ML Aggarwal Solutions for Class 10
Maths
Chapter 5 Quadratic Equations in One variable

Exercise 5.1

1. In each of the following, determine whether the given numbers are roots of the given equations or not:

(i) $x^2 - 5x + 6 = 0 : 2, -3$

Ans: Given the quadratic equation $x^2 - 5x + 6 = 0$

Now in order to check whether the given numbers are the roots of the given quadratic equation, we will replace (x) with the given numbers and check whether the result is equal to 0 or not.

That is, $x^2 - 5x + 6 = 0$

First substitute $x = 2$ in the given equation we get,

$x^2 - 5x + 6 = 0$

$\Rightarrow (2)^2 - 5(2) + 6 = 0$

$\Rightarrow 4 - 10 + 6 = 0$

$\Rightarrow -6 + 6 = 0$

$\Rightarrow 0 = 0$

$LHS = RHS$

Now substitute $x = -3$ in the given equation we get,

$x^2 - 5x + 6 = 0$

$\Rightarrow (-3)^2 - 5(-3) + 6 = 0$

$\Rightarrow 9 + 15 + 6 = 0$

$\Rightarrow 24 + 6 = 0$

$=> 30 \neq 0$

$LHS \neq RHS$

Therefore, we can see that when we substitute $x = 2$ in the equation we get the value to be 0, but $x = -3$ being substituted in the equation gives x not equal to 0.

By seeing this, we can conclude that $x = 2$ is the root of the equation $x^2 - 5x + 6 = 0$ and $x = -3$ is not the root of the given equation.

(ii) $3x^2 - 13x - 10 = 0 : 5, \frac{-2}{3}$

Ans: Given the quadratic equation $3x^2 - 13x - 10 = 0$.

In order to check whether the given numbers are the roots of the given quadratic equation, we will replace (x) with the given numbers and check whether the result is equal to 0 or not.

That is, $3x^2 - 13x - 10 = 0$

First substitute $x = 5$ in the given equation we get,

$3x^2 - 13x - 10 = 0$

$\Rightarrow 3(5)^2 - 13(5) - 10 = 0$

$\Rightarrow 3 \times 25 - 65 - 10 = 0$

$\Rightarrow 75 - 65 - 10 = 0$

$\Rightarrow 10 - 10 = 0$

$\Rightarrow 0 = 0$

$LHS = RHS$

Now substitute $x = \frac{-2}{3}$ in the given equation we get,

$3x^2 - 13x - 10 = 0$

$\Rightarrow 3(\frac{-2}{3})^2 - 13(\frac{-2}{3}) - 10 = 0$

$\Rightarrow 3 \times \frac{4}{9} + \frac{26}{3} - 10 = 0$

$\Rightarrow \frac{4}{3} + \frac{26}{3} - 10 = 0$

$\Rightarrow \frac{30}{3} - 10 = 0$

$\Rightarrow 10 - 10 = 0$

$\Rightarrow 0 = 0$

$LHS = RHS$

Therefore, we can see that when we substitute $x = 5$ and $x = \frac{-2}{3}$ in the equation we get the value to be 0.

By seeing this, we can conclude that $x=5$ and $x=\frac{-2}{3}$ are the roots of the equation $3x^2-13x-10=0$.

2. In each of the following, determine whether the given numbers are solutions of the given equations or not:

(i) $x^2-3\sqrt{3}x+6=0; x=\sqrt{3},-2\sqrt{3}$

Ans: If we have any quadratic equation like the above, $x^2-3\sqrt{3}x+6=0$ we can check whether a given value of x is the solution of the quadratic equation by just substituting the value in the equation and finding whether it is equal to 0 or not.

Given the equation: $x^2-3\sqrt{3}x+6=0$

First substitute $x=\sqrt{3}$ in the given equation and simplifying we get,

$x^2-3\sqrt{3}x+6=0$

$\Rightarrow (\sqrt{3})^2-3\sqrt{3}(\sqrt{3})+6=0$

$\Rightarrow 3-3\times 3+6=0$

$\Rightarrow 3-9+6=0$

$\Rightarrow -6+6=0$

$\Rightarrow 0=0$

$LHS=RHS$

Now substitute $x=-2\sqrt{3}$ in the given equation and simplifying we get,

$x^2-3\sqrt{3}x+6=0$

$\Rightarrow (-2\sqrt{3})^2-3\sqrt{3}(-2\sqrt{3})+6=0$

$\Rightarrow 4(3)+3\times 3\times 2+6=0$

$\Rightarrow 12+18+6=0$

$\Rightarrow 30+6=0$

$\Rightarrow 36\neq 0$

$LHS\neq RHS$

Now we can observe that the value of x at $x=\sqrt{3}$ gives the value of the equation to be 0 but not $x=-\sqrt{2}$.

Therefore, we can conclude that $x=\sqrt{3}$ is the solution of the quadratic equation $x^2-3\sqrt{3}x+6=0$ and $x=-2\sqrt{3}$ is not the solution of it.

(ii) $x^2-\sqrt{2}x-4=0;x=-\sqrt{2},2\sqrt{2}$

Ans: If we have any quadratic equation like the above, $x^2-\sqrt{2}x-4=0$, we can check whether a given value of x is the solution of the quadratic equation by just substituting the value in the equation and finding whether it is equal to 0 or not.

Given the equation: $x^2-\sqrt{2}x-4=0$

First substitute $x=-\sqrt{2}$ in the given equation and simplifying we get,

$x^2-\sqrt{2}x-4=0$

$\Rightarrow (-\sqrt{2})^2-\sqrt{2}(-\sqrt{2})-4=0$

$\Rightarrow 2+2-4=0$

$\Rightarrow 4-4=0$

$\Rightarrow 0=0$

$LHS=RHS$

Now substitute $x=2\sqrt{2}$ in the given equation and simplifying we get,

$x^2-\sqrt{2}x-4=0$

$\Rightarrow (2\sqrt{2})^2-\sqrt{2}(2\sqrt{2})-4=0$

$\Rightarrow 4\times 2-2\times 2-4=0$

$\Rightarrow 8-4-4=0$

$\Rightarrow 8-8=0$

$\Rightarrow 0=0$

$LHS=RHS$

Now we can observe that the value of x at both $x=-\sqrt{2}$ and $x=2\sqrt{2}$ gives the value of the equation to be 0.

Therefore, we can conclude that both $x=-\sqrt{2}$ and $x=2\sqrt{2}$ are the solution of the quadratic equation $x^2-\sqrt{2}x-4=0$.

3. (i) If $\frac{-1}{2}$ is the solution of the equation $3x^2+2kx-3=0$, find the value of k.

Ans: Given the quadratic equation $3x^2+2kx-3=0$ and a solution for this equation $\frac{-1}{2}$.

We need to find the value of k in the equation present. This can be found by substituting the value of the solution in the given equation and simplifying the equation.

That is, first we have $3x^2+2kx-3=0$

Substituting the value $\frac{-1}{2}$ in the quadratic equation gives,

$$3x^2+2kx-3=0$$

$$\Rightarrow 3(\frac{-1}{2})^2+2k(\frac{-1}{2})-3=0$$

$$\Rightarrow 3\times\frac{1}{4}-k-3=0$$

$$\Rightarrow \frac{3}{4}=k+3$$

Solving the equation and finding the value of k we get,

$$\frac{3}{4}=k+3$$

$$k=\frac{3}{4}-3=\frac{3-12}{4}=\frac{-9}{4}$$

$$\therefore, k=\frac{-9}{4}$$

Hence after simplification, we get the value of k is $\frac{-9}{4}$.

(ii) If $\frac{2}{3}$ is the solution of the equation $7x^2+kx-3=0$, find the value of k.

Ans: Given the quadratic equation $7x^2+kx-3=0$ and a solution for this equation $\frac{2}{3}$.

We need to find the value of k in the equation present. This can be found by substituting the value of the solution in the given equation and simplifying the equation.

That is, first we have $7x^2+kx-3=0$.

Substituting the value $\frac{2}{3}$ in the quadratic equation gives,

$7x^2 + kx - 3 = 0$

$\Rightarrow 7(\frac{2}{3})^2 + k(\frac{2}{3}) - 3 = 0$

$\Rightarrow 7(\frac{4}{9}) + k(\frac{2}{3}) - 3 = 0$

$\Rightarrow \frac{28}{9} + \frac{2k}{3} = 3$

Taking LCM and solving the equation for k we get,

$\frac{28}{9} + \frac{2k}{3} = 3$

$\Rightarrow \frac{28 + 6k}{9} = 3$

$\Rightarrow 28 + 6k = 27$

$\Rightarrow 6k = 27 - 28$

$\Rightarrow 6k = -1$

$\Rightarrow k = \frac{-1}{6}$

Hence after simplification, we get the value of k is $\frac{-1}{6}$.

4. (i) If $\sqrt{2}$ is the root of the equation $kx^2 + \sqrt{2}x - 4 = 0$, find the value of k.

Ans: Given a quadratic equation like $kx^2 + \sqrt{2}x - 4 = 0$, where we don't know the value of k but we have a root of the equation given, we can find the value of k by just substituting the value of the root to the given quadratic equation and just solving it to get the value of k.

So, we have the equation $kx^2 + \sqrt{2}x - 4 = 0$; to find the value of k we substitute the root given $\sqrt{2}$ in the given equation as follows:

$kx^2 + \sqrt{2}x - 4 = 0$

$\Rightarrow k(\sqrt{2})^2 + \sqrt{2}(\sqrt{2}) - 4 = 0$

$\Rightarrow 2k + 2 - 4 = 0$

$\Rightarrow 2k - 2 = 0$

On simplifying further, the equation, we get:

$2k - 2 = 0$

$\Rightarrow 2k = 2$

$\Rightarrow k = \frac{2}{2} = 1$

$\therefore k = 1$

Hence, the value of k in the equation $kx^2 + \sqrt{2}x - 4 = 0$ is 1.

(ii) If "a" is the root of the equation $x^2 - x(a+b) + k = 0$, find the value of k.

Ans: Given a quadratic equation like $x^2 - x(a+b) + k = 0$, where we don't know the value of k but we have a root of the equation given, we can find the value of k by just substituting the value of the root to the given quadratic equation and just solving it to get the value of k.

So, we have the equation $x^2 - x(a+b) + k = 0$; to find the value of k we substitute the root given "a" in the given equation as follows:

$x^2 - x(a+b) + k = 0$

$\Rightarrow a^2 - a(a+b) + k = 0$

$\Rightarrow a^2 - a^2 - ab + k = 0$

$\Rightarrow -ab + k = 0$

On simplifying further the equation, we get:

$-ab + k = 0$

$\Rightarrow k = ab$

Hence, the value of k in the equation $x^2 - x(a+b) + k = 0$ is ab.

5. If $\frac{2}{3}$ and -3 are the roots of the equation $px^2 + 7x + q = 0$, find the values of p and q.

Ans: Given the equation: $px^2 + 7x + q = 0$

Also, it is given that $\frac{2}{3}$ **and** -3 are the roots of the given equation.

First substitute $x = \frac{2}{3}$ in the given equation and simplifying we get,

$px^2 + 7x + q = 0$

$$\Rightarrow p(\frac{2}{3})^2 + 7(\frac{2}{3}) + q = 0$$

$$\Rightarrow p(\frac{4}{9}) + \frac{14}{3} + q = 0$$

$$\Rightarrow \frac{4p}{9} + \frac{14}{3} + q = 0$$

Further on taking LCM and simplifying it we get,

$$\frac{4p}{9} + \frac{14}{3} + q = 0$$

$$\Rightarrow \frac{4p + 42 + 9q}{9} = 0$$

$$\Rightarrow 4p + 9q + 42 = 0$$

$$\Rightarrow 4p + 9q = -42......(1)$$

Thus we got an equation for the first root, similarly simplifying for $x = -3$ by substituting in the quadratic equation we get,

$$px^2 + 7x + q = 0$$

$$\Rightarrow p(-3)^2 + 7(-3) + q = 0$$

$$\Rightarrow 9p - 21 + q = 0$$

$$\Rightarrow 9p + q = 21.....(2)$$

Now we got a simultaneous linear equation in (1) and (2), simplifying it we will get the values of p and q. Now solving them by substituting for q in equation (2) to equation (1) as follows:

$$9p + q = 21$$

$$\Rightarrow q = 21 - 9p$$

Substitute this in equation (1) we get:

$$4p + 9q = -42$$

$$\Rightarrow 4p + 9(21 - 9p) = -42$$

$$\Rightarrow 4p + 189 - 81p = -42$$

$$\Rightarrow 189 - 77p = -42$$

$$\Rightarrow 189 + 42 = 77p$$

$$\Rightarrow 231 = 77p$$

$$\Rightarrow p = \frac{231}{77} = 3$$

$\therefore p=3$

Now we got the value of p, substitute in any equation (1) or (2) to get q as:

$9p+q=21$

$\Rightarrow q=21-9p$

$\Rightarrow q=21-9(3)$

$\Rightarrow q=21-27=-6$

$\therefore q=-6$

Hence the values of p and q in the equation $px^2+7x+q=0$ are 3 and -6 respectively.

