \sin^{-1}(\sin\theta)$

$\Rightarrow f(x) = \theta$

Putting $\theta = \cot^{-1}x$

$\Rightarrow f(x) = \cot^{-1}x$

differentiating both sides w.r.t. x

$\Rightarrow \frac{df(x)}{dx} = \frac{d}{dx}(\cot^{-1}x)$

$\therefore \frac{df(x)}{dx} = -\frac{1}{1+x^2}$

Q26

$f(x) = \tan^{-1}(\frac{1+x}{1-x})$

Let $x = \tan\theta$ (i.e. $\theta = \tan^{-1}x$)

$\Rightarrow f(x) = \tan^{-1}(\frac{1+\tan\theta}{1-\tan\theta})$

$\Rightarrow f(x) = \tan^{-1}(\frac{\tan\frac{\pi}{4}+\tan\theta}{1-\tan\frac{\pi}{4}\tan\theta})$

$\Rightarrow f(x) = \tan^{-1}(\tan(\frac{\pi}{4} + \theta))$

$\Rightarrow f(x) = \frac{\pi}{4} + \theta$

Putting $\theta = \tan^{-1}x$

$\Rightarrow f(x) = \frac{\pi}{4} + \tan^{-1}x$

differentiating both sides w.r.t. x

$\Rightarrow \frac{df(x)}{dx} = \frac{d}{dx}\left(\frac{\pi}{4}\right) + \frac{d}{dx}(\tan^{-1}x)$

$\therefore \frac{df(x)}{dx} = \frac{1}{1+x^2}$

Q27

$f(x) = \cot^{-1}\left(\frac{1+x}{1-x}\right)$

Let $x = \tan\theta$ (i.e. $\theta = \tan^{-1}x$)

$\Rightarrow f(x) = \cot^{-1}\left(\frac{1+\tan\theta}{1-\tan\theta}\right)$

$\Rightarrow f(x) = \cot^{-1}\left(\frac{\tan\frac{\pi}{4}+\tan\theta}{1-\tan\frac{\pi}{4}\tan\theta}\right)$

$\Rightarrow f(x) = \cot^{-1}\{\cot\left(\frac{\pi}{2} - \left(\frac{\pi}{4} + \theta\right)\right)\}$

$\Rightarrow f(x) = \frac{\pi}{4} - \theta$

Putting $\theta = \tan^{-1}x$

$\Rightarrow f(x) = \frac{\pi}{4} - \tan^{-1}x$

differentiating both sides w.r.t. x

$\Rightarrow \frac{df(x)}{dx} = \frac{d}{dx}\left(\frac{\pi}{4}\right) + \frac{d}{dx}(\tan^{-1}x)$

$\therefore \frac{df(x)}{dx} = \frac{1}{1+x^2}$

Q28

$f(x) = \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$

Let $x = \tan\theta$ (i.e. $\theta = \tan^{-1}x$)

$\Rightarrow f(x) = \tan^{-1}\left(\frac{3\tan\theta - tan^3\theta}{1-3tan^2\theta}\right)$

$\Rightarrow f(x) = \tan^{-1}(\tan 3\theta)$

$\Rightarrow f(x) = 3\theta$

Putting $\theta = \tan^{-1}x$

$\Rightarrow f(x) = \tan^{-1}x$

differentiating both sides w.r.t. x

$\Rightarrow \frac{df(x)}{dx} = 3\frac{d}{dx}(\tan^{-1}x)$

$\therefore \frac{df(x)}{dx} = \frac{3}{1+x^2}$

Q29

$f(x) = \text{cosec}^{-1}(\frac{1+x^2}{2x})$

Let $x = \tan\theta$ (i.e. $\theta = \tan^{-1}x$)

$\Rightarrow f(x) = \tan^{-1}(\frac{1+tan^2\,\theta}{2\tan\theta})$

$\Rightarrow f(x) = \tan^{-1}(\tan 2\theta)$

$\Rightarrow f(x) = 2\theta$

Putting $\theta = \tan^{-1}x$

$\Rightarrow f(x) = \tan^{-1}x$

differentiating both sides w.r.t. x

$\Rightarrow \frac{df(x)}{dx} = 2\frac{d}{dx}(\tan^{-1}x)$

$\therefore \frac{df(x)}{dx} = \frac{2}{1+x^2}$

Q30

$f(x) = \sec^{-1}\left(\frac{1+x^2}{1-x^2}\right)$

Let $x = \tan\theta$ (i.e. $\theta = \tan^{-1}x$)

