

CHAPTER-27 - Limit

Exercise 27 A

Q.no.1**Solutions:**

It is given that $\lim_{x \rightarrow 2} (5 - x)$

As $x \rightarrow 2$,

$$\lim_{x \rightarrow 2} (5 - x) = 5 - 2 = 3$$

Q.no. 2.**Solutions:**

It is given that $\lim_{x \rightarrow 1} (x^2 - 4x + 3)$

As $x \rightarrow 1$,

$$\lim_{x \rightarrow 1} (x^2 - 4x + 3) = 1^2 - 4(1) + 3 = 0$$

Q.no. 3.**Solutions:**

It is given that $\lim_{x \rightarrow 3} \frac{x^2 + 9}{x + 3}$

As $x \rightarrow 3$,

$$\lim_{x \rightarrow 3} \frac{x^2 + 9}{x + 3} = \frac{3^2 + 9}{3 + 3} = \frac{18}{3} = 3$$

Q.no. 4.**Solutions:**

It is given that $\lim_{x \rightarrow 3} \frac{x^2 + 4x}{x - 2}$

As $x \rightarrow 3$,

$$\lim_{x \rightarrow 3} \frac{x^2 + 4x}{x - 2} = \frac{3^2 - 4(3)}{3 - 2} = -3$$

Q.no. 5.

Solutions:

It is given that $\lim_{x \rightarrow 5} \frac{x^2 - 25}{x - 5}$

As $x \rightarrow 5$,

$$\lim_{x \rightarrow 5} \frac{x^2 - 25}{x - 5} = \lim_{x \rightarrow 5} \frac{(x + 5)(x - 5)}{x - 5} = x + 5 = 5 + 5 = 10$$

Q.no. 6.

Solutions:

It is given that $\lim_{x \rightarrow 1} \frac{x^3 - 1}{x - 1}$

As $x \rightarrow 1$,

$$\begin{aligned} \lim_{x \rightarrow 1} \frac{x^3 - 1}{x - 1} &= \lim_{x \rightarrow 1} \frac{(x - 1)(x^2 + x + 1)}{x - 1} \\ &= \lim_{x \rightarrow 1} (x^2 + x + 1) = 1 + 1 + 1 = 3 \end{aligned}$$

Q.no. 7.

Solutions:

It is given that $\lim_{x \rightarrow -2} \frac{x^3 + 8}{x + 2}$

As $x \rightarrow -2$,

$$\lim_{x \rightarrow -2} \frac{x^3 + 8}{x + 2} = \lim_{x \rightarrow -2} \frac{(x + 2)(x^2 - 2x + 4)}{x + 2} = \lim_{x \rightarrow -2} (x^2 - 2x + 4)$$

$$\lim_{x \rightarrow -2} \frac{x^3 + 8}{x + 2} = 4 - 4 + 4 = 4$$

Q.no. 8.

Solutions:

It is given that $\lim_{x \rightarrow 3} \frac{x^4 - 81}{x - 3}$

As $x \rightarrow 3$,

$$\lim_{x \rightarrow 3} \frac{x^4 - 81}{x - 3} = \lim_{x \rightarrow 3} \frac{(x^2 + 9)(x^2 - 9)}{x - 3}$$

$$\lim_{x \rightarrow 3} \frac{x^4 - 81}{x - 3} = \lim_{x \rightarrow 3} \frac{(x^2 + 9)(x + 3)(x - 3)}{x - 3} = \lim_{x \rightarrow 3} (x^2 + 9)(x + 3)$$

$$\lim_{x \rightarrow 3} \frac{x^4 - 81}{x - 3} = (9 + 9)(3 + 3) = 486$$

Q.no. 9.

Solutions:

It is given that $\lim_{x \rightarrow 3} \frac{x^2 - 4x + 3}{x^2 - 2x - 3}$

$$\lim_{x \rightarrow 3} \frac{x^2 - 4x + 3}{x^2 - 2x - 3} = \lim_{x \rightarrow 3} \frac{(x - 3)(x - 1)}{(x - 3)(x + 2)} = \lim_{x \rightarrow 3} \frac{(x - 1)}{(x + 2)}$$

$$\lim_{x \rightarrow 3} \frac{x^2 - 4x + 3}{x^2 - 2x - 3} = \frac{2}{5}$$

Q.no. 10.**Solutions:**

It is given that $\lim_{x \rightarrow \frac{1}{2}} \frac{4x^2 - 1}{2x - 1}$

As $x \rightarrow 1/2$

$$\lim_{x \rightarrow \frac{1}{2}} \frac{4x^2 - 1}{2x - 1} = \lim_{x \rightarrow \frac{1}{2}} \frac{(2x + 1)(2x - 1)}{2x - 1} = \lim_{x \rightarrow \frac{1}{2}} (2x + 1)$$

$$\lim_{x \rightarrow \frac{1}{2}} \frac{4x^2 - 1}{2x - 1} = 2$$

Q.no. 11.**Solutions:**

It is given that $\lim_{x \rightarrow 4} \frac{x^3 - 64}{x^2 - 16}$

As $x \rightarrow 4$,

$$\lim_{x \rightarrow 4} \frac{x^3 - 64}{x^2 - 16} = \lim_{x \rightarrow 4} \frac{(x - 4)(x^2 + 4x + 16)}{(x + 4)(x - 4)} = \lim_{x \rightarrow 4} \frac{(x^2 + 4x + 16)}{(x + 4)}$$

$$\lim_{x \rightarrow 4} \frac{x^3 - 64}{x^2 - 16} = \frac{48}{8} = 6$$

Q.no. 12.**Solutions:**

It is given that $\lim_{x \rightarrow 2} \frac{x^5 - 32}{x^3 - 8}$

As $x \rightarrow 4$,

$$\lim_{x \rightarrow 2} \frac{x^5 - 32}{x^3 - 8} = \lim_{x \rightarrow 2} \frac{x^5 - 2^5}{x^3 - 2^3} = \lim_{x \rightarrow 2} \frac{\frac{x^5 - 2^5}{x - 2}}{\frac{x^3 - 2^3}{x - 2}}$$

$$\lim_{x \rightarrow 2} \frac{x^5 - 32}{x^3 - 8} = \lim_{x \rightarrow 2} \frac{5(2)^4}{3(2)^2} = \frac{20}{3} \quad (\text{As we know that } \frac{x^n - y^n}{x - y} = ny^{n-1})$$

Q.no. 13.

Solutions:

It is given that $\lim_{x \rightarrow a} \frac{x^{\frac{5}{2}} - a^{\frac{5}{2}}}{x - a}$

As $x \rightarrow a$,

$$\lim_{x \rightarrow a} \frac{x^{\frac{5}{2}} - a^{\frac{5}{2}}}{x - a} = \frac{5}{2} a^{\frac{5}{2} - 1} = \frac{5}{2} a^{\frac{3}{2}}$$

Q.no. 14.

Solutions:

It is given that $\lim_{x \rightarrow a} \left\{ \frac{(x+2)^{\frac{3}{5}} - (a+2)^{\frac{3}{5}}}{x - a} \right\}$

As $x \rightarrow a$,

$$\lim_{x \rightarrow a} \left\{ \frac{(x+2)^{\frac{5}{3}} - (a+2)^{\frac{5}{3}}}{x-a} \right\} = \lim_{x \rightarrow a} \left\{ \frac{(x+2)^{\frac{5}{3}} - (a+2)^{\frac{5}{3}}}{(x+2) - (a+2)} \right\}$$

$$= \frac{5}{3} (a+2)^{\frac{5}{3}-1} = \frac{5}{3} (a+2)^{\frac{2}{3}} \text{ (As we know that } \frac{x^n - y^n}{x-y} = ny^{n-1} \text{)}$$

Q.no. 15.

Solutions:

It is given that $\lim_{x \rightarrow 1} \frac{x^n - 1}{x - 1}$

$$\lim_{x \rightarrow 1} \frac{x^n - 1}{x - 1} = n \text{ (As we know that } \frac{x^n - y^n}{x - y} = ny^{n-1} \text{)}$$

Q.no. 16.

Solutions:

It is given that $\lim_{x \rightarrow a} \frac{\sqrt{x} - \sqrt{a}}{x - a}$

As $x \rightarrow a$,

$$\lim_{x \rightarrow a} \frac{x^{\frac{1}{2}} - a^{\frac{1}{2}}}{x - a} = \frac{1}{2} a^{\frac{1}{2}-1} = \frac{1}{2\sqrt{a}} \text{ (As we know that } \frac{x^n - y^n}{x - y} = ny^{n-1} \text{)}$$

Q.no. 17.

Solutions:

It is given that $\lim_{h \rightarrow 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$

As $x \rightarrow 0$,

By applying L'Hospital's rule, we get

$$\lim_{h \rightarrow 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} = \lim_{h \rightarrow 0} \frac{\frac{d}{dh}(\sqrt{x+h} - \sqrt{x})}{\frac{d}{dh}(h)} = \lim_{h \rightarrow 0} \frac{\frac{1}{2\sqrt{x+h}}}{1} = \frac{1}{2\sqrt{x}}$$

Q.no. 18.

Solutions:

It is given that $\lim_{h \rightarrow 0} \frac{1}{h} \left\{ \frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}} \right\}$

As $x \rightarrow 0$,

$$\lim_{h \rightarrow a} \left\{ \frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}} \right\} = \frac{0}{0}$$

By applying L'Hospital's rule, we get

$$\begin{aligned} \lim_{h \rightarrow 0} \frac{1}{h} \left\{ \frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}} \right\} &= \lim_{h \rightarrow 0} \frac{\frac{d}{dh} \left(\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}} \right)}{\frac{d}{dh}(h)} \\ &= \lim_{h \rightarrow 0} \frac{\frac{-1}{2\sqrt{x+h}} + \frac{1}{2\sqrt{x}}}{1} = \lim_{h \rightarrow 0} \frac{1}{h} \left\{ \frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}} \right\} = 0 \end{aligned}$$

Q.no. 19.

Solutions:

It is given that $\lim_{h \rightarrow 0} \frac{\sqrt{1+x} - 1}{x}$

As $x \rightarrow 0$,

$$\lim_{h \rightarrow 0} \frac{\sqrt{1+x} - 1}{x} = \frac{0}{0}$$

By applying L'Hospital's rule, we get

$$\lim_{x \rightarrow 0} \frac{\sqrt{1+x} - 1}{x} = \lim_{x \rightarrow 0} \frac{\frac{d}{dx} \sqrt{1+x} - 1}{\frac{d}{dx}(x)} = \lim_{x \rightarrow 0} \frac{\frac{d}{dx} \sqrt{1+x} - 1}{\frac{1}{2\sqrt{x+1}}}$$

$$\lim_{x \rightarrow 0} \frac{\sqrt{1+x} - 1}{x} = \frac{1}{2}$$

Q.no. 20.