Exercise 5.2

Solve the following questions (1 to 18) by factorization:

1. (i) $x^2-3x-10=0$

Ans: The given expression is $x^2-3x-10=0$.

Let us simplify the given expression with the help of factorization method.

$x^2-3x-10=0$

$\Rightarrow x^2-5x+2x-10=0$

$\Rightarrow x(x-5)+2(x-5)=0$

$\Rightarrow (x+2)(x-5)=0$

After further simplification,

$(x+2)=0$ Or $(x-5)=0$

$\Rightarrow x=-2$ Or $x=5$

Therefore $-2,5$ are the values of x.

(ii) $x(2x+5)=3$

Ans: The given expression is $x(2x+5)=3$.

Let us simplify the given expression with the help of factorization method.

First, expand the bracket,

$x(2x+5)=3$

$\Rightarrow 2x^2+5x=3$

$\Rightarrow 2x^2+5x-3=0$

Now, factorize the expression by splitting the terms

$2x^2+5x-3=0$

$\Rightarrow 2x^2+6x-x-3=0$

$\Rightarrow 2x(x+3)-1(x+3)=0$

$\Rightarrow (2x-1)(x+3)=0$

After further simplification,

$(2x-1)=0$ Or $(x+3)=0$

$\Rightarrow 2x=1$ Or $x=-3$

$\Rightarrow x=\frac{1}{2}$ Or $x=-3$

Therefore $\frac{1}{2},-3$ are the values of x.

2. (i) $3x^2-5x-12=0$

Ans: The given expression is $3x^2-5x-12=0$.

Let us simplify the given expression with the help of factorization method.

$3x^2-5x-12=0$

$\Rightarrow 3x^2-9x+4x-12=0$

$\Rightarrow 3x(x-3)+4(x-3)=0$

$\Rightarrow (3x+4)(x-3)=0$

After further simplification,

$(3x+4)=0$ Or $(x-3)=0$

$\Rightarrow 3x=-4$ Or $x=3$

$\Rightarrow x=\frac{-4}{3}$ Or $x=3$

Therefore $\frac{-4}{3},3$ are the values of x.

(ii) $21x^2-8x-4=0$

Ans: The given expression is $21x^2-8x-4=0$.

Let us simplify the given expression with the help of factorization method.

$21x^2 - 14x + 6x - 4 = 0$

$\Rightarrow 7x(3x-2) + 2(3x-2) = 0$

$\Rightarrow (7x+2)(3x-2) = 0$

After further simplification,

$(7x+2) = 0$ Or $(3x-2) = 0$

$\Rightarrow 7x = -2$ Or $3x = 2$

$\Rightarrow x = \frac{-2}{7}$ Or $x = \frac{2}{3}$

Therefore $\frac{-2}{7}, \frac{2}{3}$ are the values of x.

3. (i) $3x^2 = x + 4$

Ans: The given expression is $3x^2 = x + 4$.

Let us simplify the given expression with the help of factorization method.

For this, first shift the RHS values into the LHS,

$3x^2 - x - 4 = 0$

$\Rightarrow 3x^2 - 4x + 3x - 4 = 0$

$\Rightarrow x(3x-4) + 1(3x-4) = 0$

$\Rightarrow (x+1)(3x-4) = 0$

After further simplification,

$(x+1) = 0$ Or $(3x-4) = 0$

$\Rightarrow x = -1$ Or $3x = 4$

$\Rightarrow x = -1$ Or $x = \frac{4}{3}$

Therefore $-1, \frac{4}{3}$ are the values of x.

(ii) $x(6x-1) = 35$

Ans: The given expression is $x(6x-1) = 35$.

Let us simplify the given expression with the help of factorization method.

For this, first expand the bracket

$6x^2 - x = 35$

$\Rightarrow 6x^2 - x - 35 = 0$

Now, factorize the expression by splitting the terms

$6x^2 - 15x + 14x - 35 = 0$

$\Rightarrow 3x(2x-5) + 7(2x-5) = 0$

$\Rightarrow (3x+7)(2x-5) = 0$

After further simplification,

$(3x+7) = 0$ Or $(2x-5) = 0$

$\Rightarrow 3x = -7$ Or $2x = 5$

$\Rightarrow x = \frac{-7}{3}$ Or $x = \frac{5}{2}$

Therefore $\frac{-7}{3}, \frac{5}{2}$ are the values of x.

4. (i) $6p^2 + 11p - 10 = 0$

Ans: The given expression is $6p^2 + 11p - 10 = 0$.

Let us simplify the given expression with the help of factorization method.

$6p^2 + 15p - 4p - 10 = 0$

$\Rightarrow 3p(2p+5) - 2(2p+5) = 0$

$\Rightarrow (3p-2)(2p+5) = 0$

After further simplification,

$(3p-2) = 0$ Or $(2p+5) = 0$

$\Rightarrow 3p = 2$ Or $2p = -5$

$\Rightarrow p = \frac{2}{3}$ Or $x = \frac{-5}{2}$

Therefore $\frac{2}{3}, \frac{-5}{2}$ are the values of p.

(ii) $\frac{2}{3}x^2 - \frac{1}{3}x = 1$

Ans: The given expression is $\frac{2}{3}x^2-\frac{1}{3}x=1$.

Let us simplify the given expression with the help of factorization method.

For this, first take the LCM of the terms

$$\frac{2}{3}x^2-\frac{1}{3}x=1$$

$$\Rightarrow \frac{2x^2-x}{3}=1$$

$$\Rightarrow 2x^2-x=3$$

$$\Rightarrow 2x^2-x-3=0$$

Now, factorize the expression

$$2x^2-x-3=0$$

$$\Rightarrow 2x^2-3x+2x-3=0$$

$$\Rightarrow x(2x-3)+1(2x-3)=0$$

$$\Rightarrow (x+1)(2x-3)=0$$

After further simplification,

$(x+1)=0$ Or $(2x-3)=0$

$\Rightarrow x=-1$ Or $2x=3$

$\Rightarrow x=-1$ Or $x=\frac{3}{2}$

Therefore $-1,\frac{3}{2}$ are the values of x.

5. (i) $3(x-2)^2=147$

Ans: The given expression is $3(x-2)^2=147$.

Let us simplify the given expression with the help of factorization method.

For this, first expand the bracket by using the $(a-b)^2=a^2-2ab+b^2$ identity.

$$3(x^2-4x+4)=147$$

$$\Rightarrow 3x^2-12x+12=147$$

$$\Rightarrow 3x^2-12x+12-147=0$$

$\Rightarrow 3x^2 - 12x - 135 = 0$

Now, divide the above expression by 3

$$\frac{3x^2 - 12x - 135}{3} = 0$$

$\Rightarrow x^2 - 4x - 45 = 0$

Further, factorize the expression by splitting the terms

$x^2 - 9x + 5x - 45 = 0$

$\Rightarrow x(x-9) + 5(x-9) = 0$

$\Rightarrow (x+5)(x-9) = 0$

$\Rightarrow (x+5) = 0$ Or $(x-9) = 0$

$\Rightarrow x = -5$ Or $x = 9$

Therefore $-5, 9$ are the values of *x*.

(ii) $\frac{1}{7}(3x-5)^2 = 28$

Ans: The given expression is $\frac{1}{7}(3x-5)^2 = 28$

Let us simplify the given expression with the help of factorization method.

For this, first expand the bracket by using the $(a-b)^2 = a^2 - 2ab + b^2$ identity.

$(3x-5)^2 = 28 \times 7$

$\Rightarrow (3x-5)^2 = 196$

$\Rightarrow 9x^2 - 30x + 25 - 196 = 0$

$\Rightarrow 9x^2 - 30x - 171 = 0$

Now, divide the above expression by 3

$$\frac{9x^2 - 30x - 171}{3} = 0$$

$\Rightarrow 3x^2 - 10x - 57 = 0$

Further, factorize the expression by splitting the terms

$3x^2 - 19x + 9x - 57 = 0$

$\Rightarrow x(3x-19) + 3(3x-19) = 0$

$\Rightarrow (x+3)(3x-19)=0$

$\Rightarrow (x+3)=0$ Or $(3x-19)=0$

$\Rightarrow x=-3$ Or $3x=19$

$\Rightarrow x=-3$ Or $x=\frac{19}{3}$

Therefore $-3,\frac{19}{3}$ are the values of *x*.

6. $x^2-4x-12=0$**, when** $x \in \mathrm{N}$

Ans: The given expression is $x^2-4x-12=0$

Let us simplify the given expression with the help of factorization method.

$x^2-4x-12=0$

$\Rightarrow x^2-6x+2x-12=0$

$\Rightarrow x(x-6)+2(x-6)=0$

$\Rightarrow (x+2)(x-6)=0$

After further simplification,

$(x+2)=0$ Or $(x-6)=0$

$\Rightarrow x=-2$ Or $x=6$

$\therefore\ x=6$ (Because, -2 is not a natural number)

Therefore, 6 is the value of *x*.

7. $2x^2-9x+10=0$**, when**

(i) $x \in \mathrm{N}$

Ans: The given expression is $2x^2-9x+10=0$.

Let us simplify the given expression with the help of factorization method.

$2x^2-9x+10=0$

$\Rightarrow 2x^2-4x-5x+10=0$

$\Rightarrow 2x(x-2)-5(x-2)=0$

$\Rightarrow (2x-5)(x-2)=0$

After further simplification,

$(2x-5)=0$ Or $(x-2)=0$

$\Rightarrow 2x = 5$ Or $x = 2$

$\Rightarrow x = \frac{5}{2}$ Or $x = 2$

Hence, the value of x is 2 when $x \in \mathrm{N}$.

(ii) $x \in \mathrm{Q}$

Ans: From the above solution, when $x \in \mathrm{Q}$ the values of x are $x = 2, \frac{5}{2}$.

8. (i) $a^2x^2 + 2ax + 1 = 0, a \neq 0$

Ans: The given expression is $a^2x^2 + 2ax + 1 = 0, a \neq 0$.

Let us simplify the given expression with the help of factorization method.

$a^2x^2 + 2ax + 1 = 0$

$\Rightarrow a^2x^2 + ax + +ax + 1 = 0$

$\Rightarrow ax(ax+1) + 1(ax+1) = 0$

$\Rightarrow (ax+1)(ax+1) = 0$

By further simplification,

$(ax+1) = 0$ Or $(ax+1) = 0$

$\Rightarrow ax = -1$ Or $ax = -1$

$\Rightarrow x = \frac{-1}{a}$ Or $x = \frac{-1}{a}$

Therefore $\frac{-1}{a}, \frac{-1}{a}$ are the values of x.

(ii) $x^2 - (p+q)x + pq = 0$

Ans: The given expression is $x^2 - (p+q)x + pq = 0$

Let us simplify the given expression by expanding the brackets.

$x^2 - (p+q)x + pq = 0$

$\Rightarrow x^2 - px - qx + pq = 0$

$\Rightarrow x(x-p) - q(x-p) = 0$

$\Rightarrow (x-q)(x-p) = 0$

After further simplification,

$(x-q)=0$ Or $(x-p)=0$

$\Rightarrow x=q$ Or $x=p$

Therefore **q,p** are the values of *x*.

9. $a^2x^2+\left(a^2+b^2\right)x+b^2=0, a\neq 0$

Ans: The given expression is $a^2x^2+\left(a^2+b^2\right)x+b^2=0$.

Let us simplify the given expression by expanding the brackets.

$a^2x^2+\left(a^2+b^2\right)x+b^2=0$

$\Rightarrow a^2x^2+a^2x+b^2x+b^2=0$

$\Rightarrow a^2x(x+1)+b^2(x+1)=0$

$\Rightarrow \left(a^2x+b^2\right)(x+1)=0$

By further simplification,

$\left(a^2x+b^2\right)=0$ Or $(x+1)=0$

$\Rightarrow a^2x=-b^2$ Or $x=-1$

$\Rightarrow x=\frac{-b^2}{a^2}$ Or $x=-1$

Therefore $\frac{-b^2}{a^2},-1$ are the values of *x*.

10. (i) $\sqrt{3}x^2+10x+7\sqrt{3}=0$

Ans: The given expression is $\sqrt{3}x^2+10x+7\sqrt{3}=0$.

Let us simplify the given expression with the help of factorization method.

$\sqrt{3}x^2+10x+7\sqrt{3}=0$

$\Rightarrow \sqrt{3}x^2+3x+7x+7\sqrt{3}=0$

$\Rightarrow \sqrt{3}x\left(x+\sqrt{3}\right)+7\left(x+\sqrt{3}\right)=0$

$\Rightarrow \left(\sqrt{3}x+7\right)\left(x+\sqrt{3}\right)=0$

By further simplification,

$\left(\sqrt{3}x+7\right)=0$ Or $\left(x+\sqrt{3}\right)=0$

$\Rightarrow \sqrt{3}x=-7$ Or $x=-\sqrt{3}$

$\Rightarrow x=\dfrac{-7}{\sqrt{3}}$ Or $x=-\sqrt{3}$

Therefore $\dfrac{-7}{\sqrt{3}},-\sqrt{3}$ are the values of x.

(ii) $4\sqrt{3}x^2+5x-2\sqrt{3}=0$

Ans: The given expression is $4\sqrt{3}x^2+5x-2\sqrt{3}=0$.