$\Rightarrow f(x) = \sec^{-1}\left(\frac{1+tan^2\theta}{1-tan^2\theta}\right)$

dividing numerator and denominator by $1 + \tan^2\theta$

$\Rightarrow f(x) = \sec^{-1}\left(\frac{\frac{1+tan^2\theta}{1+tan^2\theta}}{\frac{1-tan^2\theta}{1+tan^2\theta}}\right)$

$\Rightarrow f(x) = \sec^{-1}\left(\frac{1}{\cos 2\theta}\right)$

$\Rightarrow f(x) = \sec^{-1}(\sec 2\theta)$

$\Rightarrow f(x) = 2\theta$

Putting $\theta = \tan^{-1}x$

$\Rightarrow f(x) = \tan^{-1}x$

differentiating both sides w.r.t. x

$\Rightarrow \frac{df(x)}{dx} = 2\frac{d}{dx}(\tan^{-1}x)$

$\therefore \frac{df(x)}{dx} = \frac{2}{1+x^2}$

Q31

$f(x) = \sin^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)$

Let $x = \cot\theta$ (i.e. $\theta = \cot^{-1}x$)

$\Rightarrow f(x) = \sin^{-1}\left(\frac{1}{\sqrt{1+\cot^2\theta}}\right)$

$\Rightarrow f(x) = \sin^{-1}\left(\frac{1}{\sqrt{\text{cosec}^2\theta}}\right)$

$\Rightarrow f(x) = \sin^{-1}\left(\frac{1}{\text{cosec}\,\theta}\right)$

$\Rightarrow f(x) = \sin^{-1}(\sin\theta)$

$\Rightarrow f(x) = \theta$

Putting $\theta = \cot^{-1}x$

$\Rightarrow f(x) = \cot^{-1}x$

differentiating both sides w.r.t. x

$\Rightarrow \frac{df(x)}{dx} = \frac{d}{dx}(\cot^{-1}x)$

$\therefore \frac{df(x)}{dx} = -\frac{1}{1+x^2}$

Q32

$f(x) = \sec^{-1}\left(\frac{x^2+1}{x^2-1}\right)$

$\Rightarrow f(x) = \sec^{-1}\left(-\frac{1+x^2}{1-x^2}\right)$

$\Rightarrow f(x) = \pi - \sec^{-1}\left(-\frac{1+x^2}{1-x^2}\right)$

Let $x = \tan\theta$ (i.e. $\theta = \tan^{-1}x$)

$\Rightarrow f(x) = \pi - \sec^{-1}\left(\frac{1+tan^2\theta}{1-tan^2\theta}\right)$

dividing numerator and denominator by $1 + \tan^2\theta$

$\Rightarrow f(x) = \pi - \sec^{-1}\left(\frac{\frac{1+\tan^2\theta}{1+\tan^2\theta}}{\frac{1-\tan^2\theta}{1+\tan^2\theta}}\right)$

$\Rightarrow f(x) = \pi - \sec^{-1}\left(\frac{1}{\cos 2\theta}\right)$

$\Rightarrow f(x) = \pi - \sec^{-1}(\sec 2\theta)$

$\Rightarrow f(x) = \pi - 2\theta$

Putting $\theta = \tan^{-1}x$

$\Rightarrow f(x) = \tan^{-1}x$

differentiating both sides w.r.t. x

$\Rightarrow \frac{df(x)}{dx} = \frac{d}{dx}\pi - 2\frac{d}{dx}(\tan^{-1}x)$

$\therefore \frac{df(x)}{dx} = \frac{-2}{1+x^2}$

Q33

$f(x) = \cos^{-1}\left(\frac{1-x^{2n}}{1+x^{2n}}\right)$

Let $x^n = \tan\theta$ (i.e. $\theta = \tan^{-1}x^n$)