Solutions:

It is given that $\lim_{x \rightarrow 0} \frac{\sqrt{2-x} - \sqrt{2+x}}{x}$

As $x \rightarrow 0$,

$$\lim_{x \rightarrow 0} \frac{\sqrt{2-x} - \sqrt{2+x}}{x} = \frac{0}{0}$$

By applying L'Hospital's rule, we get

$$\lim_{x \rightarrow 0} \frac{\sqrt{2-x} - \sqrt{2+x}}{x} = \lim_{x \rightarrow 0} \frac{\frac{d}{dx}(\sqrt{2-x} - \sqrt{2+x})}{\frac{d}{dx}(x)}$$

$$\lim_{x \rightarrow 0} \frac{\sqrt{2-x} - \sqrt{2+x}}{x} = \lim_{x \rightarrow 0} \frac{-\frac{1}{2\sqrt{2-x}} - \frac{1}{2\sqrt{2+x}}}{1} = \frac{-2}{2\sqrt{2}}$$

$$\lim_{x \rightarrow 0} \frac{\sqrt{2-x} - \sqrt{2+x}}{x} = \frac{-1}{\sqrt{2}}$$

Q.no. 21.**Solutions:**

It is given that $\lim_{x \rightarrow 0} \frac{\sqrt{1+x+x^2}-1}{x}$

As $x \rightarrow 0$,

$$\lim_{x \rightarrow 0} \frac{\sqrt{1+x+x^2}-1}{x} = \frac{0}{0}$$

By applying L'Hospital's rule, we get

$$\lim_{x \rightarrow 0} \frac{\sqrt{1+x+x^2}-1}{x} = \lim_{x \rightarrow 0} \frac{\frac{d}{dx} \sqrt{1+x+x^2}-1}{\frac{d}{dx}(x)}$$

$$\lim_{x \rightarrow 0} \frac{\sqrt{1+x+x^2}-1}{x} = \lim_{x \rightarrow 0} \frac{\frac{1+2x}{2\sqrt{1+x+x^2}}}{1} = \frac{1}{2}$$

Q.no. 22.**Solutions:**

As $x \rightarrow 0$,

$$\lim_{x \rightarrow 0} \frac{\sqrt{3-x}-1}{2-x} = \frac{\sqrt{3}-1}{2}$$

Q.no. 23.**Solutions:**

It is given that $\lim_{x \rightarrow 0} \frac{2x}{\sqrt{a+x}-\sqrt{a-x}}$

$$\lim_{x \rightarrow 0} \frac{2x}{\sqrt{a+x} - \sqrt{a-x}} = \lim_{x \rightarrow 0} \frac{2x}{\sqrt{a+x} - \sqrt{a-x}} \left(\frac{\sqrt{a+x} + \sqrt{a-x}}{\sqrt{a+x} + \sqrt{a-x}} \right)$$

$$\lim_{x \rightarrow 0} \frac{2x}{\sqrt{a+x} - \sqrt{a-x}} = \lim_{x \rightarrow 0} \frac{2x(\sqrt{a+x} + \sqrt{a-x})}{a+x-a+x}$$

$$\lim_{x \rightarrow 0} \frac{2x}{\sqrt{a+x} - \sqrt{a-x}} = \lim_{x \rightarrow 0} \sqrt{a+x} + \sqrt{a-x} = 2\sqrt{a}$$

Q.no. 24.

Solutions:

It is given that $\lim_{x \rightarrow 1} \frac{\sqrt{3+x} - \sqrt{5-x}}{x^2 - 1}$

As $x \rightarrow 0$,

$$\lim_{x \rightarrow 1} \frac{\sqrt{3+x} - \sqrt{5-x}}{x^2 - 1} = \frac{0}{0}$$

By applying L'Hospital's rule, we get

$$\lim_{x \rightarrow 1} \frac{\sqrt{3+x} - \sqrt{5-x}}{x^2 - 1} = \lim_{x \rightarrow 1} \frac{\frac{d}{dx} \sqrt{3+x} - \sqrt{5-x}}{\frac{d}{dx} (x^2 - 1)}$$

$$= \lim_{x \rightarrow 1} \frac{\frac{1}{2\sqrt{3+x}} + \frac{1}{2\sqrt{5-x}}}{2x}$$

$$\lim_{x \rightarrow 1} \frac{\sqrt{3+x} - \sqrt{5-x}}{x^2 - 1} = \frac{1}{4}$$

Q.no. 25.

Solutions:

It is given that $\lim_{x \rightarrow 2} \frac{x^2 - 4}{\sqrt{x+2} - \sqrt{3x-2}}$

As $x \rightarrow 0$,

$$\lim_{x \rightarrow 2} \frac{x^2 - 4}{\sqrt{x+2} - \sqrt{3x-2}} = \frac{0}{0}$$

By applying L'Hospital's rule, we get

$$\lim_{x \rightarrow 2} \frac{x^2 - 4}{\sqrt{x+2} - \sqrt{3x-2}} = \lim_{x \rightarrow 2} \frac{\frac{d}{dx}(x^2 - 4)}{\frac{d}{dx}(\sqrt{x+2} - \sqrt{3x-2})}$$

$$\lim_{x \rightarrow 2} \frac{x^2 - 4}{\sqrt{x+2} - \sqrt{3x-2}} = \frac{4}{\frac{1}{2\sqrt{2+2}} - \frac{3}{2\sqrt{6-2}}} = \frac{8}{\frac{1}{2} - \frac{3}{2}} = -8$$

Q.no. 26.

Solutions:

It is given that $\lim_{x \rightarrow 4} \left(\frac{3 - \sqrt{5+x}}{1 - \sqrt{5-x}} \right)$

$$\lim_{x \rightarrow 4} \left(\frac{3 - \sqrt{5+x}}{1 - \sqrt{5-x}} \right) = \lim_{x \rightarrow 4} \left(\frac{3 - \sqrt{5+x}}{1 - \sqrt{5-x}} \right) \left(\frac{1 + \sqrt{5-x}}{1 + \sqrt{5-x}} \right) \left(\frac{3 + \sqrt{5+x}}{3 + \sqrt{5+x}} \right)$$

$$\lim_{x \rightarrow 4} \left(\frac{3 - \sqrt{5+x}}{1 - \sqrt{5-x}} \right) = \lim_{x \rightarrow 4} \left(\frac{4-x}{x-4} \right) \left(\frac{1 + \sqrt{5-x}}{3 + \sqrt{5+x}} \right)$$

$$= \lim_{x \rightarrow 4} \left(\frac{1 + \sqrt{5-x}}{3 + \sqrt{5+x}} \right) = -\frac{1}{3}$$

Q.no. 27.

Solutions:

It is given that $\lim_{x \rightarrow 0} \frac{\sqrt{a+x} - \sqrt{a}}{x\sqrt{a(a+x)}}$

As $x \rightarrow 0$,

$$\lim_{x \rightarrow 0} \frac{\sqrt{a+x} - \sqrt{a}}{x\sqrt{a(a+x)}} = \frac{0}{0}$$

By applying L'Hospital's rule, we get

$$\lim_{x \rightarrow 0} \frac{\sqrt{a+x} - \sqrt{a}}{x\sqrt{a(a+x)}} = \lim_{x \rightarrow 0} \frac{\frac{d}{dx}(\sqrt{a+x} - \sqrt{a})}{\frac{d}{dx}(x\sqrt{a(a+x)})}$$

$$\lim_{x \rightarrow 0} \frac{\sqrt{a+x} - \sqrt{a}}{x\sqrt{a(a+x)}} = \lim_{x \rightarrow 0} \frac{\frac{1}{2\sqrt{a+x}}}{x \left(\frac{a}{2\sqrt{a(a+x)}} \right) + \sqrt{a(a+x)}}$$

$$\lim_{x \rightarrow 0} \frac{\sqrt{a+x} - \sqrt{a}}{x\sqrt{a(a+x)}} = \frac{1}{2\sqrt{a}} = \frac{1}{2a\sqrt{a}}$$

Q.no. 28.**Solutions:**

It is given that $\lim_{x \rightarrow 0} \frac{\sqrt{1+x^2} - \sqrt{1+x}}{\sqrt{1+x^3} - \sqrt{1+x}}$

As $x \rightarrow 0$,

$$\lim_{x \rightarrow 0} \frac{\sqrt{1+x^2} - \sqrt{1+x}}{\sqrt{1+x^3} - \sqrt{1+x}} = \frac{0}{0}$$

By applying L'Hospital's rule, we get

$$\lim_{x \rightarrow 0} \frac{\sqrt{1+x^2} - \sqrt{1+x}}{\sqrt{1+x^3} - \sqrt{1+x}} = \lim_{x \rightarrow 0} \frac{\frac{d}{dx}(\sqrt{1+x^2} - \sqrt{1+x})}{\frac{d}{dx}(\sqrt{1+x^3} - \sqrt{1+x})}$$

$$= \lim_{x \rightarrow 0} \frac{\frac{2x}{2\sqrt{1+x^2}} - \frac{1}{2\sqrt{1+x}}}{\frac{3x^2}{2\sqrt{1+x^3}} - \frac{1}{2\sqrt{1+x}}}$$

$$\lim_{x \rightarrow 0} \frac{\sqrt{1+x^2} - \sqrt{1+x}}{\sqrt{1+x^3} - \sqrt{1+x}} = \lim_{x \rightarrow 0} \frac{-\frac{1}{2}}{-\frac{1}{2}} = -1$$

Q.no. 29.

Solutions:

It is given that $\lim_{x \rightarrow 1} \left(\frac{x^4 - 3x^2 + 2}{x^3 - 5x^2 + 3x + 1} \right)$

As $x \rightarrow 0$,

$$\lim_{x \rightarrow 1} \left(\frac{x^4 - 3x^2 + 2}{x^3 - 5x^2 + 3x + 1} \right) = \frac{0}{0}$$

Therefore,

$$\lim_{x \rightarrow 1} \left(\frac{x^4 - 3x^2 + 2}{x^3 - 5x^2 + 3x + 1} \right) = \lim_{x \rightarrow 1} \frac{4x^3 - 6x}{3x^2 - 10x + 3} = \frac{4-6}{3-10+3} = -\frac{2}{-4} = -\frac{1}{2}$$

Q.no. 30.**Solutions:**

It is given that $\lim_{x \rightarrow 2} \left(\frac{3^x - 3^{3-x} - 12}{3^{3-x} - 3^{\frac{x}{2}}} \right)$

As $x \rightarrow 0$,

$$\lim_{x \rightarrow 0} \frac{2^x - 1}{x} = \frac{0}{0}$$

By applying L'Hospital's rule, we get

$$\lim_{x \rightarrow 2} \left(\frac{3^x - 3^{3-x} - 12}{3^{3-x} - 3^{\frac{x}{2}}} \right) = \lim_{x \rightarrow 2} \frac{\frac{d}{dx} (3^x - 3^{3-x} - 12)}{\frac{d}{dx} \left(3^{3-x} - 3^{\frac{x}{2}} \right)} = \lim_{x \rightarrow 2} \frac{3^x \ln 3 + 3^{3-x} \ln 3}{-3^{3-x} \ln 3 + 3^{\frac{x}{2}} \left(\frac{1}{2} \right) \ln 3}$$

$$\lim_{x \rightarrow 2} \left(\frac{3^x - 3^{3-x} - 12}{3^{3-x} - 3^{\frac{x}{2}}} \right) = \frac{\ln 3 + 27 \ln 3}{-27 \ln 3 + \left(\frac{1}{2} \right) \ln 3} = \frac{28 \ln 3}{-26.5 \ln 3}$$

Q.no. 31.**Solutions:**

It is given that $\lim_{x \rightarrow 0} \frac{e^{4x} - 1}{x}$

As $x \rightarrow 0$,

$$\lim_{x \rightarrow 0} \frac{e^{4x} - 1}{x} = \frac{0}{0}$$

By applying L'Hospital's rule, we get

$$\lim_{x \rightarrow 0} \frac{e^{4x} - 1}{x} = \lim_{x \rightarrow 0} \frac{\frac{d}{dx}(e^{4x} - 1)}{\frac{d}{dx}(x)}$$

$$\lim_{x \rightarrow 0} \frac{e^{4x} - 1}{x} = \lim_{x \rightarrow 0} \frac{4e^{4x}}{1} = 4$$

Q.no. 32.