Let us simplify the given expression with the help of factorization method.

$4\sqrt{3}x^2+5x-2\sqrt{3}=0$

$\Rightarrow 4\sqrt{3}x^2+8x-3x-2\sqrt{3}=0$

$\Rightarrow 4x\left(\sqrt{3}x+2\right)-\sqrt{3}\left(\sqrt{3}x+2\right)=0$

$\Rightarrow \left(4x-\sqrt{3}\right)\left(\sqrt{3}x+2\right)=0$

By further simplification,

$\left(4x-\sqrt{3}\right)=0$ Or $\left(\sqrt{3}x+2\right)=0$

$\Rightarrow 4x=\sqrt{3}$ Or $\sqrt{3}x=-2$

$\Rightarrow x=\dfrac{\sqrt{3}}{4}$ Or $x=\dfrac{-2}{\sqrt{3}}$

Therefore $\dfrac{\sqrt{3}}{4},\dfrac{-2}{\sqrt{3}}$ are the values of x.

11. (i) $x^2-\left(1+\sqrt{2}\right)x+\sqrt{2}=0$

Ans: The given expression is $x^2-\left(1+\sqrt{2}\right)x+\sqrt{2}=0$.

Let us first expand the given expression

$x^2-(\sqrt{2}+1)x+\sqrt{2}=0$

On further calculation, we get

$x^2-\sqrt{2}x-x+\sqrt{2}=0$

$(x-\sqrt{2})(x-1)=0$

$\therefore x=1, \quad \sqrt{2}$

(ii) $x+\frac{1}{x}=2\left(\frac{1}{20}\right)$

Ans: The given expression is $x+\frac{1}{x}=2\left(\frac{1}{20}\right)$

First, rewrite the given expression

$\frac{x^2+1}{x}=\frac{41}{20}$

By cross multiplication, we get

$20\left(x^2+1\right)=41x$

$\Rightarrow 20x^2+20=41x$

$\Rightarrow 20x^2-41x+20=0$

Now factorize the expression,

$20x^2-25x-16x+20=0$

$\Rightarrow 5x\left(4x-5\right)-4\left(4x-5\right)=0$

$\Rightarrow \left(5x-4\right)\left(4x-5\right)=0$

By further simplification,

$\left(5x-4\right)=0$ Or $\left(4x-5\right)=0$

$\Rightarrow 5x=4$ Or $4x=5$

$\Rightarrow x=\frac{4}{5}$ Or $x=\frac{5}{4}$

Therefore $\frac{4}{5},\frac{5}{4}$ are the values of *x*.

12. (i) $\frac{2}{x^2}-\frac{5}{x}+2=0, x\neq 0$

Ans: The given expression is $\frac{2}{x^2}-\frac{5}{x}+2=0$.

First, take the L.C.M for the given expression,

$$\frac{\left(2-5x+2x^2\right)}{x^2}=0$$

$\Rightarrow 2x^2-5x+2=0$

By factorizing the above expression, we get

$2x^2-4x-x+2=0$

$\Rightarrow 2x(x-2)-1(x-2)=0$

$\Rightarrow (2x-1)(x-2)=0$

By further simplification,

$(2x-1)=0$ Or $(x-2)=0$

$\Rightarrow 2x=1$ Or $x=2$

$\Rightarrow x=\frac{1}{2}$ Or $x=2$

Therefore $\frac{1}{2},2$ are the values of x.

(ii) $\frac{x^2}{15}-\frac{x}{3}-10=0$

Ans: The given expression is $\frac{x^2}{15}-\frac{x}{3}-10=0$

First, take the LCM for given expression,

$$\frac{\left(x^2-5x-150\right)}{15}=0$$

$\Rightarrow x^2-5x-150$

Now factorize the above expression, we get

$x^2-15x+10x-150=0$

$\Rightarrow x(x-15)+10(x-15)=0$

$\Rightarrow (x-15)(x+10)=0$

By further simplification,

$\left(x-15\right)=0$ Or $\left(x+10\right)=0$

$\Rightarrow x=15$ Or $x=-10$

Therefore $15,-10$ are the values of x.

13. (i) $3x-\frac{8}{x}=2$

Ans: The given expression is $3x-\frac{8}{x}=2$.

Let us simplify the given equation, by taking LCM and cross multiplying

$3x^2-8=2x$

$3x^2-2x-8=0$

By factorizing, we get

$3x^2-6x+4x-8=0$

$3x(x-2)+4(x-2)=0$

$(x-2)(3x+4)=0$

So,

$(x-2)=0$ or $(3x+4)=0$

$x=2$ or $3x=-4$

$x=2$ or $x=-\frac{4}{3}$

$\therefore$ Value of x is $2,-\frac{4}{3}$

(ii) $\frac{\left(x+2\right)}{\left(x+3\right)}=\frac{\left(2x-3\right)}{\left(3x-7\right)}$

Ans: The given expression is $\frac{\left(x+2\right)}{\left(x+3\right)}=\frac{\left(2x-3\right)}{\left(3x-7\right)}$

By cross multiplication, we get

$\left(x+2\right)\left(3x-7\right)=\left(2x-3\right)\left(x+3\right)$

$\Rightarrow 3x^2-7x+6x-14=2x^2+6x-3x-9$

$\Rightarrow 3x^2-2x^2-x-3x-14+9=0$

$\Rightarrow x^2-4x-5=0$

Now factorize the above expression, we get

$x^2 - 5x + x - 5 = 0$

$\Rightarrow x(x-5)+1(x-5)=0$

$\Rightarrow (x+1)(x-5)=0$

By further simplification,

$(x+1)=0$ Or $(x-5)=0$

$\Rightarrow x=-1$ Or $x=5$

Therefore $-1,5$ are the values of x.

14. (i) $\frac{8}{(x+3)} - \frac{3}{(2-x)} = 2$

Ans: The given expression is $\frac{8}{(x+3)} - \frac{3}{(2-x)} = 2$.

First, take the L.C.M for the given expression

$$\frac{[8(2-x)-3(x+3)]}{[(x+3)(2-x)]} = 2$$

By cross-multiplication, we get

$16-8x-3x-9=2(x+3)(2-x)$

$\Rightarrow 7-11x=2\left(2x+6-x^2-3x\right)$

$\Rightarrow 7-11x=2\left(6-x^2-x\right)$

$\Rightarrow 7-11x=12-2x^2-2x$

$\Rightarrow 2x^2-11x+2x+7-12=0$

$\Rightarrow 2x^2-9x-5=0$

By factorizing the above expression, we get

$2x^2-10x+x-5=0$

$\Rightarrow 2x(x-5)+1(x-5)=0$

$\Rightarrow (2x+1)(x-5)=0$

By further simplification,

$2x+1=0$ Or $x-5=0$

$\Rightarrow 2x=-1$ Or $x=5$

$\Rightarrow x=\frac{-1}{2}$ Or $x=5$

Therefore $\frac{-1}{2},5$ are the values of *x*.

(ii) $\frac{x}{(x-1)}+\frac{(x-1)}{x}=2\frac{1}{2}$

Ans: The given expression is $\frac{x}{(x-1)}+\frac{(x-1)}{x}=2\frac{1}{2}$

First, take the L.C.M for the given expression

$\frac{\left[x^2+(x-1)^2\right]}{x(x-1)}=\frac{5}{2}$

$\Rightarrow \frac{\left(x^2+x^2-2x+1\right)}{\left(x^2-x\right)}=\frac{5}{2}$

$\Rightarrow \frac{\left(2x^2-2x+1\right)}{\left(x^2-x\right)}=\frac{5}{2}$

By cross-multiplication, we get

$2\left(2x^2-2x+1\right)=5\left(x^2-x\right)$

$\Rightarrow 4x^2-4x+2=5x^2-5x$

$\Rightarrow 5x^2-4x^2-5x+4x-2=0$

$\Rightarrow x^2-x-2=0$

By factorizing the above expression, we get

$x^2-2x+x-2=0$

$\Rightarrow x(x-2)+1(x-2)=0$

$\Rightarrow (x+1)(x-2)=0$

By further simplification,

$x+1=0$ Or $x-2=0$

$\Rightarrow x=-1$ Or $x=2$

Therefore $-1,2$ are the values of *x*.

15. (i) $\frac{(x+1)}{(x-1)}+\frac{(x-2)}{(x+2)}=3$

Ans: The given expression is $\frac{(x+1)}{(x-1)}+\frac{(x-2)}{(x+2)}=3$.

First, take the L.C.M for the given expression

$$\frac{[(x+1)(x+2)+(x-2)(x-1)]}{[(x-1)(x+2)]}=3$$

By expanding, we get

$$x^2+3x+2+x^2-3x+2=3(x-1)(x+2)$$

$$\Rightarrow 2x^2+4=3(x^2+x-2)$$

$$\Rightarrow 2x^2+4=3x^2+3x-6$$

$$\Rightarrow 3x^2-2x^2+3x-6-4=0$$

$$\Rightarrow x^2+3x-10=0$$

Now factorize the above expression, we get

$$x^2+5x-2x-10=0$$

$$\Rightarrow x(x+5)-2(x+5)=0$$

$$\Rightarrow (x-2)(x+5)=0$$

By further simplification,

$x-2=0$ Or $x+5=0$

$\Rightarrow x=2$ Or $x=-5$

Therefore $2,-5$ are the values of x.

(ii) $\frac{1}{(x-3)}-\frac{1}{(x+5)}=\frac{1}{6}$

Ans: The given expression is $\frac{1}{(x-3)}-\frac{1}{(x+5)}=\frac{1}{6}$

First, take the L.C.M for the given expression

$$\frac{(x+5)-(x-3)}{(x-3)(x+5)}=\frac{1}{6}$$

$$\Rightarrow \frac{x+5-x+3}{(x-3)(x+5)}=\frac{1}{6}$$

$$\Rightarrow \frac{8}{(x-3)(x+5)}=\frac{1}{6}$$

By cross-multiplication, we get

$$8\times 6=(x-3)(x+5)$$

$$\Rightarrow 48=x^2+5x-3x-15$$

$$\Rightarrow x^2+2x-15-48=0$$

$$\Rightarrow x^2+2x-63=0$$

By factorizing the above expression, we get

$$x^2+9x-7x-63=0$$

$$\Rightarrow x(x+9)-7(x+9)=0$$

$$\Rightarrow (x-7)(x+9)=0$$

By further simplification,

$x-7=0$ Or $x+9=0$

$\Rightarrow x=7$ Or $x=-9$

Therefore $7,-9$ are the values of x.

16. (i) $\frac{a}{(ax-1)}+\frac{b}{(bx-1)}=a+b, a+b\neq 0, ab\neq 0$

Ans: The given expression is $\frac{a}{(ax-1)}+\frac{b}{(bx-1)}=a+b$.

First, rearrange the given expression

$$\left[\frac{a}{(ax-1)}-b\right]+\left[\frac{b}{(bx-1)}-a\right]=0$$

$$\Rightarrow \left[\frac{a-b(ax-1)}{(ax-1)}\right]+\left[\frac{b-a(bx-1)}{(bx-1)}\right]=0$$

$$\Rightarrow \frac{(a-abx+b)}{(ax-1)}+\frac{(b-abx+a)}{(bx-1)}=0$$

Now taking out the common terms

$$(a-abx+b)\left[\frac{1}{(ax-1)}+\frac{1}{(bx-1)}\right]=0$$

$$\Rightarrow (a-abx+b)\left[\frac{bx-1+ax-1}{(ax-1)(bx-1)}\right]=0$$

$$\Rightarrow (a-abx+b)\left[\frac{ax+bx-2}{(ax-1)(bx-1)}\right]=0$$

By further simplification,

$$(a-abx+b)=0 \text{ Or } \frac{ax+bx-2}{\left[(ax-1)(bx-1)\right]}=0$$

$$\Rightarrow a+b=abx \text{ Or } ax+bx-2=0$$

$$\Rightarrow x=\frac{(a+b)}{ab} \text{ Or } (a+b)x=2$$

$$\Rightarrow x=\frac{(a+b)}{ab} \text{ Or } x=\frac{2}{(a+b)}$$

Therefore $\frac{(a+b)}{ab}, \frac{2}{(a+b)}$ are the values of *x*.

(ii) $\frac{1}{(2a+b+2x)}=\frac{1}{2a}+\frac{1}{b}+\frac{1}{2x}$

Ans: The given expression is $\frac{1}{(2a+b+2x)}=\frac{1}{2a}+\frac{1}{b}+\frac{1}{2x}$

First, take the L.C.M for the given expression

$$\frac{2x-2a-b-2x}{2x(2a+b+2x)}=\frac{(2a+b)}{2ab}$$

$$\Rightarrow \frac{-2a-b}{2x(2a+b+2x)}=\frac{(2a+b)}{2ab}$$

$$\Rightarrow \frac{-(2a+b)}{2x(2a+b+2x)}=\frac{(2a+b)}{2ab}$$

$$\Rightarrow \frac{-1}{x(2a+b+2x)}=\frac{1}{ab}$$

By cross-multiplication, we get

$$\frac{-1}{x(2a+b+2x)}=\frac{1}{ab}$$

$$\Rightarrow -ab=x(2a+b)+2x^2$$

$$\Rightarrow 0=2x^2+2ax+bx+ab$$

$$\Rightarrow 2x(x+a)+b(x+a)=0$$

$$\Rightarrow (x+a)(2x+b)=0$$

By further simplification,

$x+a=0$ Or $2x+b=0$

$\Rightarrow x=-a$ Or $2x=-b$

$\Rightarrow x=-a$ Or $x=\frac{-b}{2}$

Therefore $-a,\frac{-b}{2}$ are the values of x.