$\Rightarrow f(x) = \cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$

dividing numerator and denominator by $1 + \tan^2\theta$

$\Rightarrow f(x) = \cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$

$\Rightarrow f(x) = \cos^{-1}(\cos 2\theta)$

$\Rightarrow f(x) = 2\theta$

Putting $\theta = \tan^{-1}x^n$

$\Rightarrow f(x) = \tan^{-1}x^n$

differentiating both sides w.r.t. x

$\Rightarrow \frac{df(x)}{dx} = 2\frac{d}{dx}(\tan^{-1}x^n)$

$\therefore \frac{df(x)}{dx} = \frac{-2n\,x^{n-1}}{1+x^{2n}}$

Q34

$f(x) = \tan^{-1}(\frac{x}{\sqrt{a^2-x^2}})$

Let $x = a.\sin\theta$ (i.e. $\theta = \sin^{-1}\frac{x}{a}$)

$\Rightarrow f(x) = \tan^{-1}(\frac{a\sin\theta}{\sqrt{a^2-a^2sin^2\theta}})$

dividing numerator and denominator by $1 + \tan^2\theta$

$\Rightarrow f(x) = \tan^{-1}(\frac{a\sin\theta}{\sqrt{a^2(1-sin^2\theta)}})$

$\Rightarrow f(x) = \tan^{-1}(\frac{a\sin\theta}{\sqrt{a^2(cos^2\theta)}})$

$\Rightarrow f(x) = \tan^{-1}(\frac{a\sin\theta}{a\cos\theta})$

$\Rightarrow f(x) = \tan^{-1}(\tan\theta)$

$\Rightarrow f(x) = \theta$

Putting $\theta = \sin^{-1}\frac{x}{a}$

$\Rightarrow f(x) = \sin^{-1}\frac{x}{a}$

differentiating both sides w.r.t. x

$\Rightarrow \frac{df(x)}{dx} = \frac{d}{dx}(\sin^{-1}\frac{x}{a})$

$\Rightarrow \frac{df(x)}{dx} = \frac{1}{a\sqrt{1-(\frac{x}{a})^2}}$

$\therefore \frac{df(x)}{dx} = \frac{1}{\sqrt{a^2-x^2}}$

Q35

$f(x) = \sin^{-1}(2ax\sqrt{1-a^2x^2})$

Let $ax = \sin\theta$ (i.e. $\theta = \sin^{-1}ax$)

$\Rightarrow f(x) = \sin^{-1}(2sin\theta\sqrt{1-sin^2\theta})$

$\Rightarrow f(x) = \sin^{-1}(2.\sin\theta.\cos\theta)$

$\Rightarrow f(x) = \sin^{-1}(\sin 2\theta)$

$\Rightarrow f(x) = 2\theta$

Putting $\theta = \sin^{-1}ax$

$\Rightarrow f(x) = \sin^{-1}ax$

differentiating both sides w.r.t. x

$\Rightarrow \frac{df(x)}{dx} = 2\frac{d}{dx}(\sin^{-1}ax)$

$\Rightarrow \frac{df(x)}{dx} = \frac{2a}{\sqrt{1-(ax)^2}}$

$\therefore \frac{df(x)}{dx} = \frac{2a}{\sqrt{1-a^2x^2}}$

Q36

$f(x) = \tan^{-1}\left(\frac{\sqrt{1+a^2x^2}-1}{ax}\right)$

Let $ax = \tan\theta$ (i.e. $\theta = \tan^{-1}ax$)

$\Rightarrow f(x) = \tan^{-1}\left(\frac{\sqrt{1+tan^2\theta}-1}{\tan\theta}\right)$

$\Rightarrow f(x) = \tan^{-1}\left(\frac{\sec\theta-1}{\tan\theta}\right)$

$\Rightarrow f(x) = \tan^{-1}\left(\frac{\frac{1}{\cos\theta}-1}{\frac{\sin\theta}{\cos\theta}}\right)$

$\Rightarrow f(x) = \tan^{-1}\left(\frac{1-\cos\theta}{\sin\theta}\right)$

$\Rightarrow f(x) = \tan^{-1}\left(\frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right)$

$\Rightarrow f(x) = \tan^{-1}\left(\frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}\right)$