Solutions:

It is given that $\lim_{x \rightarrow 0} \frac{e^{2+x} - e^2}{x}$

As $x \rightarrow 0$,

$$\lim_{x \rightarrow 0} \frac{e^{2+x} - e^2}{x} = \frac{0}{0}$$

By applying L'Hospital's rule, we get

$$\lim_{x \rightarrow 0} \frac{e^{2+x} - e^2}{x} = \lim_{x \rightarrow 0} \frac{\frac{d}{dx}(e^{2+x} - e^2)}{\frac{d}{dx}(x)} = \lim_{x \rightarrow 0} \frac{e^{2+x}}{1} = e^2$$

Q.no. 33.

Solutions:

It is given that $\lim_{x \rightarrow 4} \frac{e^x - e^4}{x - 4}$

As $x \rightarrow 0$,

$$\lim_{x \rightarrow 4} \frac{e^x - e^4}{x - 4} = \frac{0}{0}$$

By applying L'Hospital's rule, we get

$$\lim_{x \rightarrow 4} \frac{e^x - e^4}{x - 4} = \lim_{x \rightarrow 4} \frac{\frac{d}{dx}(e^x - e^4)}{\frac{d}{dx}(x - 4)} = \lim_{x \rightarrow 4} \frac{e^x}{1} = e^4$$

Q.no. 34.

Solutions:

It is given that $\lim_{x \rightarrow 0} \frac{e^{3x} - e^{2x}}{x}$

As $x \rightarrow 0$,

$$\lim_{x \rightarrow 0} \frac{e^{3x} - e^{2x}}{x} = \frac{0}{0}$$

By applying L'Hospital's rule, we get

$$\lim_{x \rightarrow 0} \frac{e^{3x} - e^{2x}}{x} = \lim_{x \rightarrow 0} \frac{\frac{d}{dx}(e^{3x} - e^{2x})}{\frac{d}{dx}(x)} = \lim_{x \rightarrow 0} \frac{3e^{3x} - 2e^{2x}}{1} = 3 - 2 = 1$$

Q.no. 35.

Solutions:

It is given that $= \lim_{x \rightarrow 0} \frac{e^x - x - 1}{x}$

As $x \rightarrow 0$,

$$= \lim_{x \rightarrow 0} \frac{e^x - x - 1}{x} = \frac{0}{0}$$

By applying L'Hospital's rule, we get

$$\begin{aligned}
 &= \lim_{x \rightarrow 0} \frac{e^x - x - 1}{x} = \lim_{x \rightarrow 0} \frac{\frac{d}{dx}(e^x - x - 1)}{\frac{d}{dx}(x)} = \lim_{x \rightarrow 0} \frac{e^x - 1}{1} \\
 &= \lim_{x \rightarrow 0} \frac{e^x - x - 1}{x} = 1 - 1 = 0
 \end{aligned}$$

Q.no. 36.

Solutions:

It is given that $\lim_{x \rightarrow 0} \frac{e^{bx} - e^{ax}}{x}$

By applying L'Hospital's rule, we get

$$\lim_{x \rightarrow 0} \frac{e^{bx} - e^{ax}}{x} = \lim_{x \rightarrow 0} \frac{\frac{d}{dx}(e^{bx} - e^{ax})}{\frac{d}{dx}(x)} = \lim_{x \rightarrow 0} \frac{be^{bx} - ae^{ax}}{1} = b - a$$

Therefore,, the value of $\lim_{x \rightarrow 0} \frac{e^{bx} - e^{ax}}{x}$ is $b - a$

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Solutions:

It is given that $\lim_{x \rightarrow 0} \frac{a^x - b^x}{x}$

As $x \rightarrow 0$,

$$\lim_{x \rightarrow 0} \frac{a^x - b^x}{x} = \frac{0}{0}$$

By applying L'Hospital's rule, we get

$$\lim_{x \rightarrow 0} \frac{a^x - b^x}{x} = \lim_{x \rightarrow 0} \frac{\frac{d}{dx}(a^x - b^x)}{\frac{d}{dx}(x)} = \lim_{x \rightarrow 0} \frac{a^x \ln a - b^x \ln b}{1}$$

$$\lim_{x \rightarrow 0} \frac{a^x - b^x}{x} = \ln a - \ln b = \ln \frac{a}{b}$$

Q.no. 38.

Solutions:

It is given that $\lim_{x \rightarrow 0} \frac{a^x - a^{-x}}{x}$

As $x \rightarrow 0$,

$$\lim_{x \rightarrow 0} \frac{a^x - a^{-x}}{x} = \frac{0}{0}$$

By applying L'Hospital's rule, we get

$$\lim_{x \rightarrow 0} \frac{a^x - a^{-x}}{x} = \lim_{x \rightarrow 0} \frac{\frac{d}{dx}(a^x - a^{-x})}{\frac{d}{dx}(x)} = \lim_{x \rightarrow 0} \frac{a^x \ln a + a^{-x} \ln a}{1} = 2 \ln a$$

Q.no. 39.

Solutions:

It is given that $\lim_{x \rightarrow 0} \frac{2^x - 1}{x}$

By applying L'Hospital's rule, we get

$$\lim_{x \rightarrow 0} \frac{2^x - 1}{x} = \lim_{x \rightarrow 0} \frac{\frac{d}{dx}(2^x - 1)}{\frac{d}{dx}(x)} = \lim_{x \rightarrow 0} \frac{2^x \ln 2}{1} = \ln 2$$

Q.no. 40.

Solutions:

It is given that $\lim_{x \rightarrow 0} \frac{3^{2+x} - 9}{x}$

As $x \rightarrow 0$,

$$\lim_{x \rightarrow 0} \frac{3^{2+x} - 9}{x} = \frac{0}{0}$$

By applying L'Hospital's rule, we get

$$\lim_{x \rightarrow 0} \frac{3^{2+x} - 9}{x} = \lim_{x \rightarrow 0} \frac{\frac{d}{dx}(3^{2+x} - 9)}{\frac{d}{dx}(x)} = \lim_{x \rightarrow 0} \frac{(2+x)\ln 3}{1} = 2\ln 3$$

Exercise 27 B

Q.no. 1.

Solutions:

$$\lim_{x \rightarrow 0} \frac{\sin 4x}{6x} = \lim_{x \rightarrow 0} \left(\frac{\sin 4x}{4x} \right) \times \frac{4}{6} = \frac{4}{6} = \frac{2}{3} \left(\because \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \right)$$

Q.no. 2.**Solutions:**

$$\lim_{x \rightarrow 0} \frac{\sin 5x}{\sin 8x} = \lim_{x \rightarrow 0} \left(\frac{\sin 5x}{5x} \times \frac{8x}{\sin 8x} \times \frac{5x}{8x} \right) = \frac{5x}{8x} = \frac{5}{8} \left(\because \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \right)$$

Q.no. 3.**Solutions:**

$$\lim_{x \rightarrow 0} \frac{\tan 3x}{\tan 5x} = \lim_{x \rightarrow 0} \left(\frac{\tan 3x}{3x} \times \frac{5x}{\sin 5x} \times \frac{3x}{5x} \right) = \frac{3x}{5x} = \frac{3}{5}$$

$$\left(\because \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \text{ and } \lim_{x \rightarrow 0} \frac{\tan x}{x} = 1 \right)$$

Q.no. 4.**Solutions:**

$$\lim_{x \rightarrow 0} \frac{\tan \alpha x}{\tan \beta x} = \lim_{x \rightarrow 0} \left(\frac{\tan \alpha x}{\alpha x} \times \frac{\beta x}{\sin \beta x} \times \frac{\alpha x}{\beta x} \right) = \frac{\alpha x}{\beta x} = \frac{\alpha}{\beta}$$

$$\left(\because \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \text{ and } \lim_{x \rightarrow 0} \frac{\tan x}{x} = 1 \right)$$

Q.no. 5.**Solutions:**

$$\lim_{x \rightarrow 0} \frac{\sin 4x}{\tan 7x} = \lim_{x \rightarrow 0} \left(\frac{\sin 4x}{4x} \times \frac{7x}{\sin 7x} \times \frac{4x}{7x} \right) = \frac{4x}{7x} = \frac{4}{7}$$

$$\left(\because \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \text{ and } \lim_{x \rightarrow 0} \frac{\tan x}{x} = 1 \right)$$

Q.no. 6.**Solutions:**

$$\lim_{x \rightarrow 0} \frac{\tan 3x}{\sin 4x} = \lim_{x \rightarrow 0} \left(\frac{\tan 3x}{3x} \times \frac{4x}{\sin 4x} \times \frac{3x}{4x} \right) = \frac{3x}{4x} = \frac{3}{4}$$

$$\left(\therefore \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \text{ and } \lim_{x \rightarrow 0} \frac{\tan x}{x} = 1 \right)$$

Q.no. 7.**Solutions:**

$$\lim_{x \rightarrow 0} \frac{\sin mx}{\tan nx} = \lim_{x \rightarrow 0} \left(\frac{\sin mx}{mx} \times \frac{nx}{\tan nx} \times \frac{mx}{nx} \right) = \frac{mx}{nx} = \frac{m}{n}$$

$$\left(\therefore \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \text{ and } \lim_{x \rightarrow 0} \frac{\tan x}{x} = 1 \right)$$

Q.no. 8.**Solutions:**

$$\lim_{x \rightarrow 0} \frac{\sin x - 2\sin 3x + \sin 5x}{x} = \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} - \frac{2\sin 3x}{x} + \frac{\sin 5x}{x} \right)$$

$$\lim_{x \rightarrow 0} \left(\frac{\sin x}{x} - \frac{2\sin 3x}{3x} \times 3 + \frac{5\sin 5x}{5x} \right)$$

By using the above formula,

$$\therefore \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} - \frac{2\sin 3x}{3x} \times 3 + \frac{5\sin 5x}{5x} \right) = 1 - 2 \times 3 + 5 = 0$$

$$\left(\therefore \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \right)$$

Q.no. 9.**Solutions:**

$$\lim_{x \rightarrow \frac{\pi}{6}} \frac{4\sin x \cos x + \cos x}{4\sin x \cos x - 3\cos x} = \lim_{x \rightarrow \frac{\pi}{6}} \frac{4\sin x + 1}{4\sin x - 3} = \frac{2+1}{2-2} = -3$$

Q.no. 10.