17. $\frac{1}{(x+6)}+\frac{1}{(x-10)}=\frac{3}{(x-4)}$

Ans: The given expression is $\frac{1}{(x+6)}+\frac{1}{(x-10)}=\frac{3}{(x-4)}$

First, take the L.C.M for the given expression

$$\frac{(x-10)+(x+6)}{(x+6)(x-10)}=\frac{3}{(x-4)}$$

$$\Rightarrow \frac{2x-4}{x^2-4x-60}=\frac{3}{(x-4)}$$

By cross-multiplication, we get

$$(2x-4)(x-4)=3(x^2-4x-60)$$

$$\Rightarrow 2x^2-8x-4x+16=3x^2-12x-180$$

$$\Rightarrow 2x^2-12x+16=3x^2-12x-180$$

$$\Rightarrow 3x^2-2x^2-12x+12x-180-16=0$$

$$\Rightarrow x^2-196=0$$

$$\Rightarrow x^2=196$$

$\Rightarrow x = \sqrt{196}$

$\therefore x = \pm 14$

Therefore ± 14 are the values of *x*.

18. (i) $\sqrt{(3x+4)} = x$

Ans: The given expression is $\sqrt{(3x+4)} = x$.

When we squaring on both sides, we get

$(3x+4) = x^2$

$\Rightarrow x^2 - 3x - 4 = 0$

Now factorize the above expression,

$x^2 - 4x + x - 4 = 0$

$\Rightarrow x(x-4) + 1(x-4) = 0$

$\Rightarrow (x-4)(x+1) = 0$

By further simplification,

$x - 4 = 0$ Or $x + 1 = 0$

$\Rightarrow x = 4$ Or $x = -1$

Therefore $4, -1$ are the values of *x*.

(ii) $\sqrt{x(x-7)} = 3\sqrt{2}$

Ans: The given expression is $\sqrt{x(x-7)} = 3\sqrt{2}$.

When we squaring on both sides, we get

$x(x-7) = \left(3\sqrt{2}\right)^2$

$\Rightarrow x^2 - 7x - 18 = 0$

Now factorize the above expression,

$x^2 - 9x + 2x - 18 = 0$

$\Rightarrow x(x-9) + 2(x-9) = 0$

$\Rightarrow (x-9)(x+2) = 0$

By further simplification,

$x - 9 = 0$ Or $x + 2 = 0$

$\Rightarrow x = 9$ Or $x = -2$

Therefore $9, -2$ are the values of x.

19. Use the substitution $y = 3x + 1$ to solve for x: $5(3x+1)^2 + 6(3x+1) - 8 = 0$

Ans: The given equation is $5(3x+1)^2 + 6(3x+1) - 8 = 0$

By substituting $y = 3x + 1$ in the given equation, we get

$5y^2 + 6y - 8 = 0$

Now we will find the value of y by using the factorization method.

$5y^2 + 10y - 4y - 8 = 0$

$\Rightarrow 5y(y+2) - 4(y+2) = 0$

$\Rightarrow (5y-4)(y+2) = 0$

After further simplification, we get

$5y - 4 = 0$ Or $y + 2 = 0$

$\Rightarrow 5y = 4$ Or $y = -2$

$\Rightarrow y = \frac{4}{5}$ Or $y = -2$

We will now find the value of x by substituting the value y in $y = 3x + 1$.

$3x + 1 = \frac{4}{5}$ Or $3x + 1 = -2$

$\Rightarrow 3x = \frac{4}{5} - 1$ Or $3x = -2 - 1$

$\Rightarrow 3x = \frac{-1}{5}$ Or $3x = -3$

$\Rightarrow x = \frac{-1}{15}$ Or $x = -1$

Therefore $-1, \frac{-1}{15}$ are the values of x.

20. Find the value of x if $p + 1 = 0$ and $x^2 + px - 6 = 0$

Ans: The given quadratic equation is $x^2 + px - 6 = 0$.

And also we have

$p+1=0$

$\Rightarrow p=-1$

By substituting $p=-1$ in the given equation, we get

$x^2+(-1)x-6=0$

$\Rightarrow x^2-x-6=0$

Now we will find the value of x by using the factorization method.

$x^2-3x+2x-6=0$

$\Rightarrow x(x-3)+2(x-3)=0$

$\Rightarrow (x+2)(x-3)=0$

After further simplification, we get

$x+2=0$ Or $x-3=0$

$\Rightarrow x=-2$ Or $x=3$

Therefore $-2,3$ are the values of x.

21. Find the value of x if $p+7=0, q-12=0$ and $x^2+px+q=0$

Ans: The given quadratic equation is $x^2+px+q=0$.

And also we have

$p+7=0$ and $q-12=0$

$\Rightarrow p=-7$ and $q=12$

By substituting the value of p and q in the given equation, we get

$x^2+(-7)x+12=0$

$\Rightarrow x^2-7x+12=0$

Now we will find the value of x by using the factorization method.

$x^2-4x-3x+12=0$

$\Rightarrow x(x-4)-3(x-4)=0$

$\Rightarrow (x-3)(x-4)=0$

After further simplification, we get

$x-3=0$ Or $x-4=0$

$\Rightarrow x=3$ Or $x=4$

Therefore 3,4 are the values of x.

22. If $x = p$ is a solution of the equation $x(2x+5) = 3$ then find the value of *p*.

Ans: It is given that $x = p$ is a solution of the equation is $x(2x+5) = 3$.

By substituting $x = p$ in the given equation, we get

$p(2p+5) = 3$

$\Rightarrow 2p^2 + 5p = 3$

$\Rightarrow 2p^2 + 5p - 3 = 0$

Now we will find the value of *p* by using the factorization method.

$2p^2 + 6p - p - 3 = 0$

$\Rightarrow 2p(p+3) - 1(p+3) = 0$

$\Rightarrow (2p-1)(p+3) = 0$

After further simplification, we get

$2p - 1 = 0$ Or $p + 3 = 0$

$\Rightarrow 2p = 1$ Or $p = -3$

$\Rightarrow p = \frac{1}{2}$ Or $p = -3$

Therefore $p = \frac{1}{2}, -3$ are the values of *p*.

23. If $x = 3$ is a solution of the equation $(k+2)x^2 - kx + 6 = 0$, find the value of *k*. Hence, find the other root of the equation.

Ans: It is given that $x = 3$ is a solution of the equation is $(k+2)x^2 - kx + 6 = 0$.

By substituting $x = 3$ in the given equation, we get

$(k+2)(3)^2 - k(3) + 6 = 0$

$\Rightarrow (k+2)9 - 3k + 6 = 0$

$\Rightarrow 9k + 18 - 3k + 6 = 0$

$\Rightarrow 6k + 24 = 0$

$\Rightarrow 6(k+4) = 0$

So, we get

$\Rightarrow k + 4 = 0$

$\Rightarrow k = -4$

By substituting the value of k in the given equation, we get

$$(-4+2)x^2-(-4)x+6=0$$

$$\Rightarrow -2x^2+4x+6=0$$

Now divide the above equation by -2

$$\frac{-2x^2+4x+6}{-2}=0$$

$$\Rightarrow x^2-2x-3=0$$

Further, factorizing the above equation

$$x^2-3x+x-3=0$$

$$\Rightarrow x(x-3)+1(x-3)=0$$

$$\Rightarrow (x+1)(x-3)=0$$

$$\Rightarrow x+1=0 \text{ Or } x-3=0$$

$$\Rightarrow x=-1 \text{ Or } x=3$$

Hence, -1 is the other root of the given equation.

Exercise 5.3

Solve the following (1 to 6) equations using formula:

1.(i) $2x^2-7\mathbf{x}+6=0$

Ans: Comparing this equation with $\mathrm{a}x^2+\mathrm{bx}+\mathrm{c}=0$

It gives, $a=2, b=-7, c=6$

Now by substituting these values in $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$$\Rightarrow x=\frac{-(-7)\pm\sqrt{(-7)^2-4\times2\times6}}{2\times2}$$

$$\Rightarrow x=\frac{7\pm1}{4}$$

$$\Rightarrow x=\frac{7+1}{4},\frac{7-1}{4}$$

$$\Rightarrow x=\frac{8}{4},\frac{6}{4}$$

$$\Rightarrow x=2,\frac{3}{2}$$

So, $x = 2, \frac{3}{2}$.

(ii) $2x^2 - 6\mathbf{x} + 3 = 0$

Ans: $2x^2 - 6x + 3 = 0$

Here $a = 2, b = -6, c = 3$

then $D = b^2 - 4ac$

$= (-6)^2 - 4 \times 2 \times 3$

$= 36 - 24$

$=12$

Now

$$x = \frac{-b \pm \sqrt{\mathrm{D}}}{2a}$$

$$= \frac{(-6) \pm \sqrt{12}}{2 \times 2}$$

$$= \frac{6 \pm 2\sqrt{3}}{4}$$

$$\therefore x_1 = \frac{6 + 2\sqrt{3}}{4}$$

$$= \frac{2(3+\sqrt{3})}{4} = \frac{3+\sqrt{3}}{2}$$

$$x_2 = \frac{6 - 2\sqrt{3}}{4}$$

$$= \frac{2(3-\sqrt{3})}{4}$$

$$= \frac{3-\sqrt{3}}{2}$$

Hence $x = \frac{3+\sqrt{3}}{2}, \frac{3-\sqrt{3}}{2}$.

2. (i) $256x^2 - 32\mathbf{x} + 1 = 0$

Ans: Comparing this equation with $\mathrm{a}x^2 + \mathrm{bx} + \mathrm{c} = 0$.

It gives $a=256, b=-32, c=1$

Substitute these values in $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$$\Rightarrow x=\frac{-(-32)\pm\sqrt{(-32)^2-4\times256\times1}}{2\times256}$$

$$\Rightarrow x=\frac{32\pm\sqrt{0}}{512}$$

Hence, $x=\dfrac{1}{16}$

(ii) $25x^2+30\mathbf{x}+7=0$

Ans: $25x^2+30x+7=0$

Here $a=25, b=30, c=7$

$D=b^2-4ac$

$=(30)^2-4\times25\times7$

$=900-700$

=200

$$\because x=\frac{-b\pm\sqrt{D}}{2a}$$

$$=\frac{-30\pm\sqrt{200}}{2\times25}$$

$$=\frac{-30\pm\sqrt{100\times2}}{50}$$

$$=\frac{-30\pm10\sqrt{2}}{50}$$

$$=\frac{-3\pm\sqrt{2}}{5}$$

Hence $x=\dfrac{-3+\sqrt{2}}{5},\dfrac{-3-\sqrt{2}}{5}$.

3.(i) $2x^2+\sqrt{5}x-5=0$

Ans: The given expression is

Comparing this equation with $ax^2+bx+c=0$

Here $a=2, b=\sqrt{5}, c=-5$

$D=b^2-4ac$

$=(\sqrt{5})^2-4\times 2\times(-5)$

=5+40

=45

$\because x=\dfrac{-b\pm\sqrt{D}}{2a}$

$=\dfrac{-\sqrt{5}\pm\sqrt{45}}{2\times 2}$

$=\dfrac{-\sqrt{5}\pm\sqrt{9\times 5}}{4}$

$=\dfrac{-\sqrt{5}\pm 3\sqrt{5}}{4}$

$\therefore x_1=\dfrac{-\sqrt{5}+3\sqrt{5}}{4}$

$=\dfrac{2\sqrt{5}}{4}$

$=\dfrac{\sqrt{5}}{2}$

$x_2=\dfrac{-\sqrt{5}-3\sqrt{5}}{4}$

$=\dfrac{-4\sqrt{5}}{4}$

$=-\sqrt{5}$

Hence $x=\dfrac{\sqrt{5}}{2},-\sqrt{5}$.

(ii) $\sqrt{3}x^2+10x-8\sqrt{3}=0$

Ans: The given expression is

Comparing this equation with $ax^2 + bx + c = 0$

Here $a = \sqrt{3}, b = 10, c = -8\sqrt{3}$

$$D = b^2 - 4ac$$
$$= (10)^2 - 4 \times \sqrt{3} \times (-8\sqrt{3})$$
$$= 100 + 96$$
$$= 196$$
$$\because x = \frac{-b \pm \sqrt{D}}{2a}$$
$$= \frac{-10 \pm \sqrt{196}}{2 \times \sqrt{3}}$$
$$= \frac{-10 \pm 14}{2\sqrt{3}}$$
$$\therefore x_1 = \frac{-10 + 14}{2\sqrt{3}}$$
$$= \frac{4}{2\sqrt{3}}$$
$$= \frac{2 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}}$$
$$= \frac{2\sqrt{3}}{3}$$
$$x_2 = \frac{-10 - 14}{2\sqrt{3}}$$
$$= \frac{-24}{2\sqrt{3}}$$
$$= \frac{-12 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}}$$
$$= \frac{-12\sqrt{3}}{3}$$
$$= -4\sqrt{3}$$

Hence $x = \frac{2\sqrt{3}}{3}, -4\sqrt{3}$.