$\Rightarrow f(x) = \tan^{-1}\left(\tan\frac{\theta}{2}\right)$

$\Rightarrow f(x) = \frac{\theta}{2}$

Putting $\theta = \tan^{-1}ax$

$\Rightarrow f(x) = \frac{1}{2}\tan^{-1}ax$

differentiating both sides w.r.t. x

$\Rightarrow \frac{df(x)}{dx} = \frac{1}{2}\frac{d}{dx}(\tan^{-1}ax)$

$\therefore \frac{df(x)}{dx} = \frac{a}{2(1-(ax)^2)}$

Q37

$f(x) = \sin^{-1}(\frac{x^2}{\sqrt{a^4+x^4}})$

Let $x^2 = a^2.\cot\theta$ (i.e. $\theta = \cot^{-1}\frac{x^2}{a^2}$)

$\Rightarrow f(x) = \sin^{-1}(\frac{a^2\cot\theta}{\sqrt{a^4+a^4cot^4\theta}})$

$\Rightarrow f(x) = \sin^{-1}(\frac{a^2\cot\theta}{\sqrt{a^4(1+cot^4\theta)}})$

$\Rightarrow f(x) = \sin^{-1}(\frac{a^2\cot\theta}{\sqrt{a^4(cosec^4\theta)}})$

$\Rightarrow f(x) = \sin^{-1}(\frac{a^2\cot\theta}{a^2cosec^2\theta})$

$\Rightarrow f(x) = \sin^{-1}(\frac{cos\,\theta}{\sin\theta}\times\sin\theta)$

$\Rightarrow f(x) = \sin^{-1}(\cos\theta)$

$\Rightarrow f(x) = \sin^{-1}(\sin(\frac{\pi}{2}-\theta))$

$\Rightarrow f(x) = \frac{\pi}{2}-\theta$

Putting $\theta = \cot^{-1}\frac{x^2}{a^2}$

$\Rightarrow f(x) = \frac{\pi}{2} - \cot^{-1}\frac{x^2}{a^2}$

differentiating both sides w.r.t. x

$\Rightarrow \frac{df(x)}{dx} = \frac{d}{dx}(\frac{\pi}{2} - \cot^{-1}\frac{x^2}{a^2})$

$\Rightarrow \frac{df(x)}{dx} = \frac{2x}{a^2(1+(\frac{x^2}{a^2})^2)}$

$\therefore \frac{df(x)}{dx} = \frac{2a^2x}{a^4+x^4}$

Q38

$f(x) = \tan^{-1}(\frac{e^{2x}+1}{e^{2x}-1})$

$\Rightarrow f(x) = \tan^{-1}(-\frac{1+e^{2x}}{1-e^{2x}})$

$\Rightarrow f(x) = -\tan^{-1}(\frac{1+e^{2x}}{1-e^{2x}})$

Let $e^{2x} = \tan\theta$ (i.e. $\theta = \tan^{-1}e^{2x}$)

$\Rightarrow f(x) = -\tan^{-1}(\frac{1+\tan\theta}{1-\tan\theta})$

$\Rightarrow f(x) = -\tan^{-1}(\frac{\tan\frac{\pi}{4}+\tan\theta}{1-\tan\frac{\pi}{4}\tan\theta})$