Solutions:

$$\lim_{x \rightarrow 0} \frac{\sin 2x + 3x}{2x + \sin 3x} = \lim_{x \rightarrow 0} \frac{\frac{\sin 2x + 3x}{x}}{\frac{2x + \sin 3x}{x}} = \lim_{x \rightarrow 0} \frac{\frac{\sin 2x}{x} + 3}{2 + \frac{\sin 3x}{x}}$$

$$\lim_{x \rightarrow 0} \frac{\frac{2\sin 2x}{2x} + 3}{2 + \frac{3\sin 3x}{3x}} = \frac{2+3}{2+3} = 1 \left(\because \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \right)$$

Q.no. 11.

Solutions:

$$\lim_{x \rightarrow 0} \frac{\tan 2x + 3x}{3x + \tan x} = \lim_{x \rightarrow 0} \frac{\frac{\tan 2x + x}{x}}{\frac{3x + \tan x}{x}} = \lim_{x \rightarrow 0} \frac{\frac{\tan 2x}{x} - 1}{3 - \frac{\tan x}{x}}$$

$$= \lim_{x \rightarrow 0} \frac{\frac{2\tan 2x}{2x} - 1}{3 - \frac{\tan x}{x}} = \frac{2-1}{3-1} = \frac{1}{2} \left(\because \lim_{x \rightarrow 0} \frac{\tan x}{x} = 1 \right)$$

Q.no. 12.

Solutions:

$$\begin{aligned}
 \lim_{x \rightarrow 0} \frac{x^2 - \tan 2x}{\tan x} &= \lim_{x \rightarrow 0} \frac{\frac{x^2 - \tan 2x}{x}}{\frac{\tan x}{x}} = \lim_{x \rightarrow 0} \frac{x - \tan 2x}{\tan x} \\
 &= \lim_{x \rightarrow 0} \frac{x - \frac{2 \tan 2x}{x}}{\frac{\tan x}{x}} = \frac{0 - 2}{1} = -2 \left(\because \lim_{x \rightarrow 0} \frac{\tan x}{x} = 1 \right)
 \end{aligned}$$

Q.no. 13.

Solutions:

$$\lim_{x \rightarrow 0} \frac{x \cos x + \sin x}{x^2 + \tan x} = \lim_{x \rightarrow 0} \frac{\frac{x \cos x + \sin x}{x}}{\frac{x^2 + \tan x}{x}} = \lim_{x \rightarrow 0} \frac{\cos x + \frac{\sin x}{x}}{x + \frac{\tan x}{x}} = \frac{1+1}{0+1} = 2$$

Q.no. 14.

Solutions:

Now, $\tan x - \sin x =$

$$\frac{\sin x}{\cos x} - \sin x = \frac{\sin x - \sin x \cos x}{\cos x} = \sin x \left(\frac{1 - \cos x}{\cos x} \right)$$

$$\lim_{x \rightarrow 0} \frac{\tan x - \sin x}{\sin^3 x} = \lim_{x \rightarrow 0} \frac{\frac{1 - \cos x}{\cos x}}{\sin^2 x} = \lim_{x \rightarrow 0} \frac{1 - \cos x}{\sin^2 x \cos x}$$

$$= \lim_{x \rightarrow 0} \frac{\frac{1 - \cos x}{x^2}}{\frac{\sin^2 x \cos x}{x^2}} = \frac{1}{1} = \frac{1}{2}$$

Q.no. 15.

Solutions:

$$\lim_{x \rightarrow 0} \operatorname{cosec} x = \lim_{x \rightarrow 0} \frac{x}{\sin x} = 1$$

Q.no. 16.**Solutions:**

$$\lim_{x \rightarrow 0} x \cot 2x = \lim_{x \rightarrow 0} \frac{2x}{2 \tan 2x} = \frac{1}{2}$$

Q.no. 17.**Solutions:**

$$\lim_{x \rightarrow 0} \frac{\sin x \cos x}{3x} = \lim_{x \rightarrow 0} \frac{\sin x}{x} \times \frac{\cos x}{3} = \frac{1}{3}$$

Q.no. 18.**Solutions:**

$$\lim_{x \rightarrow 0} \frac{\sin(x/4)}{x} = \lim_{x \rightarrow 0} \frac{\sin(x/4)}{4(x/4)} = \frac{1}{4}$$

Therefore, $\lim_{x \rightarrow 0} \frac{\sin(x/4)}{x} = \frac{1}{4}$

Q.no. 19.**Solutions:**

$$\lim_{x \rightarrow 0} \frac{\tan(x/2)}{3x} = \lim_{x \rightarrow 0} \frac{\tan(x/2)}{6(x/2)} = \frac{1}{6}$$

Therefore, $\lim_{x \rightarrow 0} \frac{\tan(x/2)}{3x} = \frac{1}{6}$

Q.no. 20.

Solutions:

$$\lim_{x \rightarrow 0} \frac{1 - \cos x}{\sin^2 x} = \lim_{x \rightarrow 0} \frac{2 \sin^2(x/2)}{\sin^2 x}$$

$$\lim_{x \rightarrow 0} \frac{2 \sin^2(x/2)}{\sin^2 x} = \lim_{x \rightarrow 0} \frac{2 \sin^2\left(\frac{x}{2}\right)}{\frac{\sin^2 x}{x^2}} = \lim_{x \rightarrow 0} \frac{2 \sin^2\left(\frac{x}{2}\right)}{4 \frac{x^2}{4}} = \frac{2}{1} = \frac{2}{4} = \frac{1}{2}$$

$$[\text{Since, } \lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} = \frac{1}{2}]$$

Q.no. 21.**Solutions:**

$$\lim_{x \rightarrow 0} \frac{1 - \cos 3x}{x^2} = \lim_{x \rightarrow 0} \frac{9[1 - \cos 3x]}{(3x)^2} = \frac{9}{2} \quad [\because \lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} = \frac{1}{2}]$$

Q.no. 22.**Solutions:**

$$\lim_{x \rightarrow 0} \frac{1 - \cos x}{\sin^2 2x} = \lim_{x \rightarrow 0} \frac{\frac{1 - \cos x}{x^2}}{\frac{\sin^2 2x}{x^2}} = \frac{1}{2} \quad [\because \lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} = \frac{1}{2}] \quad \text{and}$$

$$\left(\because \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \right)$$

Q.no. 23.**Solutions:**

$$\lim_{x \rightarrow 0} \frac{1 - \cos 2x}{3 \tan^2 x} = \lim_{x \rightarrow 0} \frac{\frac{4[1 - \cos 2x]}{(4x)^2}}{\frac{3 \tan^2 x}{x^2}} = \frac{4}{6} = \frac{2}{3}$$

$$\left(\because \lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} = \frac{1}{2} \text{ and } \lim_{x \rightarrow 0} \frac{\tan x}{x} = 1 \right)$$

Q.no. 24.

Solutions:

So, by using the above formula,

$$\lim_{x \rightarrow 0} \frac{1 - \cos 4x}{1 - \cos 6x} = \lim_{x \rightarrow 0} \frac{\frac{16[1 - \cos 4x]}{(4x)^2}}{\frac{36[1 - \cos 6x]}{(6x)^2}} = \frac{\frac{16}{36}}{\frac{1}{2}} = \frac{8}{18} = \frac{4}{9}$$

$$\left[\because \lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} = \frac{1}{2} \right]$$

Q.no. 25.

Solutions:

$$\lim_{x \rightarrow 0} \frac{1 - \cos mx}{1 - \cos nx} = \lim_{x \rightarrow 0} \frac{\frac{m^2[1 - \cos mx]}{(mx)^2}}{\frac{n^2[1 - \cos nx]}{(nx)^2}} = \frac{m^2}{n^2}$$

$$\left[\because \lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} = \frac{1}{2} \right]$$

Q.no. 26.**Solutions:**

We know that $\sin 2x = 2 \sin x \cos x$

$$\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} = \frac{1}{2} \quad \text{and} \quad \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

By using the above formula,

$$\text{Therefore, } \lim_{x \rightarrow 0} \frac{2 \sin x - \sin 2x}{x^3} = 1$$

Q.no. 27.**Solutions:**

$$\begin{aligned} \text{Now, } \tan x - \sin x &= \frac{\sin x}{\cos x} - \sin x = \frac{\sin x - \sin x \cos x}{\cos x} \\ &= \sin x \left(\frac{1 - \cos x}{\cos x} \right) \end{aligned}$$

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{\tan x - \sin x}{x^3} &= \lim_{x \rightarrow 0} \frac{\left(\frac{1 - \cos x}{\cos x} \right) \sin x}{x^3} = \lim_{x \rightarrow 0} \frac{2 \sin x [1 - \cos x]}{x^3} \\ &= \lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2 \cos x} \times \frac{\sin x}{x} = \frac{1}{2} \\ \left[\because \lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} = \frac{1}{2} \right] \quad \text{and} \quad \left(\because \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \right) \end{aligned}$$

Q.no. 28.**Solutions:**

$$\begin{aligned}\lim_{x \rightarrow 0} \frac{\tan 2x - \sin 2x}{x^3} &= \lim_{x \rightarrow 0} \frac{\sin 2x + \sin 2x \cos 2x}{x^3} \\ &= \lim_{x \rightarrow 0} \frac{\sin 2x [1 - \cos 2x]}{x^3} = \lim_{x \rightarrow 0} \frac{2 \sin 2x}{2x} \times \frac{4(1 - \cos 2x)}{(2x)^2} = 4\end{aligned}$$

Q.no. 29.

Solutions:

Now, $\operatorname{cosec} x - \cot x = (1 - \cos x)/\sin x$

$$\lim_{x \rightarrow 0} \frac{\operatorname{cosec} x - \cot x}{x} = \lim_{x \rightarrow 0} \frac{1 - \cos x}{x \sin x} = \lim_{x \rightarrow 0} \frac{\frac{1 - \cos x}{x^2}}{\frac{x \sin x}{x^2}} = \lim_{x \rightarrow 0} \frac{\frac{1 - \cos x}{x^2}}{\frac{\sin x}{x}} = \frac{1}{2}$$

$$\left[\because \lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} = \frac{1}{2} \right] \text{ and } \left(\because \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \right)$$

Q.no. 30.