4.(i) $\frac{x-2}{x+2}+\frac{x+2}{x-2}=4$

Ans: $\frac{x-2}{x+2}+\frac{x+2}{x-2}=4$

$\Rightarrow \frac{(x-2)^2+(x+2)^2}{(x+2)(x-2)}=4$

$\Rightarrow \frac{x^2-4x+4+x^2+4x+4}{x^2-4}$

$\Rightarrow 2x^2+8=4x^2-16$

$\Rightarrow 2x^2+8-4x^2+16=0$

$\Rightarrow -2x^2+24=0$

$\Rightarrow x^2-12=0$

Here $a=1, b=0, c=-12$

$D=b^2-4ac$

$=(0)^2-4\times1(-12)$

$=0+48$

=48

$\because x=\frac{-b\pm\sqrt{D}}{2a}$

$=\frac{0\pm\sqrt{48}}{2\times1}$

$=\frac{\pm\sqrt{48}}{2}$

$=\frac{\pm\sqrt{16\times3}}{2}$

$=\pm\frac{4\sqrt{3}}{2}$

$=\pm2\sqrt{3}$

Hence roots are $2\sqrt{3}, -2\sqrt{3}$.

(ii) $\frac{x+1}{x+3}=\frac{3x+2}{2x+3}$

Ans: $\frac{x+1}{x+3}=\frac{3x+2}{2x+3}$

$(x+1)(2x+3)=(3x+2)(x+3)$

$=2x^2+3x+2x+3=3x^2+9x+2x+6$

$=2x^2+5x+3-3x^2-11x-6=0$

$=-x^2-6x-3=0$

$=x^2+6x+3=0$

Here $a=1, b=6, c=3$

$D=b^2-4ac$

$=(6)^2-4\times1\times3$

$=36-12$

$=24$

$\because x=\frac{-b\pm\sqrt{D}}{2a}$

$=\frac{-6\pm\sqrt{24}}{2\times1}$

$=\frac{6\pm\sqrt{4\times6}}{2}$

$=\frac{-6\pm2\sqrt{6}}{2}$

$=-3\pm\sqrt{6}$

$\therefore x_1=-3+\sqrt{6}, x_2=-3-\sqrt{6}$

Hence $x=-3+\sqrt{6},-3-\sqrt{6}$.

5.(i) $\mathbf{A\left(x^2+1\right)=\left(A^2+1\right)x}$

Ans: The given expression is $A(x^2+1)=\left(A^2+1\right)x$

$\Rightarrow Ax^2-\left(A^2+1\right)x+A=0$

Comparing this equation with $ax^2+bx+c=0$

It gives, $a = A, b = -\left(A^2 + 1\right), c = A$

So, by using the formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$\Rightarrow x = \frac{A^2 + 1 \pm \sqrt{A^2 \times A^2 + 2A^2 + 1 - 4A^2}}{2A}$$

$$\Rightarrow x = \frac{A^2 + 1 \pm \left(A^2 - 1\right)}{2A}$$

So, $x = \dfrac{A^2 + 1 - A^2 + 1}{2A}, \dfrac{A^2 + 1 + A^2 - 1}{2A}$

$$\Rightarrow x = \frac{1}{A}, A$$

(ii) $4x^2 - 4\mathbf{Ax} + \left(\mathbf{A}^2 - \mathbf{B}^2\right) = 0$

Ans: The given expression is $4x^2 - 4Ax + \left(A^2 - B^2\right) = 0$

Comparing this equation with $ax^2 + bx + c = 0$

It gives, $a = 4, b = -4A, C = A^2 - B^2$

So, by using the formula, $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$\Rightarrow x = \frac{-(-4A) \pm \sqrt{(-4A)^2 - 4 \times 4 \times \left(A^2 - B^2\right)}}{2 \times 4}$$

$$\Rightarrow x = \frac{4A \pm \sqrt{16A^2 - 16A^2 + 16B^2}}{8}$$

$$\Rightarrow x = \frac{4A \pm 4B}{8}$$

So $x = \dfrac{A + B}{2}, \dfrac{A - B}{2}$

6.(i) $x - \dfrac{1}{x} = 3$

Ans: The given expression is $x-\frac{1}{x}=3$

$\Rightarrow \frac{x^2-1}{x}=3$

$\Rightarrow x^2-1=3x$

$\Rightarrow x^2-3x-1=0$

Comparing this equation with $ax^2+bx+c=0$

It gives, $a=1, b=-3, c=-1$

By using formula, $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$\Rightarrow x=\frac{-(-3)\pm\sqrt{(-3)^2-4\times1\times(-1)}}{2\times1}$

$\Rightarrow x=\frac{3\pm\sqrt{13}}{2}$

So, $x=\frac{3+\sqrt{13}}{2},\frac{3-\sqrt{13}}{2}$

(ii) $\frac{1}{x}+\frac{1}{x-2}=3$

Ans: The given expression is $\frac{1}{x}+\frac{1}{x-2}=3$

By taking LCM,

$\Rightarrow \frac{x-2+x}{x(x-2)}=3$

$\Rightarrow 2x-2=3x^2-6x$

$\Rightarrow 3x^2-8x+2=0$

Comparing this equation with $ax^2+bx+c=0$

It gives, $a=3, b=-8, c=2$

By using formula, $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$$\Rightarrow x=\frac{-(-8)\pm\sqrt{(-8)^2-4\times3\times2}}{2\times3}$$

$$\Rightarrow x=\frac{8\pm2\sqrt{10}}{6}$$

So, $x=\frac{4+\sqrt{10}}{3},\frac{4-\sqrt{10}}{3}$.

7. Solve for x: $2\left(\frac{2x-1}{x+3}\right)-3\left(\frac{x+3}{2x-1}\right)=5$

Ans: Let $\frac{2x-1}{x+3}=y$ then $\left(\frac{x+3}{2x-1}\right)=\frac{1}{y}$

$\therefore 2y-\frac{3}{y}=5$

$2y^2-3=5y$

$=2y^2-5y-3=0$

Here a=2, b=-5, c=-3

b^2-4ac

$=(-5)^2-4\times2\times(-3)$

$=25+24$

$=49$

Now, $y=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$=\frac{-(-5)\pm\sqrt{49}}{2\times2}$

$=\frac{5\pm7}{4}$

$y=\frac{5+7}{4}$

$=\frac{12}{4}$

$=3$

or

$y = \frac{5-7}{4}$

$= \frac{-2}{4}$

$= \frac{-1}{2}$

$\therefore y = 3, \frac{-1}{2}$

When y=3, then $\frac{2x-1}{x+3} = 3$

$\Rightarrow 3x+9 = 2x-1$

$\Rightarrow 3x-2x = -1-9$

$\Rightarrow x = -10$

When $y = \frac{-1}{2}$, then

or

$\frac{2x-1}{x+3} = \frac{-1}{2}$

$4x-2 = -x-3$

$4x+x = -3+2$

$\Rightarrow 5x = -1$

$x = \frac{-1}{5}$

$\therefore x = -10, \frac{-1}{5}.$

8. Solve the following quadratic equation for x and give your answer correct to 2 decimal places:

(i) $x^2 - 5x - 10 = 0$

Ans: $x^2 - 5x - 10 = 0$

On comparing with, $ax^2 + bx + c = 0$ $a = 1, b = -5, c = -10$

$\because \text{x} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(-10)}}{2 \times 1}$$

$$\therefore x = \frac{5 \pm \sqrt{25 + 40}}{2}$$

$$\Rightarrow \text{x} = \frac{5 \pm \sqrt{65}}{2}$$

$$= \frac{5 \pm 8.06}{2}$$

Either $x = \frac{5 + 8.06}{2}$

$$= \frac{13.06}{2}$$

$$= 6.53$$

or

$$x = \frac{5 - 8.06}{2}$$

$$= \frac{-3.06}{2}$$

$$= 1.53$$

$$\therefore x = 6.53, x = -1.53$$

(ii) $x^2 + 7x = 7$

Ans: Given quadratic equation is $x^2 + 7x = 7$

$$\Rightarrow x^2 + 7x - 7 = 0$$

Comparing with $ax^2 + bx + c = 0$ we have $a = 1, b = 7$ and $c = -7$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow x = \frac{-7 \pm \sqrt{7^2 - 4 \times 1 \times (-7)}}{2 \times 1}$$

$$\Rightarrow x = \frac{-7 \pm \sqrt{77}}{2}$$

$$\Rightarrow x=\frac{-7\pm 8.77}{2}$$

$$\Rightarrow x=\frac{-7+8.77}{2} \text{ and } x=\frac{-7-8.77}{2}$$

$$\Rightarrow x=\frac{1.77}{2} \text{ and } x=\frac{-15.77}{2}$$

$\Rightarrow x=0.885$ and $x=-7.885$

$\Rightarrow x=0.89$ and $x=-7.89$ (Correct to two decimal places)

Therefore, the value of x is 0.89 and -7.89.

9. Solve the following equation by using quadratic formula and give your answer correct to 2 decimal places:

(i) $4x^2-5\mathbf{x}-3=0$

Ans: Given equation $4x^2-5x-3=0$

Comparing with $ax^2+bx+c=0$, we have

$a=4, b=-5, c=-3$

$$\because x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{-(-5)\pm\sqrt{(-5)^2-4\times 4\times(-3)}}{2\times 4}$$

$$=\frac{5\pm\sqrt{25+48}}{8}$$

$$=\frac{5\pm\sqrt{73}}{8}$$

$$=\frac{5\pm 8.544}{8}$$

$$=\frac{5+8.544}{8} \text{ or } \frac{5-8.544}{8}$$

$$=\frac{13.544}{8} \text{ or } \frac{-3.544}{8}$$

$=1.693$ or -0.443

$=1.69$ or -0.44. ...(correct to 2 decimal places)

(ii) $2x-\frac{1}{x}=7$

Ans: $2x-\frac{1}{x}=1$

$\Rightarrow 2x^2-1=7x$

$\Rightarrow 2x^2-7x-1=0 \quad \ldots(i)$

Comparing (i) with ax^2+bx+c, we get,

$a=2, b=-7, c=-1$

$\because x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$\Rightarrow x=\frac{-(-7)\pm\sqrt{(-7)^2-4(2)\times(-1)}}{2\times 2}$

$\Rightarrow \frac{7\pm\sqrt{49+8}}{4}$

$\Rightarrow \frac{7\pm\sqrt{57}}{4}$

$\Rightarrow x=\frac{7+\sqrt{57}}{4}$ or $x=\frac{7-\sqrt{57}}{4}$

$\Rightarrow x=\frac{7+7.55}{4}$ or $x=\frac{7-7.55}{4}$

$\Rightarrow x=\frac{14.55}{4}$ or $x=\frac{-0.55}{4}$

$\Rightarrow x=3.64$ or $x=-0.14.$

Therefore, the value of x is 3.64 and -0.14

10. Solve the following quadratic equation for x and give your answer correct to two significant figures:

(i) $x^2-4x-8=0$

Ans: Given quadratic equation is $x^2-4x-8=0$

We have a formula for solving quadratic equation $ax^2+bx+c=0$ is

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

By using quadratic formula, we have

$$x=\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(-8)}}{2(1)}=\frac{4\pm\sqrt{16+32}}{2}$$

$$=\frac{4\pm\sqrt{48}}{2}=\frac{4\pm4\sqrt{3}}{2}=2\pm2\sqrt{3}$$

$$=2(1\pm\sqrt{3})=2(1\pm1.73205)=2(2.73205)\text{ or }2(-0.73205)$$

$$=5.46410\text{ or }-1.4641$$

$$=5.46\text{ or }-1.46$$

Hence, the value of x is 5.46 or -1.46.

(ii) $\mathbf{x-\frac{18}{x}=6}$

Ans: The given expression is: $x-\frac{18}{x}=6$

$$\Rightarrow x^2-6x-18=0$$

$$a=1,b=-6,c=-18$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{6\pm\sqrt{36+72}}{2}$$

$$=\frac{6\pm\sqrt{108}}{2}$$

$$=\frac{6\pm6\sqrt{3}}{2}\text{ or }\frac{6(1-1.73)}{2}$$

$$=3\times2.73\text{ or }3\times-0.73$$

$$=8.19\text{ or }-2.19.$$

Hence, the value of x is 8.19 or -2.19.

11. Solve for the equation $\mathbf{5x^2-3x-4=0}$ and give your answer to 3 significant figures.

Ans: We have $5x^2-3x-4=0$

Here $a=5,b=-3,c=-4$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{3 \pm \sqrt{9 + 4 \times 5 \times 4}}{2 \times 5}$$
$$= \frac{3 \pm \sqrt{89}}{10}$$
$$x = \frac{3 + 9.43}{10} \text{ or } x = \frac{3 - 9.43}{10}$$
$$\Rightarrow x = \frac{12.43}{10} \text{ or } x = \frac{-6.43}{10}$$
$$\Rightarrow x = 1.24 \text{ or } x = -0.643.$$

Hence, the value of x is 1.24 or -0.64.