$\Rightarrow f(x) = -\tan^{-1}(\tan(\frac{\pi}{4} + \theta))$

$\Rightarrow f(x) = -\frac{\pi}{4} - \theta$

Putting $\theta = \tan^{-1}e^{2x}$

$\Rightarrow f(x) = -\frac{\pi}{4} - \tan^{-1}e^{2x}$

differentiating both sides w.r.t. x

$\Rightarrow \frac{df(x)}{dx} = \frac{d}{dx}(-\frac{\pi}{4} - \tan^{-1}e^{2x})$

$\Rightarrow \frac{df(x)}{dx} = 0 - \frac{2e^{2x}}{1+(e^{2x})^2}$

$\therefore \frac{df(x)}{dx} = \frac{-2e^{2x}}{1+e^{4x}}$

Q39

$f(x) = \cos^{-1}(2x) + 2\cos^{-1}(\sqrt{1-4x^2})$

Let $2x = \cos\theta$ (i.e. $\theta = \cos^{-1}(2x)$

$\Rightarrow f(x) = \cos^{-1}(\cos\theta) + 2\cos^{-1}(\sqrt{1-cos^2\theta})$

$\Rightarrow f(x) = \theta + 2\cos^{-1}(\sin\theta)$

$\Rightarrow f(x) = \theta + 2\cos^{-1}(\cos(\frac{\pi}{2} - \theta)$

$\Rightarrow f(x) = \theta + 2(\frac{\pi}{2} - \theta)$

$\Rightarrow f(x) = \theta + \pi - 2\theta$

$\Rightarrow f(x) = \pi - \theta$

Putting $\theta = \cos^{-1}(2x)$

$\Rightarrow f(x) = \pi - \cos^{-1}(2x)$

differentiating both sides w.r.t. x

$\Rightarrow \frac{df(x)}{dx} = \frac{d}{dx}(\pi - \cos^{-1}(2x))$

$\Rightarrow \frac{df(x)}{dx} = \frac{2}{\sqrt{1-(2x)^2}}$

$\therefore \frac{df(x)}{dx} = \frac{2}{\sqrt{1-4x^2}}$

Q40

$f(x) = \tan^{-1}(\frac{\sqrt{x}-x}{1+x^{3/2}})$

$\Rightarrow f(x) = \tan^{-1}(\frac{\sqrt{x}-x}{1+x\sqrt{x}})$

$\Rightarrow f(x) = \tan^{-1}\sqrt{x} - \tan^{-1}x$

differentiating both sides w.r.t. x

$\Rightarrow \frac{df(x)}{dx} = \frac{d}{dx}(\tan^{-1}\sqrt{x} - \tan^{-1}x)$

$\Rightarrow \frac{df(x)}{dx} = \frac{1}{1+(\sqrt{x})^2}.\frac{1}{2\sqrt{x}} - \frac{1}{1+x^2}$

$\therefore \frac{df(x)}{dx} = \frac{1}{2\sqrt{x}(1+x)} - \frac{1}{1+x^2}$

Q42

$f(x) = \tan^{-1}(\frac{\sqrt{a}+\sqrt{x}}{1-\sqrt{ax}})$

$\Rightarrow f(x) = \tan^{-1}\sqrt{a} + \tan^{-1}\sqrt{x}$

differentiating both sides w.r.t. x

$\Rightarrow \frac{df(x)}{dx} = \frac{d}{dx}(\tan^{-1}\sqrt{a} + \tan^{-1}\sqrt{x})$

$\Rightarrow \frac{df(x)}{dx} = \frac{1}{1+(\sqrt{x})^2}.\frac{1}{2\sqrt{x}}$

$\therefore \frac{df(x)}{dx} = \frac{1}{2\sqrt{x}(1+x)}$

Q43

$f(x) = \tan^{-1}(\frac{3-2x}{1+6x})$

$\Rightarrow f(x) = \tan^{-1}3 - \tan^{-1}2x$

differentiating both sides w.r.t. x

$\Rightarrow \frac{df(x)}{dx} = \frac{d}{dx}(\tan^{-1}3 - \tan^{-1}2x)$

$\Rightarrow \frac{df(x)}{dx} = 0 - \frac{1}{1+(2x)^2}.2$

$\therefore \frac{df(x)}{dx} = \frac{-2}{(1+4x^2)}$

Q44

$f(x) = \tan^{-1}(\frac{5x}{1-6x^2})$

$\Rightarrow f(x) = \tan^{-1}(\frac{3x+2x}{1-3x.2x})$

Let $3x = \tan A$ (i.e. $A = \tan^{-1}3x$)

and $2x = \tan B$ (i.e. $B = \tan^{-1}2x$)