Solutions:

Now, $\cot 2x - \operatorname{cosec} 2x = (1 - \cos 2x)/\sin 2x$

$$\lim_{x \rightarrow 0} \frac{\cot 2x - \operatorname{cosec} 2x}{x} = \lim_{x \rightarrow 0} \frac{\cos 2x - 1}{x \sin 2x} = \frac{-4}{2} = -2$$

$$= \lim_{x \rightarrow 0} \frac{\frac{\cos 2x - 1}{x^2}}{\frac{x \sin 2x}{x^2}} = \lim_{x \rightarrow 0} \frac{\frac{4[\cos 2x - 1]}{(2x)^2}}{\frac{2 \sin 2x}{2x}}$$

$$\left[\because \lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} = \frac{1}{2} \right] \text{ and } \left(\because \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \right)$$

Q.no. 31.**Solutions:**

$$\begin{aligned}\lim_{x \rightarrow 0} \frac{\sin 2x(1 - \cos 2x)}{x^3} &= \lim_{x \rightarrow 0} \frac{\sin 2x}{x} \times \frac{(1 - \cos 2x)}{x^2} \\ &= \lim_{x \rightarrow 0} \frac{2 \sin 2x}{2x} \times \frac{4(1 - \cos 2x)}{(2x)^2} = 4 \\ \left[\because \lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} = \frac{1}{2} \right] \text{ and } \left(\because \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \right)\end{aligned}$$

Q.no. 32.**Solutions:**

$$\begin{aligned}\lim_{x \rightarrow \frac{\pi}{4}} \frac{\sec^2 x - 2}{\tan x - 1} &= \lim_{x \rightarrow \frac{\pi}{4}} \frac{2 \sec x (\sec x \tan x) - 0}{\sec^2 x - 0} \\ &= \lim_{x \rightarrow \frac{\pi}{4}} \frac{2 \sec x (\sec x \tan x)}{\sec^2 x} = \lim_{x \rightarrow \frac{\pi}{4}} 2 \tan x = 2\end{aligned}$$

Q.no. 33.**Solutions:**

$$\begin{aligned}\lim_{x \rightarrow \frac{\pi}{4}} \frac{\operatorname{cosec}^2 x - 2}{\cot x - 1} &= \lim_{x \rightarrow \frac{\pi}{4}} \frac{2 \operatorname{cosec} x (-\operatorname{cosec} x \cot x) - 0}{-\operatorname{cosec}^2 x - 0} \\ &= \lim_{x \rightarrow \frac{\pi}{4}} \frac{2 \operatorname{cosec} x (-\operatorname{cosec} x \cot x)}{-\operatorname{cosec}^2 x} = \lim_{x \rightarrow \frac{\pi}{4}} 2 \cot x = 2\end{aligned}$$

Q.no. 34.**Solutions:**

$$\lim_{x \rightarrow \frac{\pi}{4}} \frac{1 - \tan x}{x - \frac{\pi}{4}} = \lim_{x \rightarrow \frac{\pi}{4}} \frac{0 - \sec^2 x}{1 - 0} = \lim_{x \rightarrow \frac{\pi}{4}} \frac{-\sec^2 x}{1} = -2$$

Q.no. 35.

Solutions:

$$\lim_{x \rightarrow \pi} \frac{\sin 3x - 3\sin x}{(\pi - x)^3} = \lim_{x \rightarrow \pi} \frac{3\cos 3x - 3\cos x}{-3(\pi - x)^2}$$

Differtiate both sides w.r.t x again,

$$\lim_{x \rightarrow \pi} \frac{\sin 3x - 3\sin x}{(\pi - x)^3} = \lim_{x \rightarrow \pi} \frac{-9\sin 3x + 3\sin x}{6(\pi - x)}$$

Differtiate both sides w.r.t x again,

$$\begin{aligned} \lim_{x \rightarrow \pi} \frac{\sin 3x - 3\sin x}{(\pi - x)^3} &= \lim_{x \rightarrow \pi} \frac{-27\cos 3x + 3\cos x}{-6} \\ &= \frac{-27\cos 3\pi + 3\cos \pi}{-6} = \frac{27 - 3}{-6} = -4 \end{aligned}$$

Q.no. 36.

Solutions:

$$\begin{aligned} \lim_{x \rightarrow \frac{\pi}{2}} \frac{1 + \cos 2x}{(\pi - 2x)^2} &= \lim_{x \rightarrow \frac{\pi}{2}} \frac{0 + (-2\sin 2x)}{2(\pi - 2x)(-2)} \\ &= \lim_{x \rightarrow \frac{\pi}{2}} \frac{-\sin 2x}{-4(\pi - 2x)} = \lim_{x \rightarrow \frac{\pi}{2}} \frac{2\sin 2x}{4(\pi - 2x)} \end{aligned}$$

So, Differtiate both sides w.r.t x again,

$$\lim_{x \rightarrow \frac{\pi}{2}} \frac{1 + \cos 2x}{(\pi - 2x)^2} = \lim_{x \rightarrow \frac{\pi}{2}} \frac{4\cos 2x}{4(-2)} = \lim_{x \rightarrow \frac{\pi}{2}} \frac{\cos 2x}{(-2)} = \frac{\cos \pi}{(-2)} = \frac{-1}{(-2)} = \frac{1}{2}$$

Q.no. 37.**Solutions:** It is given that

$$\begin{aligned}
 & \lim_{x \rightarrow a} \frac{(\cos x - \cos a)}{(x - a)} \\
 &= \lim_{x \rightarrow a} \frac{-2 \times \sin\left(\frac{x+a}{2}\right) \sin\left(\frac{x-a}{2}\right)}{x - a} \\
 & \left[\because \cos x - \cos a = -2 \times \sin \frac{x+a}{2} \sin \frac{x-a}{2} \right] \\
 &= \lim_{x \rightarrow a} \sin \frac{x+a}{2} \times \frac{\sin\left(\frac{x-a}{2}\right)}{(x-a)} \left[\because \lim_{x \rightarrow a} \frac{\sin \theta}{\theta} = 1 \right] \\
 & \qquad \qquad \qquad 2 \\
 &= -1 \times \sin \frac{(a+a)}{2} = -1 \times \sin \frac{2a}{2} = -\sin(a)
 \end{aligned}$$

Q.no. 38.**Solutions:** It is given that

$$\begin{aligned}
 \lim_{x \rightarrow a} \frac{(\sin x - \sin a)}{(x - a)} &= \lim_{x \rightarrow a} \frac{\left(2 \times \cos \frac{x+a}{2} \sin \frac{x-a}{2}\right)}{(x-a)} \\
 &= 1 \times \lim_{x \rightarrow a} \cos \frac{x+a}{2} \left[\because \lim_{x \rightarrow a} \frac{\sin \theta}{\theta} = 1 \right] \\
 &= \cos \frac{a+a}{2} = \cos a
 \end{aligned}$$

Q.no. 39.

Solutions:

$$\begin{aligned}
 \lim_{x \rightarrow a} \frac{(\sin x - \sin a)}{(\sqrt{x} - \sqrt{a})} &= \lim_{x \rightarrow a} \frac{(\sin x - \sin a)}{(\sqrt{x} - \sqrt{a})} \times \frac{(\sqrt{x} + \sqrt{a})}{(\sqrt{x} + \sqrt{a})} \\
 &\left[\times \text{ and divide by } \sqrt{x} - \sqrt{a} \right] \\
 &= \lim_{x \rightarrow a} \frac{(\sin x - \sin a) \times (\sqrt{x} \times \sqrt{a})}{(x - a)} \\
 &= \cos a \times \lim_{x \rightarrow a} (\sqrt{x} + \sqrt{a}) \left[\because \lim_{x \rightarrow a} \frac{\sin x - \sin a}{x - a} = \cos a \right] \\
 &= 2\sqrt{a} \times \cos a = 2\sqrt{a} \cos a
 \end{aligned}$$

Q.no. 40.

Solutions:

$$\lim_{x \rightarrow 0} \frac{\sin 5x - \sin 3x}{\sin x} = \lim_{x \rightarrow 0} \frac{\left(2 \sin \frac{5x - 3x}{2} \cos \frac{5x + 3x}{2} \right)}{\sin x}$$

$$\left[\text{Applying } \sin C - \sin D = 2 \sin \frac{C - D}{2} \cos \frac{C + D}{2} \right]$$

$$= \lim_{x \rightarrow 0} 2 \cos 4x = 2 \times 1 = 2$$

Q.no. 41.

Solutions:

$$\lim_{x \rightarrow 0} \frac{(\cos 3x - \cos 5x)}{x^2} = \lim_{x \rightarrow 0} \frac{(1 - \cos 5x - (1 - \cos 3x))}{x^2}$$

$$= \lim_{x \rightarrow 0} \left(\frac{1 - \cos 5x}{x^2} - \frac{1 - \cos 3x}{x^2} \right)$$

$$= \left(\lim_{x \rightarrow 0} \frac{1 - \cos 5x}{x^2} - \frac{1 - \cos 3x}{x^2} \right) - \left(\lim_{x \rightarrow 0} \frac{1 - \cos 3x}{x^2} \times \frac{9}{9} \right)$$

$$= \frac{25}{2} - \frac{9}{2} = \frac{16}{2} = 8 \left[\because \lim_{x \rightarrow 0} \frac{1 - \cos ax}{(ax)^2} = \frac{1}{2} \right]$$

Q.no. 42.

Solutions:

$$\lim_{x \rightarrow 0} \frac{(\sin 3x + \sin 5x)}{(\sin 6x - \sin 4x)}$$

$$= \lim_{x \rightarrow 0} \frac{\left(2 \times \sin \frac{3x+5x}{2} \times \cos \frac{3x-5x}{2} \right)}{\left(2 \times \cos \frac{6x+4x}{2} \sin \frac{6x-4x}{2} \right)} = \lim_{x \rightarrow 0} \frac{\sin 4x \cos x}{\cos 5x \sin x}$$

$$= \lim_{x \rightarrow 0} \frac{\sin 4x}{\cos 5x} \times \frac{4x}{\frac{\sin x}{\cos x}}$$

$$= 4 \times \lim_{x \rightarrow 0} \frac{\sin 4x}{4x} \times \frac{1}{\cos 5x} \times \frac{x}{\tan x} = 4 \left[\begin{array}{l} \because \lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1 \\ \lim_{\theta \rightarrow 0} \frac{\theta}{\tan \theta} = 1 \end{array} \right]$$

Q.no. 43.

Solutions: It is given that

$$\lim_{x \rightarrow 0} \frac{[\sin(2+x) - \sin(2-x)]}{x}$$

$$= \lim_{x \rightarrow 0} \frac{\left[2 \times \cos \frac{(2+x+2-x)}{2} \times \sin \frac{(2+x-2+x)}{2} \right]}{x}$$

$$= \lim_{x \rightarrow 0} \frac{(2 \times \cos 2 \times \sin x)}{x} = 2 \cos 2 \lim_{x \rightarrow 0} \frac{\sin x}{x} = 2 \cos 2$$

Q.no. 44.