Exercise 5.4

1. Find the discriminant of the following equations and hence find the nature of roots:

(i) $3x^2 - 5x - 2 = 0$

Ans: Compare the given equation with $ax^2 + bx + c = 0$, to get

$a = 3$
$b = -5$
$c = -2$

By the use of discriminant formula,

$$D = b^2 - 4ac$$
$$\Rightarrow (-5)^2 - 4(3)(-2)$$
$$\Rightarrow 25 + 24 = 49$$

So,

Discriminate, $D = 49$

$D > 0$

$\therefore$ Roots are real and distinct.

(ii) $2x^2 - 3x + 5 = 0$

Ans: Compare the given equation with $ax^2 + bx + c = 0$, to get

$a = 2$

$b = -3$
$c = 5$
By the use of discriminant formula,
$D = b^2 - 4ac$
$\Rightarrow (-3)^2 - 4(2)(5)$
$\Rightarrow 9 - 40 = -31$
So,
Discriminate, $D = -31$
$D < 0$
$\therefore$ Roots are not real.

(iii) $16x^2 - 40x + 25 = 0$

Ans: Compare the given equation with $ax^2 + bx + c = 0$, to get
$a = 16$
$b = -40$
$c = 25$
By the use of discriminant formula,
$D = b^2 - 4ac$
$\Rightarrow (-40)^2 - 4(16)(25)$
$\Rightarrow 1600 - 1600 = 0$
So,
Discriminate, $D = 0$
$D = 0$
$\therefore$ Roots are real and equal.

(iv) $2x^2 + 15x + 30 = 0$

Ans: Compare the given equation with $ax^2 + bx + c = 0$, to get
$a = 2$
$b = 15$
$c = 30$
By the use of discriminant formula,
$D = b^2 - 4ac$
$\Rightarrow (15)^2 - 4(2)(30)$

$\Rightarrow 225-240=-15$
So,
Discriminate, $D=-15$
$D<0$
$\therefore$ Roots are not real.

2. Discuss the nature of the roots of the following quadratic equations:

(i) $3x^2-4\sqrt{3}x+4=0$

Ans: Compare the given equation with $ax^2+bx+c=0$, to get
$a=3$
$b=-4\sqrt{3}$,
$c=4$
By using the discriminant formula,
$D=b^2-4ac$
$\Rightarrow -4\sqrt{3}^2-4(3)(4)$
$\Rightarrow 16(3)-48=48-48=0$
Discriminate, $D=0$
$D=0$
$\therefore$ Roots are real and equal.

(ii) $x^2-\frac{1}{2x}+4=0$

Ans: Compare the given equation with $ax^2+bx+c=0$, to get
$a=1$
$b=-\frac{1}{2}$
$c=4$
By using the discriminant formula,
$D=b^2-4ac$
$\Rightarrow \left(\frac{-1}{2}\right)^2-4(1)(4)$
$\Rightarrow \frac{1}{4}-16=\frac{-63}{4}$

So,

Discriminate, $D=-\frac{63}{4}$

D < 0

∴ Roots are not real.

(iii) $-2x^2+x+1=0$

Ans: Compare the given equation with $ax^2+bx+c=0$, to get

$a=-2$

$b=1$

$c=1$

By using the discriminant formula,

$D=b^2-4ac$

$\Rightarrow 1^2-4(-2)(1)$

$\Rightarrow 1+8=9$

So,

Discriminate, $D=9$

D > 0

∴ Roots are real and distinct.

(iv) $2\sqrt{3}x^2-5x+\sqrt{3}=0$

Ans: Compare the given equation with $ax^2+bx+c=0$, to get

$a=2\sqrt{3}$,

$b=-5$

$c=\sqrt{3}$

By using the discriminant formula,

$D=b^2-4ac$

$\Rightarrow (-5)^2-4(2\sqrt{3})(\sqrt{3})$

$\Rightarrow 25-24=1$

So,

Discriminate, $D=1$

$D>0$

∴ Roots are real and distinct.

3. Find the nature of the roots of the following quadratic equations:
If real roots exist, find them.

(i) $x^2 - \frac{1}{2x} - \frac{1}{2} = 0$

Ans: Compare the given equation with $ax^2 + bx + c = 0$, to get

$a = 1$

$b = -\frac{1}{2}$

$c = -\frac{1}{2}$

By using the discriminant formula,

$D = b^2 - 4ac$

$\Rightarrow \left(-\frac{1}{2}\right)^2 - 4(1)(\frac{-1}{2})$

$\Rightarrow \frac{1}{4} + 2 = \frac{9}{4}$

So,

Discriminate, $D = \frac{9}{4}$

D > 0

∴ Roots are real and unequal.

(ii) $x^2 - 2\sqrt{3}x - 1 = 0$

Ans: Compare the given equation with $ax^2 + bx + c = 0$, to get

$a = 1$

$b = -2\sqrt{3}$

$c = -1$

By using the discriminant formula,

$D = b^2 - 4ac$

$\Rightarrow (2\sqrt{3})^2 - 4(1)(-1)$

$\Rightarrow 12 + 4 = 16$

So,

Discriminate, $D=16$

$D>0$

$\therefore$ Roots are real and unequal.

4. Without solving the following quadratic equation, find the value of 'p' for which the given equations have real and equal roots:

(i) $px^2-4x+3=0$

Ans: Compare the given equation with $ax^2+bx+c=0$, to get

$a=p$

$b=-4$

$c=3$

By using the discriminant formula**,**

$D=b^2-4ac$

$\Rightarrow(-4)^2-4(p)(3)$

$\Rightarrow 16-12p$

Since, roots are real.

$16-12p=0$

$\Rightarrow 16=12p$

$\Rightarrow p=\frac{16}{12}=\frac{4}{3}$

$\therefore p=\frac{4}{3}$

(ii) $x^2+(p-3)x+p=0$

Ans: Compare the given equation with $ax^2+bx+c=0$, to get

$a=1$

$b=p-3$

$c=p$

By using the discriminant formula,

$D=b^2-4ac$

$\Rightarrow(p-3)^2-4(1)(p)$

$\Rightarrow p^2-3^2-2(3)(p)-4p$

$\Rightarrow p^2+9-6p-4p$

$\Rightarrow p^2-10p+9$

Since, roots are real and have equal roots.

$p^2-10p+9=0$

Now let us factorise,

$\Rightarrow p^2-9p-p+9=0$

$\Rightarrow p(p-9)-1(p-9)=0$

$\Rightarrow (p-9)(p-1)=0$

So,

$\Rightarrow (p-9)=0$ or $(p-1)=0$

$\Rightarrow p=9$ or $p=1$

$\therefore p=1,9$

5. Find the value (s) of k for which each of the following quadratic equation has equal roots:

1.(i) $x^2+4kx+(k^2-k+2)=0$

Ans: Compare the given equation with $ax^2+bx+c=0$, to get

$a=1$

$b=4k$

$c=k^2-k+2$

By using the discriminant formula,

$D=b^2-4ac$

$\Rightarrow (4k)^2-4(1)\left(k^2-k+2\right)$

$\Rightarrow 16k^2-4k^2+4k-8$

$\Rightarrow 12k^2+4k-8$

As, roots are equal, $D=0$

$\Rightarrow 12k^2+4k-8=0$

Dividing by 4 on both sides, we get

$\Rightarrow 3k^2+k-2=0$

$\Rightarrow 3k^2+3k-k-2=0$

$\Rightarrow 3k(k+1)-1(k+2)=0$

$\Rightarrow (3k-1)(k+2)=0$

So,

$\Rightarrow 3k-1=0$ or $k+2=0$

$\Rightarrow k=\frac{1}{3}$ or $k=-2$

$\therefore k=\frac{1}{3},-2$

(ii) $(k-4)x^2+2(k-4)x+4=0$

Ans: Compare the given equation with $ax^2+bx+c=0$, to get

$a=k-4$

$b=2(k-4)$

$c=4$

By using the discriminant formula,

$D=b^2-4ac$

$\Rightarrow \left(2\left(k-4\right)\right)^2-4(k-4)(4)$

$\Rightarrow \left[4\left(k^2+16-8k\right)\right]-16\left(k-4\right)$

$\Rightarrow 4\left(k^2-8k+16\right)-16k+64$

$\Rightarrow 4\left(k^2-8k+16-4k+16\right)$

$\Rightarrow 4\left(k^2-12k+32\right)$

Since, roots are equal.

$\Rightarrow 4\left(k^2-12k+32\right)=0$

$\Rightarrow k^2-12k+32=0$

Now let us factorise,

$\Rightarrow k^2-8k-4k+32=0$

$\Rightarrow k\left(k-8\right)-4\left(k-8\right)=0$

$\Rightarrow \left(k-8\right)\left(k-4\right)=0$

So,

$\Rightarrow (k-8)=0$ or $(k-4)\neq 0$

$\Rightarrow k=8$ or $k \neq 4$
$\therefore k=8$

6. Find the value(s) of m for which each of the following quadratic equation has real and equal roots:

1.(i) $(3m+1)x^2+2(m+1)x+m=0$

Ans: Compare the given equation with $ax^2+bx+c=0$, to get

$a=3m+1$
$b=2(m+1)$
$c=m$

By using the discriminant formula,

$D=b^2-4ac$

$\Rightarrow [2(m+1)]^2-4(3m+1)(m)$

$\Rightarrow 4(m^2+1+2m)-4m(3m+1)$

$\Rightarrow 4(m^2+2m+1)-12m^2-4m$

$\Rightarrow 4m^2+8m+4-12m^2-4m$

$\Rightarrow -8m^2+4m+4$

Since, roots are equal.

$D=0$

$\Rightarrow -8m^2+4m+4=0$

Divide by 4, we get

$\Rightarrow -2m^2+m+1=0$

$\Rightarrow 2m^2-m-1=0$

Now let us factorise,

$\Rightarrow 2m^2-2m+m-1=0$

$\Rightarrow 2m(m-1)+1(m-1)=0$

$\Rightarrow (m-1)(2m+1)=0$

So,

$\Rightarrow (m-1)=0 or (2m+1)=0$

$\Rightarrow m=1 or 2m=-1$

$\Rightarrow m = 1 or m = -\frac{1}{2}$

$\therefore m = 1, \frac{-1}{2}$

(ii) $x^2 + 2(m-1)x + (m+5) = 0$

Ans: Compare the given equation with $ax^2 + bx + c = 0$, to get

$a = 1$

$b = 2(m-1)$

$c = m+5$

By using the discriminant formula,

$D = b^2 - 4ac$

$\Rightarrow [2(m-1)]^2 - 4(1)(m+5)$

$\Rightarrow \left[4\left(m^2 + 1 - 2m\right)\right] - 4m - 20$

$\Rightarrow 4m^2 - 8m + 4 - 4m - 20$

$\Rightarrow 4m^2 - 12m - 16$

Since, roots are equal.

$D = 0$

$\Rightarrow 4m^2 - 12m - 16 = 0$

Divide by 4, we get.

$\Rightarrow m^2 - 3m - 4 = 0$

Now let us factorise,

$\Rightarrow m^2 - 4m + m - 4 = 0$

$\Rightarrow m(m-4) + 1(m-4) = 0$

$\Rightarrow (m-4)(m+1) = 0$

So,

$\Rightarrow (m-4) = 0 or (m+1) = 0$

$\Rightarrow m = 4 or m = -1$

$\therefore m = 4, -1$

7. Find the values of k for which each of the following quadratic equation has equal roots:

Also, find the roots for those values of k in each case.

(i) $9x^2+kx+1=0$

Ans: Compare the given equation with $ax^2+bx+c=0$, to get

$a=9$

$b=k$

$c=1$

By using the discriminant formula,

$D=b^2-4ac$

$\Rightarrow k^2-4(9)(1)$

$\Rightarrow k^2-36$

Since, roots are equal.

$D=0$

$\Rightarrow k^2-36=0$

$\Rightarrow (k+6)(k-6)=0$

So,

$\Rightarrow (k+6)=0$ or $(k-6)=0$

$\Rightarrow k=-6$ or $k=6$

$\therefore k=6,-6$

Now, let us substitute in the equation

When $k=6$, then

$\Rightarrow 9x^2+kx+1=0$

$\Rightarrow 9x^2+6x+1=0$

$\Rightarrow (3x)^2+2(3x)(1)+12=0$

$\Rightarrow (3x+1)^2=0$

$\Rightarrow 3x+1=0$

$\Rightarrow 3x=-1$

$\therefore x=-\frac{1}{3},-\frac{1}{3}$

When $k=-6$, then

$\Rightarrow 9x^2+kx+1=0$

$\Rightarrow 9x^2-6x+1=0$

$\Rightarrow (3x)^2 - 2(3x)(1) + 12 = 0$

$\Rightarrow (3x - 1)^2 = 0$

$\Rightarrow 3x - 1 = 0$

$\Rightarrow 3x = 1$

$\Rightarrow x = \frac{1}{3}, \frac{1}{3}$

(ii) $x^2 - 2kx + 7k - 12 = 0$

Ans: Compare the given equation with $ax^2 + bx + c = 0$, to get

$a = 1$

$b = -2k$

$c = 7k - 12$

By using the discriminant formula,

$D = b^2 - 4ac$

$\Rightarrow (-2k)^2 - 4(1)(7k - 12)$

$\Rightarrow (4k)^2 - 28k + 48$

Since, roots are equal.