$\Rightarrow f(x) = \tan^{-1}(\frac{tanA+tanB}{1-tanA.tanB})$

$\Rightarrow f(x) = \tan^{-1}(\tan(A + B))$

$\Rightarrow f(x) = (A + B)$

Put back value of A and B in above equation,

$\Rightarrow f(x) = \tan^{-1}3x + \tan^{-1}2x$

differentiating both sides w.r.t. x

$\Rightarrow \frac{df(x)}{dx} = \frac{d}{dx}(\tan^{-1}3x + \tan^{-1}2x)$

$\Rightarrow \frac{df(x)}{dx} = \frac{1}{1+(3x)^2}.3 + \frac{1}{1+(2x)^2}.2$

$\therefore \frac{df(x)}{dx} = \frac{3}{(1+9x^2)} + \frac{2}{(1+4x^2)}$

Q45

$f(x) = \tan^{-1}(\frac{2x}{1+15x^2})$

$\Rightarrow f(x) = \tan^{-1}(\frac{5x-3x}{1+5x.3x})$

$\Rightarrow f(x) = \tan^{-1}5x - \tan^{-1}3x$

differentiating both sides w.r.t. x

$\Rightarrow \frac{df(x)}{dx} = \frac{d}{dx}(\tan^{-1}5x + \tan^{-1}3x)$

$\Rightarrow \frac{df(x)}{dx} = \frac{1}{1+(5x)^2}.5 - \frac{1}{1+(3x)^2}.3$

$\therefore \frac{df(x)}{dx} = \frac{5}{(1+25x^2)} - \frac{3}{(1+9x^2)}$

Q46

To prove: $\frac{dy}{dx} = \frac{1}{1+x^2}$

Given: $y = \tan^{-1}(\frac{ax-b}{a+bx})$

dividing numerator and denominator by a,

$\Rightarrow y = \tan^{-1}(\frac{\frac{ax-b}{a}}{\frac{a+bx}{a}})$

$\Rightarrow y = \tan^{-1}(\frac{x-\frac{b}{a}}{1+\frac{b}{a}.x})$

$\Rightarrow y = \tan^{-1}x - \tan^{-1}\frac{b}{a}$

differentiating both sides w.r.t. x

$\Rightarrow \frac{df(x)}{dx} = \frac{d}{dx}(\tan^{-1}x - \tan^{-1}\frac{b}{a})$

$\Rightarrow \frac{df(x)}{dx} = \frac{1}{1+(x)^2} - 0$

$\therefore \frac{df(x)}{dx} = \frac{1}{1+x^2}$

Hence, Proved

Q47

To show: $\frac{dy}{dx} = \frac{4}{1+x^2}$

Given: $y = \sin^{-1}(\frac{2x}{1+x^2}) + \sec^{-1}(\frac{1+x^2}{1-x^2})$

Let $x = \tan\theta$ (i.e. $\theta = \tan^{-1} x$)

$\Rightarrow y = \sin^{-1}(\frac{2\tan\theta}{1+\tan^2\theta}) + \sec^{-1}(\frac{1+\tan^2\theta}{1-\tan^2\theta})$

$\Rightarrow y = \sin^{-1}(\sin 2\theta) + \sec^{-1}(\sec 2\theta)$

$\Rightarrow y = 2\theta + 2\theta$

$\Rightarrow y = 4\theta$

Putting $\theta = \tan^{-1}(x)$

$\Rightarrow f(x) = 4\tan^{-1}(x)$

differentiating both sides w.r.t. x

$\Rightarrow \frac{df(x)}{dx} = \frac{d}{dx}(4\tan^{-1}(x))$

$\therefore \frac{df(x)}{dx} = \frac{4}{1+x^2}$

Q48

To show: $\frac{dy}{dx} = 0$

Given: $y = \sec^{-1}(\frac{x+1}{x-1}) + \sin^{-1}(\frac{x-1}{x+1})$

$\Rightarrow y = \cos^{-1}(\frac{x+1}{x-1}) + \sin^{-1}(\frac{x-1}{x+1})$

$\Rightarrow y = \frac{\pi}{2}$

differentiating both sides w.r.t. x

$\Rightarrow \frac{df(x)}{dx} = \frac{d}{dx}(\frac{\pi}{2})$

$\therefore \frac{df(x)}{dx} = 0$

Q49

To show: $\frac{dy}{dx} = -\frac{x}{\sqrt{1-x^2}}$

Given: $y = \sin\{2\tan^{-1}\sqrt{\frac{1-x}{1+x}})\}$

Let $x = \cos\theta$ (i.e. $\theta = \cos^{-1} x$)

$\Rightarrow y = \sin\{2\tan^{-1}\sqrt{\frac{1-\cos\theta}{1+\cos\theta}})\}$

$\Rightarrow y = \sin\{2\tan^{-1}\sqrt{\frac{2\sin^2\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}}})\}$