Solutions: It is given that

$$\begin{aligned}
& \lim_{x \rightarrow 0} \frac{1 - \cos 2x}{(\cos 2x - \cos 8x)} \\
&= \lim_{x \rightarrow 0} \frac{2 \times \sin x \times \sin x}{2 \times \sin 3 \times \sin 5x} \times \frac{5x \times 3x}{x \times x} \times \frac{1}{15} \\
&= \frac{1}{15} \times 1 \times 1 \times 1 \times 1 = \frac{1}{15} \left[\because \lim_{x \rightarrow 0} \frac{\sin \theta}{\theta} = 1 \right]
\end{aligned}$$

Q.no. 45.

Solutions: It is given that

$$\begin{aligned}
& \lim_{x \rightarrow \frac{\pi}{2}} \left(\frac{\pi}{2} - x \right) \tan x \\
&= -1 \times \lim_{y \rightarrow 0} y \tan \left(y + \frac{\pi}{2} \right) \left[x - \frac{\pi}{2} = y \right] \\
&= -1 \times \lim_{y \rightarrow 0} y \cot y \times -1 = \lim_{y \rightarrow 0} \frac{y}{\tan y} = 1
\end{aligned}$$

Q.no. 46.

Solutions:

It is given that

$$\begin{aligned}
& \lim_{x \rightarrow 0} \frac{(\sqrt{1+2x} - \sqrt{1-2x})}{\sin x} \\
&= \lim_{x \rightarrow 0} \frac{(\sqrt{1+2x} - \sqrt{1-2x})}{\sin x} \times \frac{\sqrt{1+2x} + \sqrt{1-2x}}{(\sqrt{1+2x} + \sqrt{1-2x})} \\
&= 4 \times \lim_{x \rightarrow 0} \frac{x}{\sin x} \times \frac{1}{\sqrt{1+2x} + \sqrt{1-2x}} = 4 \times \frac{1}{2} \times 1 = 2
\end{aligned}$$

Q.no. 48.

Solutions: It is given that

$$\begin{aligned}
& \lim_{h \rightarrow 0} \frac{(a+h)^2 \sin(a+h) - a^2 \sin a}{h} \\
&= \lim_{h \rightarrow 0} \frac{a^2 (\sin(a+h) - \sin a) + 2ah \sin(a+h) + h^2 \sin(a+h)}{h} \\
&= 2a \sin a + 0 + \lim_{h \rightarrow 0} \frac{a^2 \times 2 \times \cos\left(a + \frac{h}{2}\right) \times \sin h}{h} \\
&= 2a \sin a + 2a^2 \cos a = 2a^2 \cos a + 2a \sin a
\end{aligned}$$

Q.no. 48. .

Solutions: It is given that

$$\lim_{x \rightarrow 0} \frac{(e^{3+x} - \sin x - e^3)}{x}$$

$$= -\lim_{x \rightarrow 0} \frac{\sin x}{x} + \lim_{x \rightarrow 0} \frac{e^{3+x} - e^3}{x} = -1 + e^3$$

Q.no. 49.

Solutions: It is given that

$$\lim_{x \rightarrow 0} \frac{(e^{\tan x} - 1)}{\tan x}$$

As x tends to 0, $\tan(x)$ also tends to zero,

So,

$$\lim_{x \rightarrow 0} \frac{(e^{\tan x} - 1)}{\tan x} = \lim_{\tan x \rightarrow 0} \frac{(e^{\tan x} - 1)}{\tan x} = 1$$

Q.no. 50.

Solutions: It is given that

$$\lim_{x \rightarrow 0} \frac{e^{\tan x} - 1}{x}$$

$$= \lim_{x \rightarrow 0} \frac{e^{\tan x} - 1}{x} \times \frac{\tan x}{\tan x} = \lim_{x \rightarrow 0} \frac{e^{\tan x} - 1}{\tan x} \times \frac{\tan x}{x}$$

$$= 1 \times 1 = 1$$

Q.no. 51.

Solutions: It is given that

$$\begin{aligned}
 \lim_{x \rightarrow 0} \frac{ax + x \cos x}{b \sin x} &= \lim_{x \rightarrow 0} \frac{ax}{b \sin x} + \frac{x \cos x}{b \sin x} \\
 &= \lim_{x \rightarrow 0} \frac{ax}{b \sin x} + \lim_{x \rightarrow 0} \frac{x \cos x}{b \sin x} = \frac{a}{b} + \frac{1}{b} \left[\begin{array}{l} \because \lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1 \\ \lim_{x \rightarrow 0} \cos \theta = 1 \end{array} \right] \\
 &= \frac{a+1}{b}
 \end{aligned}$$

Q.no. 52.

Solutions: It is given that

$$\begin{aligned}
 \lim_{x \rightarrow 0} \frac{\sin(ax) + bx}{ax + \sin(bx)} \\
 &= \lim_{x \rightarrow 0} \frac{\sin(ax) + bx}{ax + \sin(bx)} \times \frac{bx}{ax} \times \frac{a}{b} \\
 &= \lim_{x \rightarrow 0} \frac{\frac{\sin ax + bx}{bx}}{\frac{ax + \sin bx}{bx}} \times \frac{a}{b} = \frac{a}{b} \times \frac{\lim_{x \rightarrow 0} \frac{\sin ax + bx}{bx}}{\lim_{x \rightarrow 0} \frac{ax + \sin bx}{bx}} \\
 &= \frac{a}{b} \times \frac{1 + \frac{b}{a}}{1 + \frac{a}{b}} = \frac{a}{b} \times \frac{b}{a} = 1
 \end{aligned}$$

Q.no. 53.

Solutions: It is given that

$$\begin{aligned}
 \lim_{x \rightarrow 0} \frac{\sin(\pi - x)}{\pi(\pi - x)} &= \lim_{x \rightarrow 0} \frac{\sin x}{\pi(\pi - x)} \times \frac{x}{x} \\
 &= \lim_{x \rightarrow 0} \frac{\sin x}{x} \times \lim_{x \rightarrow 0} \frac{\pi - (\pi - x)}{\pi(\pi - x)} \\
 &= 1 \times \lim_{x \rightarrow 0} \left(\frac{1}{\pi - x} - \frac{1}{\pi} \right) = \frac{1}{\pi} - \frac{1}{\pi} = 0
 \end{aligned}$$

Q.no. 54. .

Solutions: It is given that

$$\lim_{x \rightarrow \frac{\pi}{2}} \frac{\tan 2x}{x - \frac{\pi}{2}}$$

As x tends to $\pi/2$, $x - \pi/2$ tends to zero.

Let, $y = x - \pi/2$

$$= \lim_{y \rightarrow 0} \frac{\tan\left(2y + \frac{\pi}{2} \times 2\right)}{y} = \lim_{y \rightarrow 0} \frac{\tan(\pi + 2y)}{y}$$

$$= \lim_{y \rightarrow 0} \frac{\tan 2y}{2y} \times 2 = 2$$

Q.no. 55.

Solutions: It is given that

$$\begin{aligned}
 &= \lim_{x \rightarrow 0} \frac{\cos 2x - 1}{\cos x - 1} = \lim_{x \rightarrow 0} \frac{\cos 2x - 1}{\cos x - 1} \times \frac{4 \times x \times x}{2x \times 2x} \\
 &= 4 \times \lim_{x \rightarrow 0} \frac{\frac{\cos 2x - 1}{2x \times 2x}}{\frac{\cos x - 1}{x \times x}} = 4 \times \frac{\lim_{x \rightarrow 0} \frac{\cos 2x - 1}{(2x)^2}}{\lim_{x \rightarrow 0} \frac{\cos x - 1}{x^2}} = 4 \times \frac{\frac{1}{2}}{\frac{1}{2}} = 4
 \end{aligned}$$

Q.no. 56.

Solutions: It is given that

$$\begin{aligned}
 \lim_{x \rightarrow 0} (\operatorname{cosec} x - \cot x) &= \lim_{x \rightarrow 0} \left(\frac{1}{\sin x} - \frac{\cos x}{\sin x} \right) \\
 &= \lim_{x \rightarrow 0} \left(\frac{2 \times \sin \frac{x}{2} \times \sin \frac{x}{2}}{2 \times \sin \frac{x}{2} \times \cos \frac{x}{2}} \right) \left[\because 1 - \cos \theta = 2 \sin \theta \times \sin \theta \right] \\
 &= \lim_{x \rightarrow 0} \left(\tan \frac{x}{2} \right) = 0
 \end{aligned}$$

Q.no. 57.

Solutions: It is given that

$$\lim_{x \rightarrow 0} \frac{1 - \cos 2mx}{1 - \cos 2nx} = \lim_{x \rightarrow 0} \frac{\frac{1 - \cos 2mx}{(2mx)^2} \times (2mx)^2}{\frac{1 - \cos 2nx}{(2nx)^2} \times (2nx)^2}$$

$$= \frac{\frac{1}{2}}{\frac{1}{2}} \times \frac{m \times m}{n \times n} \left[\because \frac{1 - \cos \theta}{\theta \times \theta} = \frac{1}{2} \right]$$

$$= \frac{m^2}{n^2}$$

Q.no. 58.

Solutions: It is given that

$$\lim_{x \rightarrow 0} \frac{1 - \cos mx}{1 - \cos nx} = \lim_{x \rightarrow 0} \frac{\frac{1 - \cos mx}{mx \times mx} \times \frac{mx \times mx}{n \times n}}{\frac{1 - \cos nx}{nx \times nx} \times \frac{nx \times nx}{n \times n}} = \frac{m^2}{n^2}$$

Q.no. 59.

Solutions: It is given that

$$\lim_{x \rightarrow 0} \frac{\sin mx \times \sin mx}{\sin nx \times \sin nx} = \lim_{x \rightarrow 0} \frac{\frac{\sin mx \times \sin mx}{mx \times mx} \times \frac{mx \times mx}{n \times n}}{\frac{\sin nx \times \sin nx}{nx \times nx} \times \frac{nx \times nx}{n \times n}} \times \frac{m^2}{n^2} = \frac{m^2}{n^2}$$

Q.no. 60.

Solutions: It is given that

$$\lim_{x \rightarrow 0} \frac{\sin 2x \times \sin 3x}{2x \times \sin 3x} \times \frac{3x}{3x} = \lim_{x \rightarrow 0} \frac{\frac{\sin 2x \times \sin 3x}{2x \times \sin 3x} \times \frac{3x}{3x}}{\frac{3x}{3x}} =$$

Q.no. 61.