$D = 0$

$\Rightarrow (4k)^2 - 28k + 48 = 0$

Divide by 4, we get

$\Rightarrow k^2 - 7k + 12 = 0$

Now let us factorise,

$\Rightarrow k^2 - 3k - 4k + 12 = 0$

$\Rightarrow k(k - 3) - 4(k - 3) = 0$

$\Rightarrow (k - 3)(k - 4) = 0$

$\Rightarrow k - 3 = 0$ or $k - 4 = 0$

$\Rightarrow k = 3$ or $k = 4$

$\therefore k = 3, 4$

Now, let us substitute in the equation

When $k = 3$, then

By using the quadratic formula,

$$\Rightarrow x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$\Rightarrow \frac{[-(-2k)\pm\sqrt{0}]}{2(1)}$$

$$\Rightarrow \frac{2(3)}{2}=3$$

$\therefore x=3,3$

When $k=4$, then

By using the quadratic formula,

$$\Rightarrow x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$\Rightarrow x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Let $b^2 \ -4ac=D$

$$\Rightarrow \frac{[-(-2k)\pm\sqrt{0}]}{2(1)}$$

$$\Rightarrow \frac{2(4)}{2}=4$$

$x=4,4$

8. Find the value(s) of p for which the quadratic equation has equal roots. Also find these roots.

Ans: Compare the given equation with $ax^2+bx+c=0$, to get

$a=2p+1$

$b=-(7p+2)$

$c=(7p-3)$

By using the discriminant formula,

$D=b^2-4ac$

$$\Rightarrow\left[-(7p+2)\right]^2-4(2p+1)(7p-3)$$

$$\Rightarrow 49p^2+4+28p-4\left(14p^2-6p+7p-3\right)$$

$\Rightarrow 49p^2 + 4 + 28p - 56p^2 - 4p + 12$

$\Rightarrow -7p^2 + 24p + 16$

By factorising,

$\Rightarrow -7p^2 + 28p - 4p + 16 = 0$

$\Rightarrow -7p(p-4) - 4(p-4) = 0$

$\Rightarrow (p-4)(-7p-4) = 0$

So,

$\Rightarrow (p-4) = 0$ or $(-7p-4) = 0$

$\Rightarrow p = 4$ or $-7p = 4$

$\Rightarrow p = 4$ or $p = \frac{-4}{7}$

$\therefore$ Value of $p = 4, \frac{-4}{7}$

9. Find the value(s) of p for which the equation $2x^2 + 3x + p = 0$ has real roots.

Ans: Compare the given equation with $ax^2 + bx + c = 0$, to get

$a = 2$

$b = 3$

$c = p$

By using the discriminant formula,

$D = b^2 - 4ac$

$\Rightarrow 3^2 - 4(2)(p)$

$\Rightarrow 9 - 8p$

Since, roots are real.

$\Rightarrow 9 - 8p \geqslant 0$

$\Rightarrow 9 \geqslant 8p$

$\Rightarrow 8p \leqslant 9$

$\Rightarrow p \leqslant \frac{9}{8}$

10. Find the least positive value of k for which the equation $x^2 + kx + 4 = 0$ has real roots.

Ans: Compare the given equation with $ax^2+bx+c=0$, to get

$a=1$

$b=k$

$c=4$

By using the discriminant formula,

$D=b^2-4ac$

$\Rightarrow k^2-16$

Since, roots are real and positive.

$\Rightarrow k^2-16\geqslant 0$

$\Rightarrow k^2\geqslant 16$

$\Rightarrow k\geqslant 4$

$k=4$

$\therefore$ Value of $k=4$

11. Find the values of p for which the equation $3x^2-px+5=0$ **has real roots.**

Ans: Compare the given equation with $ax^2+bx+c=0$, to get

$a=3$

$b=-p$

$c=5$

By using the discriminant formula,

$D=b^2-4ac$

$\Rightarrow (-p)^2-4(3)(5)$

$\Rightarrow p^2-60$

Since, roots are real.

$\Rightarrow p^2-60\geqslant 0$

$\Rightarrow p^2\geqslant 60$

$\Rightarrow p\geqslant \pm\sqrt{60}$

$\Rightarrow p\geqslant \pm 2\sqrt{15}$

$\therefore$ Value of $p=+2\sqrt{15}$ or $-2\sqrt{15}$

Exercise 5.5

1. Find two consecutive natural numbers such that the sum of their squares is 61.

Ans: Let us assume the first natural number be x

Then the Second natural number will be $x+1$

Now using the data given in the question, we have

$x^2+(x+1)^2=61$

By simplification,

$\Rightarrow x^2+x^2+1^2+2x-61=0$

$\Rightarrow 2x^2+2x-60=0$

Dividing the equation by 2

$\Rightarrow x^2+x-30=0$

$x^2+6x-5x-30=0$

$\Rightarrow x(x+6)-5(x+6)=0$

$\Rightarrow (x+6)(x-5)=0$

So,

$\Rightarrow x=-6 \text{or } x=5$

As the natural numbers are only positive numbers

So, $x=5$

Therefore, the first natural number will be 5

And the second natural number will be $5+1=6$

(ii) Find two consecutive integers such that the sum of their squares is 61.

Ans: Let's assume the first integer number to be 'x'

Then the Second integer number be 'x + 1'

Using the data given in above question, we have

$x^2+(x+1)=61$

By simplifying the equation,

$\Rightarrow x^2+x^2+1^2+2x-61=0$

$\Rightarrow 2x^2+2x-60=0$

Dividing the equation by 2 ,

$\Rightarrow x^2+x-30=0$

$\Rightarrow x^2+6x-5x-30=0$

$\Rightarrow x(x+6)-5(x+6)=0$

$\Rightarrow (x+6)(x-5)=0$

Therefore,

$\Rightarrow x=-6$ or $x=5$

So, as integer can be both positive or negative

Therefore, if $x=-6$, then

The First integer number will be -6

And the Second integer number will be $(-6+1)=-5$

And if $x=5$, then

The First integer number will be 5

And the Second integer number will be $5+1=6$

2. (i) If the product of two positive consecutive even integers is 288, find the integers.

Ans: Let's assume the first positive even integer number to be **2x**

Then the Second even integer number will be $2x+2$

Using the data given in question, we have

$2x\times(2x+2)=288$

$\Rightarrow 4x^2+4x-288=0$

Dividing the equation by 4 , we get

$\Rightarrow x^2+x-72=0$

$\Rightarrow x^2+9x-8x-72=0$

$\Rightarrow x(x+9)-8(x+9)=0$

$\Rightarrow (x+9)(x-8)=0$

Therefore, the value of $x=8 \text{or} x=-9$

Since

x cannot be negative so the value of x will be 8

So, the First even integer will be $2x=2(8)=16$

And the Second even integer will be $2x+2=2(8)+2=18$

(ii) If the product of two consecutive even integers is 224, find the integers.

Ans: Let's assume the first positive even integer number to be **2x**

And the Second even integer number to be $2x+2$

Use the data which is given in question, we have

$2x\times(2x+2)=224$

$\Rightarrow 4x^2+4x-224=0$

Dividing the equation by 4

$\Rightarrow x^2+8x-7x-56=0$

$\Rightarrow x(x+8)-7(x+8)=0$

$\Rightarrow (x+8)(x-7)=0$

Therefore $-8 \text{ or } x=7$

Since negative value of x is not possible the value of x will be 7

So, the First even integer will be $2x=2(7)=14$

And the Second even integer will be $2x+2=2(7)+2=16$

(iii) Find two consecutive even natural numbers such that the sum of their squares is 340.

Ans: Let's assume the first positive even natural number to be **2x**

And the Second even number to be $2x+2$

Using the data given in the question, we have

$(2x)^2+(2x+2)^2=340$

$\Rightarrow 4x^2+4x^2+8x+4-340=0$

$\Rightarrow 8x^2+8x-336=0$

Dividing the equation by 8

$\Rightarrow x^2+x-42=0$

$\Rightarrow x^2+7x-6x-56=0$

$\Rightarrow x(x+7)-6(x+7)=0$

$\Rightarrow (x+7)(x-6)=0$

Therefore, the value of $x=-7 \text{ or } x=6$

since the value of x cannot be negative so the value of x will be 6

So, the First even natural number will be $2x=2\times6=12$

And the Second even natural number will be $2x+2=2\times6+2=14$

(iv) Find two consecutive odd integers such that the sum of their squares is 394.

Ans: Let us assume the first odd integer number to be $2x+1$

And the Second odd integer number to be $2x+3$

Use the data which is given in question, we have

$(2x+1)^2+(2x+3)^2=394$

$\Rightarrow 4x^2+4x+1+4x^2+12x+9-394=0$

$\Rightarrow 8x^2+16x-384=0$

Dividing the equation by 8

$\Rightarrow x^2+2x-48=0$

$\Rightarrow x^2+8x-6x-48=0$

$\Rightarrow x(x+8)-6(x+8)=0$

$\Rightarrow (x+8)(x-6)=0$

Therefore, the value of $x=-8 \text{or} x=6$

So, when x will be -8, then

First odd integer will be $2x+1=2(-8)+1=-16+1=-15$

And second odd integer will be $2x+3=2(-8)+3=-13$

When x will be 6, then

First odd integer will be $2x+1=2\times6+1=13$

Second odd integer will be $2x+3=2\times6+3=15$

So, the odd integers are $-15,-13,15,13$

3. The sum of two numbers is 9 and the sum of their squares is 41. Taking one number as x, form ail equation in x and solve it to find the numbers.

Ans: It is given that,

Sum of two number = 9

Let's assume first number to be x

Second number be $9-x$

Now using the information given in question,

$\Rightarrow x^2+(9-x)^2=41$

$\Rightarrow x^2+81-18x+x^2-41=0$

$\Rightarrow 2x^2-18x+40=0$

Dividing the equation by 2 ,

$\Rightarrow x^2-9x+20=0$

$\Rightarrow x^2-4x-5x+20=0$

$\Rightarrow x(x-4)-5(x-4)=0$

$\Rightarrow (x-4)(x-5)=0$

Therefore $4 or x=5$

When the value of x = 4, then

First number = $x=4$

Second number = $9-x=9-4=5$

And when the value of x = 5, then

First number = $x=5$

Second number = $9-x=9-5=4$

So, the required numbers are 4 and 5.

4. Five times a certain whole number is equal to three less than twice the square of the number. Find the number.

Ans: Let's assume the number to be 'x'

Using the information given in question,

$5x=2x^2-3$

$\Rightarrow 2x^2-5x-3=0$

$\Rightarrow 2x^2-6x+x-3=0$

$\Rightarrow 2x(x-3)+1(x-3)=0$

$\Rightarrow (x-3)(2x+1)=0$

Therefore $3 or x=\left(-\frac{1}{2}\right)$

Since the whole number cannot be negative

∴ The number is 3

5. Sum of two natural numbers is 8 and the difference of their reciprocals is 15. Find the numbers.

Ans: Assume the two numbers to be 'x' and 'y'

Using the data given in the question,

$\frac{1}{x}-\frac{1}{y}=\frac{2}{15}$ ….(1)

Given that $x+y=8$

So, $y=8-x$

Substitute value of y from equation (1), we get

$\Rightarrow \frac{1}{x} - \frac{1}{(8-x)} = \frac{2}{15}$

$\Rightarrow \frac{8-x-x}{x(8-x)} = \frac{2}{15}$

$\Rightarrow \frac{8-2x}{x(8-x)} = \frac{2}{15}$

$\Rightarrow 15(8-2x) = 2x(8-x)$

$\Rightarrow 120-30x = 16x-2x^2$

$\Rightarrow 120-30x-16x+2x^2 = 0$

$\Rightarrow 2x^2-46x+120 = 0$

Dividing the equation by 2

$\Rightarrow x^2-23x+60 = 0$

$\Rightarrow x^2-20x-3x+60 = 0$

$\Rightarrow x(x-20)-3(x-20) = 0$

$\Rightarrow (x-20)(x-3) = 0$

Therefore $20 \text{or} x = 3$

Sum of two natural numbers,

$\Rightarrow y = 8-x = 8-20 = -12$

which is a negative value.

So, value of $x = 3$,

$\Rightarrow y = 8-x = 8-3 = 5$

Hence, the value of x and y are 3 and 5.

6. The difference between the squares of two numbers is 45. The square of the smaller number is 4 times the larger number. Determine the numbers.

Ans: Let's assume larger number will be 'x'

And the smaller number be 'y'

Use the data which is given in question, we have

$x^2 - y^2 = 45$

We are given that

$y^2 = 4x$

So,

$\Rightarrow x^2 - 4x = 45$

$\Rightarrow x^2 - 4x - 45 = 0$

$\Rightarrow x^2 - 9x + 5x - 45 = 0$

$\Rightarrow x(x-9) + 5(x-9) = 0$

$\Rightarrow (x-9)(x+5) = 0$

Therefore $9 \text{or} x = -5$

If x = 9, then

The larger number = x = 9

Smaller number = y

$\Rightarrow y^2 = 4x$

$\Rightarrow y = \sqrt{4x} = \sqrt{4} \times 9 = \sqrt{36} = 6$

If $x = -5$, then

The larger number

$= x = -5$

Smaller number = y

$\Rightarrow y^2 = 4x$

$\Rightarrow y = \sqrt{4x} = \sqrt{4(-5)} = \sqrt{-20}$

Since this is not possible

Hence, the value of x and y are 9, 6.