$\Rightarrow y = \sin\{2\tan^{-1}(\tan\frac{\theta}{2})\}$

$\Rightarrow y = \sin 2 \times \frac{\theta}{2}$

$\Rightarrow y = \sin\theta$

Putting $\theta = \cos^{-1}x$

$\Rightarrow f(x) = \sin(\cos^{-1}x)$

differentiating both sides w.r.t. x

$\Rightarrow \frac{df(x)}{dx} = \frac{d}{dx}(\sin(\cos^{-1}x))$

$\Rightarrow \frac{df(x)}{dx} = -\cos(\cos^{-1} x).\ \frac{1}{\sqrt{1-x^2}}$

$\therefore \frac{df(x)}{dx} = -\frac{x}{\sqrt{1-x^2}}$

Q50

To Prove: $\frac{dy}{dx} = \frac{1}{2\sqrt{1-x^2}}$

Given: $y = \tan^{-1}\{\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\}$

Let $x = \cos 2\theta$ (i.e. $\theta = \frac{1}{2}\cos^{-1} x$)

$\Rightarrow y = \tan^{-1}\{\frac{\sqrt{1+\cos 2\theta}-\sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta}+\sqrt{1-\cos 2\theta}}\}$

$\Rightarrow y = \tan^{-1}\{\frac{\sqrt{2\cos^2\theta}-\sqrt{2\sin^2\theta}}{\sqrt{2\cos^2\theta}+\sqrt{2\sin^2\theta}}\}$

$\Rightarrow y = \tan^{-1}\{\frac{\sqrt{2}(\cos\theta-\sin\theta)}{\sqrt{2}(\cos\theta+\sin\theta)}\}$

$\Rightarrow y = \tan^{-1}\{\frac{(\cos\theta-\sin\theta)}{(\cos\theta+\sin\theta)}\}$

Dividing numerator and denominator by $\cos\theta$

$\Rightarrow y = \tan^{-1}\{\frac{\frac{\cos\theta-\sin\theta}{\cos\theta}}{\frac{\cos\theta+\sin\theta}{\cos\theta}}\}$

$\Rightarrow y = \tan^{-1}\{\frac{1-\frac{\sin\theta}{\cos\theta}}{1+\frac{\sin\theta}{\cos\theta}}\}$

$\Rightarrow y = \tan^{-1}\{\frac{1-tan\theta}{1+tan\theta}\}$

$\Rightarrow y = \tan^{-1}\{\frac{tan\frac{\pi}{4}-tan\theta}{1+\tan\frac{\pi}{4}tan\theta}\}$

$\Rightarrow y = \tan^{-1}\{\tan\frac{\pi}{4} - \theta\}$

$\Rightarrow y = \frac{\pi}{4} - \theta$

Putting $\theta = \frac{1}{2}\cos^{-1}x$

$\Rightarrow f(x) = \frac{\pi}{4} - \frac{1}{2}\cos^{-1}x$

differentiating both sides w.r.t. x

$\Rightarrow \frac{df(x)}{dx} = \frac{d}{dx}(\frac{\pi}{4} - \frac{1}{2}\cos^{-1}x)$

$\therefore \frac{df(x)}{dx} = \frac{1}{2\sqrt{1-x^2}}$

Q51

$f(x) = \sin^{-1}(\frac{2^{x+1}}{1+4^x})$

$\Rightarrow f(x) = \sin^{-1}(\frac{2^x.2}{1+(2^2)^x})$

Let $2^x = \tan\theta$ (i.e. $\theta = \tan^{-1}(2^x)$)

$\Rightarrow f(x) = \sin^{-1}(\frac{2\tan\theta}{1+\tan^2\theta})$

$\Rightarrow f(x) = \sin^{-1}(\sin 2\theta)$

$\Rightarrow f(x) = 2\theta$

Putting $\theta = \tan^{-1}(2^x)$

$\Rightarrow f(x) = 2\tan^{-1}(2^x)$

differentiating both sides w.r.t. x

$\Rightarrow \frac{df(x)}{dx} = \frac{d}{dx}(2\tan^{-1}(2^x))$

$$\Rightarrow \frac{df(x)}{dx} = 2.\frac{1}{1+(2^x)^2}.2^x\log 2$$

$$\therefore \frac{df(x)}{dx} = \frac{2^{1+x}\log 2}{1+4^x}$$