Solutions: It is given that

$$\lim_{x \rightarrow 0} \frac{\sec 4x - \sec 2x}{\sec 3x - \sec x} = \lim_{x \rightarrow 0} \frac{\frac{1}{\cos 4x} - \frac{1}{\cos 2x}}{\frac{1}{\cos 3x} - \frac{1}{\cos x}}$$

$$= \lim_{x \rightarrow 0} \frac{(\cos 2x - \cos 4x) \times \cos x \times \cos 3x}{(\cos x - \cos 3x) \times \cos 2x \times \cos 4x}$$

$$= \lim_{x \rightarrow 0} \frac{2 \times \sin 3x \times \sin x \times \cos x \times \cos 3x}{2 \times \sin 2x \times \sin x \times \cos 2x \times \cos 4x}$$

$$= \lim_{x \rightarrow 0} \frac{\sin 6x \times \cos x}{\sin 4x \times \cos 4x} \times \frac{2}{2}$$

$$= 2 \times \lim_{x \rightarrow 0} \frac{\sin 6x \times \cos x}{\sin 8x} \times \frac{8x}{6x} \times \frac{6}{8}$$

$$= \frac{3}{2} \times \lim_{x \rightarrow 0} \frac{\frac{\sin 6x}{6x} \times \cos x}{\frac{\sin 8x}{8x}} = \frac{3}{2} \times \frac{1 \times 1}{1} = \frac{3}{2}$$

Q.no. 62.

Solutions: It is given that

$$\begin{aligned}
& \lim_{x \rightarrow 0} \frac{\sqrt{2} - \sqrt{1 + \cos x}}{\sin x \times \sin x} \\
&= \lim_{x \rightarrow 0} \frac{\sqrt{2} - \sqrt{1 + \cos x}}{\sin x \times \sin x} \times \frac{\sqrt{2} + \sqrt{1 + \cos x}}{\sqrt{2} + \sqrt{1 + \cos x}} \\
&= \lim_{x \rightarrow 0} \frac{2 - (1 - \cos x)}{\sin x \times \sin x \times \sqrt{2} + \sqrt{1 + \cos x}} \\
&= \lim_{x \rightarrow 0} \frac{1 - \cos x}{2 \times \sin \frac{x}{2} \cos \frac{x}{2} \sin x (\sqrt{2} + \sqrt{1 + \cos x})} \\
&= \lim_{x \rightarrow 0} \frac{2 \times \sin \frac{x}{2} \times \sin \frac{x}{2}}{2 \times \sin \frac{x}{2} \cos \frac{x}{2} \sin x (\sqrt{2} + \sqrt{1 + \cos x})} \\
&= \lim_{x \rightarrow 0} \frac{\sin \frac{x}{2} \times \frac{1}{2}}{\frac{x}{2}} \times \lim_{x \rightarrow 0} \frac{1}{\cos \frac{x}{2} \times (\sqrt{2} + \sqrt{1 + \cos x})} \\
&= \frac{1}{2} \times \frac{1}{(\sqrt{2} + \sqrt{2})} = \frac{1}{4\sqrt{2}}
\end{aligned}$$

Q.no. 63.

Solutions: It is given that

$$\begin{aligned}
& \lim_{x \rightarrow 0} \frac{\sqrt{1 + \sin x} - \sqrt{1 - \sin x}}{x} \\
&= \lim_{x \rightarrow 0} \frac{\sqrt{1 + \sin x} - \sqrt{1 - \sin x}}{x} \times \frac{\sqrt{1 + \sin x} + \sqrt{1 - \sin x}}{\sqrt{1 + \sin x} + \sqrt{1 - \sin x}} \\
&= \lim_{x \rightarrow 0} \frac{2 \times \sin x}{x(\sqrt{1 + \sin x} + \sqrt{1 - \sin x})} \\
&= 2 \times \lim_{x \rightarrow 0} \frac{\sin x}{x} \times \lim_{x \rightarrow 0} \frac{1}{(\sqrt{1 + \sin x} + \sqrt{1 - \sin x})} \\
&= 2 \times 1 \times \frac{1}{2} = 1
\end{aligned}$$

Q.no. 64.

Solutions: It is given that

$$\begin{aligned}
& \lim_{x \rightarrow \frac{\pi}{6}} \frac{2 - \sqrt{3} \cos x - \sin x}{(6x - \pi)^2} = \lim_{y \rightarrow 0} \frac{2 - \sqrt{3} \cos \left(y + \frac{\pi}{6}\right) - \sin \left(y + \frac{\pi}{6}\right)}{y^2 \times 36} \\
&= \frac{1}{36} \times \lim_{y \rightarrow 0} \frac{2 - \frac{3}{2} \cos y + \frac{\sqrt{3}}{2} \sin y - \frac{1}{2} \cos y - \frac{\sqrt{3}}{2} \sin y}{y^2} \\
&= \frac{1}{36} \times \lim_{y \rightarrow 0} \frac{2(1 - \cos y)}{y^2} = 2 \times \frac{1}{2} \times \frac{1}{36} = \frac{1}{36}
\end{aligned}$$

Q.no. 65.

Solutions: It is given that

$$\begin{aligned}
 \lim_{x \rightarrow 0} \frac{\cos ax - \cos bx}{\cos cx - 1} &= \lim_{x \rightarrow 0} \frac{1 - \cos bx - (1 - \cos ax)}{\cos cx - 1} \\
 &= \lim_{x \rightarrow 0} \frac{\frac{(1 - \cos bx)}{(bx)^2} \times b^2 - \frac{(1 - \cos ax)}{(ax)^2} \times a^2}{\frac{-(1 - \cos cx)}{(cx)^2} \times c^2} = \frac{a^2 - b^2}{c^2} \\
 \therefore \lim_{x \rightarrow 0} \frac{\cos ax - \cos bx}{\cos cx - 1} &= \frac{a^2 - b^2}{c^2}
 \end{aligned}$$

Q.no. 66.

Solutions: It is given that

$$\begin{aligned}
 \lim_{x \rightarrow a} \frac{\cos x - \cos a}{\cot x - \cot a} &= \lim_{x \rightarrow a} \frac{(\cos x - \cos a)}{\frac{\sin(a - x)}{\sin x \sin a}} \\
 &= \sin a \times \lim_{x \rightarrow a} \frac{\sin\left(\frac{x + a}{2}\right) \times \sin x}{\cos\left(\frac{x - a}{2}\right)} = \sin 3a
 \end{aligned}$$

$$\therefore \lim_{x \rightarrow a} \frac{\cos x - \cos a}{\cot x - \cot a} = \sin a \times \sin a \times \sin a$$

Q.no. 67.

Solutions: It is given that

$$\begin{aligned}
&= \lim_{x \rightarrow \frac{\pi}{4}} \frac{\left(\frac{-\sin x (\cos 2x)}{\cos x \cos x \cos x} \right)}{\left(\frac{\cos x - \sin x}{\sqrt{2}} \right)} = -\sqrt{2} \times \lim_{x \rightarrow \frac{\pi}{4}} \frac{\sin x (\cos x + \sin x)}{\cos x \times \cos x \times \cos x} \\
&= -\sqrt{2} \times \frac{\frac{1}{\sqrt{2}} \times \sqrt{2}}{\left(\frac{1}{\sqrt{2}} \right)^3} = -4
\end{aligned}$$

Q.no. 68.

Solutions: It is given that

$$\begin{aligned}
&\lim_{x \rightarrow \frac{\pi}{2}} \frac{\sqrt{2} - \sqrt{1 + \sin x}}{\sqrt{2} \cos x \cos x} \\
&= \lim_{x \rightarrow \frac{\pi}{2}} \frac{\sqrt{2} - \sqrt{1 + \sin x}}{\sqrt{2} \cos x \cos x} \times \frac{\sqrt{2} + \sqrt{1 + \sin x}}{\sqrt{2} + \sqrt{1 + \sin x}} \\
&= \lim_{x \rightarrow \frac{\pi}{2}} \frac{1 - \sin x}{\sqrt{2} + \sqrt{1 + \sin x} (\sqrt{2} \cos x \cos x)}
\end{aligned}$$

$$\text{Let, } y = x - \frac{\pi}{2}$$

$$\begin{aligned}
&= \lim_{y \rightarrow 0} \frac{1 - \cos y}{\sqrt{2} + \sqrt{1 + \cos y} (\sqrt{2} \sin y \sin y)} \\
&= \frac{1}{2\sqrt{2}} \times \frac{1}{2\sqrt{2}} = \frac{1}{8}
\end{aligned}$$

Q.no. 69.**Solutions:** It is given that

$$\begin{aligned}
 & \lim_{x \rightarrow \frac{\pi}{6}} \frac{\cot x \times \cot x - 3}{\csc x - 2} \\
 &= \lim_{x \rightarrow \frac{\pi}{6}} \frac{(\cos x \times \cos x) - 3 \times \sin x \times \sin x}{\sin x (1 - 2 \sin x)} \\
 &= \lim_{x \rightarrow \frac{\pi}{6}} \frac{(1 - 2 \sin x) \times (1 + 2 \sin x)}{\sin x (1 - 2 \sin x)} = 4
 \end{aligned}$$

Q.no. 70.**Solutions:** It is given that

$$\begin{aligned}
 & \lim_{x \rightarrow \pi} \frac{\sqrt{2 + \cos x} - 1}{(\pi - x)^2} = \lim_{x \rightarrow \pi} \frac{\sqrt{2 + \cos x} - 1}{(\pi - x)^2} \times \frac{\sqrt{2 + \cos x} + 1}{\sqrt{2 + \cos x} + 1} \\
 &= \lim_{x \rightarrow \pi} \frac{1 + \cos x}{(\pi - x)^2} \times \frac{1}{\sqrt{2 + \cos x} + 1}
 \end{aligned}$$

Let, $y = x - \pi$

$$= \lim_{y \rightarrow 0} \frac{1 - \cos y}{y^2 \times \sqrt{2 - \cos y} + 1} = \frac{1}{4}$$

Q.no. 71.

Solutions: It is given that

$$\lim_{x \rightarrow \frac{\pi}{4}} \frac{1 - \tan x}{1 - \sqrt{2} \sin x}$$

By Applying **L'Hospital's Rule**

$$\begin{aligned} &= \lim_{x \rightarrow \frac{\pi}{4}} \frac{\frac{d}{dx}(1 - \tan x)}{\frac{d}{dx}(1 - \sqrt{2} \sin x)} \\ &= \lim_{x \rightarrow \frac{\pi}{4}} \frac{-\sec^2 x}{\sqrt{2} \cos x} = \frac{1}{\sqrt{2}} \lim_{x \rightarrow \frac{\pi}{4}} \sec^3 x \\ &= \frac{1}{\sqrt{2}} \sec^3 \frac{\pi}{4} = \frac{1}{\sqrt{2}} \cdot (\sqrt{2})^3 = 2 \end{aligned}$$

Q.no. 72.

Solutions: It is given that

$$\begin{aligned} &\lim_{x \rightarrow \frac{\pi}{2}} \frac{2 \sin^2 x + \sin x - 1}{2 \sin^2 x - 3 \sin x + 1} = \\ &\lim_{x \rightarrow \frac{\pi}{6}} \frac{2 \times \sin x \times \sin x + \sin x - 1}{2 \times \sin x \times \sin x - 3 \sin x + 1} \\ &\lim_{x \rightarrow \frac{\pi}{6}} \frac{(2 \sin x - 1) \times (\sin x + 1)}{(2 \sin x - 1)(\sin x - 1)} = -3 \end{aligned}$$

Exercise 27 C

Q.no. 1.

Solutions:

$$\text{L.H.L : } \lim_{x \rightarrow 3^-} f(x) = \lim_{x \rightarrow 3^-} |x| - 3 = \lim_{x \rightarrow 3^-} -(x - 3) = -(3 - 3) = 0$$

$$\text{R.H.L : } \lim_{x \rightarrow 3^+} f(x) = \lim_{x \rightarrow 3^+} |x| - 3 = \lim_{x \rightarrow 3^+} (x - 3) = 3 - 3 = 0$$

Since,

$$\lim_{x \rightarrow 3^-} f(x) = \lim_{x \rightarrow 3^+} f(x)$$

So, the limit exists and $\lim_{x \rightarrow 3} f(x) = 0$

Q.no. 2.

Solutions:

$$\text{L.H.L: } \lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} \frac{x}{|x|} = \lim_{x \rightarrow 0^-} \frac{x}{(-x)} = \lim_{x \rightarrow 0^-} -1 = -1$$

$$\text{R.H.L: } \lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} \frac{x}{|x|} = \lim_{x \rightarrow 0^+} \frac{x}{(+x)} = \lim_{x \rightarrow 0^+} 1 = 1$$

Now, $\lim_{x \rightarrow 0^-} f(x) \neq \lim_{x \rightarrow 0^+} f(x)$, $\therefore \lim_{x \rightarrow 0} f(x)$ does not exist

Q.no. 3.

Solutions:

$$\text{L.H.L: } \lim_{x \rightarrow 3^-} f(x) = \lim_{x \rightarrow 3^-} \frac{|x-3|}{x-3} \lim_{x \rightarrow 3^-} \frac{-(x-3)}{x-3} = \lim_{x \rightarrow 3^-} -1 = -1$$

$$\text{R.H.L: } \lim_{x \rightarrow 3^+} f(x) = \lim_{x \rightarrow 3^+} \frac{|x-3|}{x-3} = \lim_{x \rightarrow 3^+} \frac{(x-3)}{x-3} = \lim_{x \rightarrow 3^+} 1 = 1$$

Now

$\lim_{x \rightarrow 3^-} f(x) \neq \lim_{x \rightarrow 3^+} f(x)$ Therefore, $\lim_{x \rightarrow 3} f(x)$ does not exist.

Q.no. 4.

Solutions:

L.H.L:

$$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} 1 + x^2 = 2$$

R.H.L:

$$\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} 2 - x = 1$$

$$\lim_{x \rightarrow 1^-} f(x) \neq \lim_{x \rightarrow 1^+} f(x)$$

Therefore, Limit does not exist

Q.no. 5.

Solutions:

L.H.L:

$$\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} \frac{x - |x|}{x} = \lim_{x \rightarrow 0^-} \frac{x - (-x)}{x} = \lim_{x \rightarrow 0^-} \frac{x + x}{x}$$

$$= \lim_{x \rightarrow 0^-} \frac{2x}{x} = \lim_{x \rightarrow 0^-} 2 = 2$$

R.H.L:

$$\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} \frac{x - |x|}{x} = \lim_{x \rightarrow 0^+} \frac{x - (x)}{x} = \lim_{x \rightarrow 0^+} \frac{0}{x} = \lim_{x \rightarrow 0^+} 0 = 0$$

$$\lim_{x \rightarrow 0^-} f(x) \neq \lim_{x \rightarrow 0^+} f(x)$$

Therefore, Limit does not exist

Q.no. 6.

Solutions:

$$\text{L.H.L: } \lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} 5x - 4 = 5 - 4 = 1$$

R.H.L:

$$\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} 4x^3 - 3x = 4 - 3 = 1$$

$$\therefore \lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^+} f(x), \text{ Therefore,, } \lim_{x \rightarrow 1} f(x) = 1$$

Q.no. 7.

Solutions:

$$\text{L.H.L: } \lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^-} 4x - 5 = 8 - 5 = 3$$

$$\text{R.H.L: } \lim_{x \rightarrow 2^+} f(x) = \lim_{x \rightarrow 2^+} x - a = 2 - a$$

Since $\lim_{x \rightarrow 2} f(x)$ exists,

$$\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^+} f(x)$$

$$\Rightarrow 3 = 2 - a \rightarrow a = 2 - 3 \rightarrow a = -1$$

Q.no. 8.

Solutions:

L.H.L:

$$\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} \frac{3x}{|x| + 2x} = \lim_{x \rightarrow 0^-} \frac{3x}{(-x) + 2x} = \lim_{x \rightarrow 0^-} \frac{3x}{x} = \lim_{x \rightarrow 0^-} 3 = 3$$

R.H.L:

$$\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} \frac{3x}{|x| + 2x} = \lim_{x \rightarrow 0^+} \frac{3x}{(x) + 2x} = \lim_{x \rightarrow 0^+} \frac{3x}{3x} = \lim_{x \rightarrow 0^+} 1 = 1$$

Since

$$\lim_{x \rightarrow 0^-} f(x) \neq \lim_{x \rightarrow 0^+} f(x)$$

Therefore, Limit does not exist

Q.no. 9.

Solutions:

L.H.L:

$$\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} x + k = 0 + k = k$$

R.H.L:

$$\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} \cos x = \cos(0) = 1$$

It is given that $\lim_{x \rightarrow 0} f(x)$ exists.

Therefore,

$$\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^+} f(x)$$

$$\Rightarrow k = 1$$

Q.no. 10.

Solutions:

R.H.L:

$$\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} \frac{1}{x}$$

$$= \lim_{x \rightarrow 0^+} \frac{1}{0+h} = \lim_{x \rightarrow 0^+} \frac{1}{+h} = +\frac{1}{0} = +\infty$$

L.H.L:

$$\lim_{x \rightarrow 0^-} f(x)$$

$$\lim_{x \rightarrow 0^-} \frac{1}{x} = \lim_{h \rightarrow 0^-} \frac{1}{0+h} = \lim_{h \rightarrow 0^-} \frac{1}{-h} = -\frac{1}{0} = -\infty$$

Since,

$$\lim_{x \rightarrow 0^-} f(x) \neq \lim_{x \rightarrow 0^+} f(x)$$

Therefore, Limit does not exist

Q.no. 11.

Solutions:

R.H.L:

$$\lim_{x \rightarrow 0^+} f(x)$$

$$\lim_{x \rightarrow 0^+} \frac{1}{|x|} = \lim_{x \rightarrow 0^+} \frac{1}{(x)} = \lim_{h \rightarrow 0^+} \frac{1}{(0+h)} = \frac{1}{0} = \infty$$

L.H.L:

$$\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} \frac{1}{|x|} = \lim_{x \rightarrow 0^-} \frac{1}{(-x)} = \lim_{h \rightarrow 0^-} \frac{1}{-(0-h)} = \lim_{h \rightarrow 0^-} \frac{1}{h}$$

$$= \frac{1}{0} = -\infty$$

Therefore,

$$\lim_{x \rightarrow 0^-} f(x) \neq \lim_{x \rightarrow 0^+} f(x) \quad \therefore \lim_{x \rightarrow 0} \frac{1}{|x|} = \infty$$

Q.no. 12.**Solutions:****L.H.L:**

$$\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} e^{\frac{-1}{(-x)}} = \lim_{x \rightarrow 0^-} e^{\frac{1}{x}} = e^{\frac{1}{0^-}}$$

$$= e^{\infty}$$

$$\lim_{x \rightarrow 0^+} f(x)$$

R.H.L:

$$\lim_{x \rightarrow 0^+} e^{\frac{-1}{x}} = e^{\frac{-1}{0^+}} = e^{-\infty} = e^{\frac{1}{\infty}}$$

Since

$$\lim_{x \rightarrow 0^-} f(x) \neq \lim_{x \rightarrow 0^+} f(x)$$

Therefore, Limit does not exist

Q.no. 13.**Solutions:****R.H.L:**

$$\lim_{x \rightarrow 0^+} f(x)$$

$$= \lim_{x \rightarrow 0^+} \sin \frac{1}{x} = \lim_{h \rightarrow 0^+} \sin \frac{1}{0+h} = \sin \frac{1}{0} = \sin \infty = \infty$$

L.H.L :

$$= \lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} \sin \frac{1}{x} = \lim_{h \rightarrow 0^-} \sin \frac{1}{0-h}$$

$$= \sin \frac{1}{-0} = -\sin \frac{1}{0} = -\sin \infty = -\infty$$

Since,

$$= \lim_{x \rightarrow 0^-} f(x) \neq \lim_{x \rightarrow 0^+} f(x)$$

Therefore, Limit does not exist

Q.no. 14.

Solutions:

L.H.L.:

$$= \lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^+} \frac{x}{|x|} = \lim_{x \rightarrow 0^-} \frac{x}{(-x)} = \lim_{x \rightarrow 0^-} -1 = -1$$

R.H.L:

$$= \lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} \frac{x}{|x|} = \lim_{x \rightarrow 0^+} \frac{x}{(x)} = \lim_{x \rightarrow 0^+} 1 = 1$$

Since

$$= \lim_{x \rightarrow 0^-} f(x) \neq \lim_{x \rightarrow 0^+} f(x)$$

Therefore, Limit does not exist.

Q.no. 15.

Solutions:

It is given that

$$f(x) = \begin{cases} \frac{k \cos x}{\pi - 2x}, & x \neq \frac{\pi}{2} \\ 3, & x = \frac{\pi}{2} \end{cases}$$

$$\lim_{x \rightarrow \frac{\pi}{2}} f(x) = \lim_{x \rightarrow \frac{\pi}{2}} \frac{k \cos x}{\pi - 2x}$$

$$\text{Let } h = x - \frac{\pi}{2} \Rightarrow x = h + \frac{\pi}{2}$$

$$x \rightarrow \frac{\pi}{2} \text{ or, } h + \frac{\pi}{2} \rightarrow \frac{\pi}{2}$$

$$\text{or, } h \rightarrow 0$$

$$= \lim_{h \rightarrow 0} \frac{k \cos\left(\frac{\pi}{2} + h\right)}{\pi - 2\left(\frac{\pi}{2} + h\right)} = \lim_{h \rightarrow 0} \frac{-k \sinh}{\pi - \pi + h} = \lim_{h \rightarrow 0} \frac{-k \sinh}{h} = -k \times 1 = -k$$

$$f\left(\frac{\pi}{2}\right) = 3$$

NOW,

$$\lim_{x \rightarrow \frac{\pi}{2}} f(x) = f\left(\frac{\pi}{2}\right)$$

$$\therefore -k = 3 \rightarrow k = -3$$