7. There are three consecutive positive integers such that the sum of the square of the first and the product of the other two is 154. What are the integers?

Ans: Let's assume the first integer to be x

The second integer be $x+1$

And the third integer be $x+2$

Using the data given in question, we have

$x^2 + (x+1)(x+2) = 154$

$\Rightarrow x^2 + x^2 + 3x + 2 - 154 = 0$

$\Rightarrow 2x^2 + 3x - 152 = 0$

$\Rightarrow 2x^2 + 19x - 16x - 152 = 0$

$\Rightarrow x(2x+19) - 8(2x+19) = 0$

$\Rightarrow (2x+19)(x-8)=0$

Therefore $8 \text{or} x=\frac{-19}{2}$

Since the value of x cannot be negative

So, the value of x will be 8

Therefore

First integer $=x=8$

Second integer $=x+1=8+1=9$

Third integer $=x+2=8+2=10$

So, the numbers are **8,9,10**

8. (i) Find three successive even natural numbers, the sum of whose squares is 308.

Ans: Let's assume first even natural number to be 2x

Second even number be $2x+2$

Third even number be $2x+4$

Use the data which is given in question, we have

$\left(2x^2\right)+\left(2x+2\right)^2+\left(2x+4\right)^2=308$

$\Rightarrow 4x^2+4x^2+8x+4+4x^2+16x+16-308=0$

$\Rightarrow 12x^2+24x-288=0$

Dividing the equation by 12

$\Rightarrow x^2+2x-24=0$

$\Rightarrow x^2+6x-4x-24=0$

$\Rightarrow x(x+6)-4(x+6)=0$

$\Rightarrow (x+6)(x-4)=0$

Therefore $4 \text{or} x=-6$

The value cannot be negative

Value of x will be 4

First even natural number

$=2x=2\times 4=8$

Second even natural number $=2x+2=2\times 4+2=10$

Third even natural number $=2x+4=2\times 4+4=12$

So, the numbers are **8,10,12**

(ii) Find three consecutive odd integers, the sum of whose squares is 83.

Ans: Let's assume the three numbers to be $x, x+2, x+4$

Using the data given in question, we have

$$x^2+(x+2)^2+(x+4)^2=83$$

$$\Rightarrow x^2+x^2+4x+4+x^2+8x+16-83=0$$

$$\Rightarrow 3x^2+12x-63=0$$

Dividing the equation by 3

$$\Rightarrow x^2+4x-21=0$$

$$\Rightarrow x^2+7x-3x-21=0$$

$$\Rightarrow x(x+7)-3(x+7)=0$$

$$\Rightarrow (x+7)(x-3)=0$$

Therefore $3 \text{or} x=-7$

So, the numbers will be

$$x, x+2, x+4$$

$$\Rightarrow -7,-7+2,-7+4$$

$$\Rightarrow -7,-5,-3$$

Or the numbers will be $x, x+2, x+4$

$$\Rightarrow 3,3+2,3+4$$

$$\Rightarrow 3,5,7$$

9. In a certain positive fraction, the denominator is greater than the numerator by 3. If 1 is subtracted from both the numerator and denominator, the fraction is decreased by 1/14. Find the fraction.

Ans: Let the numerator be x

Denominator be

$$x+3$$

So, the fraction is $\dfrac{x}{x+3}$

Use the data which is given in question, we have

$$\frac{x-1}{x+3-1}=\frac{x}{x+3}-\frac{1}{14}$$

$\Rightarrow \frac{x-1}{x+2}=\frac{14x-x-3}{14(x+3)}$

$\Rightarrow \frac{x-1}{x+2}=\frac{13x-3}{14x+42}$

$\Rightarrow (x-1)(14x+42)=(x+2)(13x-3)$

$\Rightarrow 14x^2+42x-14x-42=13x^2-3x+26x-6$

$\Rightarrow x^2+5x-36=0$

$\Rightarrow x^2+9x-4x-36=0$

$\Rightarrow x(x+9)-4(x+9)=0$

$\Rightarrow (x+9)(x-4)=0$

Therefore $-9 \text{or} x=4$

Since it cannot have negative value

Value of x will be 4

The fraction would be

$\frac{x}{x+3}=\frac{4}{4+3}=\frac{4}{7}$

So, the required fraction is $\frac{4}{7}$

10. The sum of the numerator and denominator of a certain positive fraction is 8. If 2 is added to both the numerator and denominator, the fraction is increased by 4/35. Find the fraction.

Ans: Let the denominator be x

So, the numerator will be $8-x$

The fraction would be $\frac{x}{8-x}$

Using the data given in the question, we have

$\frac{8-x+2}{x+2}=\frac{8-x}{x}+\frac{4}{35}$

$\Rightarrow \frac{10-x}{x+2}=\frac{8-x}{x}+\frac{4}{35}$

$\Rightarrow \frac{10x-x^2-8x+x^2-16+2x}{x(x+2)}=\frac{4}{35}$

$\Rightarrow \dfrac{4x-16}{x^2+2x} = \dfrac{4}{35}$

$\Rightarrow 35(4x-16) = 4\left(x^2+2x\right)$

$\Rightarrow 140x - 560 = 4x^2 + 8x$

$\Rightarrow 4x^2 - 132x + 560 = 0$

Dividing the equation by 4

$\Rightarrow x^2 - 33x + 140 = 0$

$\Rightarrow x^2 - 28x - 5x + 140 = 0$

$\Rightarrow x(x-28) - 5(x-28) = 0$

$\Rightarrow (x-28)(x-5) = 0$

Therefore $28 \text{or} x = 5$

Since the sum of numerator and denominator is **8x** cannot be 28

So, the value of x will be 5

Now $\dfrac{8-x}{x} = \dfrac{8-5}{5} = \dfrac{3}{5}$

So, the required fraction is $\dfrac{3}{5}$

11. A two-digit number contains the bigger at ten's place. The product of the digits is 27 and the difference between two digits is 6. Find the number.

Ans: Let us assume unit's digit to be x

Ten's digit will be $x+6$

Number $= x + 10\left(x+6\right)$

$= x + 10x + 60$

$= 11x + 60$

Use the data which is given in question, we have

$\Rightarrow x(x+6) = 27$

$\Rightarrow x^2 + 6x - 27 = 0$

$\Rightarrow x^2 + 9x - 3x - 27 = 0$

$\Rightarrow x(x+9) - 3(x+9) = 0$

$\Rightarrow (x+9)(x-3) = 0$

Therefore $-9 \text{or} x = 3$

Since the value cannot be negative
Value of x will be 3
So, the number
$=11x+60=11\times 3+60=93$

12. A two-digit positive number is such that the product of its digits is 6. If 9 is added to the number, the digits interchange their places. Find the number. (2014)

Ans: Let us assume the 2-digit number be $xy=10x+y$

And Reversed digits be $yx=10y+x$

Use the data which is given in the question, we have
$10x+y+9=10y+x$

We are given that,
$\Rightarrow xy=6$

$\Rightarrow y=\frac{6}{x}$

Using the value of y we get

$\Rightarrow 10x+\frac{6}{x}+9=10\left(\frac{6}{x}\right)+x$

$\Rightarrow 10x^2+6+9x=60+x^2$

$\Rightarrow 9x^2+9x-54=0$

Dividing the equation by 9

$\Rightarrow x^2+x-6=0$

$\Rightarrow x^2+3x-2x-6=0$

$\Rightarrow x(x+3)-2(x+3)=0$

$\Rightarrow (x+3)(x-2)$

Therefore $-3 \text{or} x=2$

Value of x cannot be negative

The value of x will be 2

$\Rightarrow y=\frac{6}{x}=\frac{6}{2}=3$

Therefore 2-digit number
$=10x+y=10\times 2+3=23$

13. A rectangle of area 105 cm² has its length equal to x cm. Write down its breadth in terms of x. Given that the perimeter is 44 cm, write down an equation in x and solve it to determine the dimensions of the rectangle.

Ans: Perimeter of rectangle = 44cm

Length + breadth = $\frac{44}{2}$ = 22cm

Let us assume the length to be x

Then Breadth will be $22-x$

Use the data which is given in the question, we have

$x(22-x)=105$

$\Rightarrow 22x-x^2-105=0$

$\Rightarrow x^2-22x+105=0$

$\Rightarrow x^2-15x-7x+105=0$

$\Rightarrow x(x-15)-7(x-15)=0$

$\Rightarrow (x-15)(x-7)=0$

Therefore

$x=15 \text{or} x=7$

Since length > breadth,

$x=7$ is not admissible.

$\therefore$ Length $=15cm$

Breadth $=22-x=22-15=7cm$

14. A rectangular garden 10 m by 16 m is to be surrounded by a concrete walk of uniform width. Given that the area of the walk is 120 square meters, assuming the width of the walk to be x, form an equation in x and solve it to find the value of x.

Ans: Length of garden $=16cm$

Width $=10cm$

Let the width of walk be 'x' meter

Outer length $=16+2x$

Outer width $=10x+2x$

Using the data given in the question, we have

$(16+2x)(10+2x)-16(10)=120$

$\Rightarrow 160+32x+20x+4x^2-160-1201=0$

$\Rightarrow 4x^2+52x-120=0$

Dividing the equation by 4

$\Rightarrow x^2+13x-30=0$

$\Rightarrow x^2+15x-2x-30=0$

$\Rightarrow x(x+15)-2(x+15)=0$

$\Rightarrow (x+15)(x-2)=0$

Therefore $-15 or x=2$

Since x cannot can negative value

So, the value of x is 2

15. The length of a rectangle exceeds its breadth by 5 m. If the breadth was doubled and the length reduced by 9 m, the area of the rectangle would have increased by 140 m². Find its dimensions.

Ans: In first case:

Let us assume the length of the rectangle to be ‘x’ meter

Width = $(x-5)$ m

Area = $l\times b$

$=x(x-5)m^2$

In second case:

Length $=x-9m$

Width = $2(x-5)m$

Area = $(x-9)\times 2(x-5)m^2$

Use the data whichh is given in the question, we have

$2(x-9)(x-5)=x(x-5)+140$

$\Rightarrow 2(x^2-14x+45)=x^2-5x+140$

$\Rightarrow 2x^2-28x+90-x^2+5x-140=0$

$\Rightarrow x^2-23x-50=0$

$\Rightarrow x^2-25x+2x-50=0$

$\Rightarrow x(x-25)+2(x-25)=0$

$\Rightarrow (x-25)(x+2)=0$

Therefore $25 \text{or} x=-2$

Since x cannot have negative value

So, Length of the first rectangle will be 25meters.

Width

$=x-5=25-5=20m$

Area = l×b

$=25\times 20=500\ m^2$

16. The perimeter of a rectangular plot is 180 m and its area is 1800 m². Take the length of the plot as x m. Use the perimeter 180 m to write the value of the breadth in terms of x. Use the values of length, breadth and the area to write an equation in x. Solve the equation to calculate the length and breadth of the plot.

Ans: Perimeter of a rectangular field $=180m$

And area $=1800\ m^2$

Length of rectangular field be x

We know that,

Perimeter of rectangular field = 2 (length + breadth)

$\Rightarrow (\text{ length } + \text{ breadth }) = \frac{\text{perimeter}}{2}$

$\Rightarrow x + \text{ breadth } = \frac{180}{2}$

$\Rightarrow \text{ breadth } = 90-x$

area of the rectangular field is

$length \times breadth = 1800$

$\Rightarrow x\times(90-x)=1800$

$\Rightarrow x^2-90x+1800=0$

$\Rightarrow x^2-60x-30x+1800=0$

$\Rightarrow x(x-60)-30(x-60)=0$

$\Rightarrow (x-30)(x-60)=0$

Therefore $x=30 \text{or} x=60$

It is given that length is greater than its breadth,

So, for the rectangular field

Length $=60m$ and breadth $=90-60=30m$

17. The lengths of the parallel sides of a trapezium are (x + 9) cm and (2x – 3) cm and the distance between them is (x + 4) cm. If its area is 540 cm², find x.

Ans: $\text{Areaoftrapezium} = \frac{1}{2}\times(\text{sumofparallelsides})\times(height)$

We are given that,

the length of parallel sides is $(x+9)$ and $(2x-3)$

And height $=x+4$

Use the data which is given in question, we have

$\frac{1}{2}(x+9+2x-3)(x+4)=540$

$\Rightarrow (3x+6)(x+4)=1080$

$\Rightarrow 3x^2+12x+6x+24=1080$

$\Rightarrow 3x^2+18x-1056=0$

Dividing the equation by 3

$\Rightarrow x^2+6x-352=0$

$\Rightarrow x^2+22x-16x-352=0$

$\Rightarrow x(x+22)-16(x+22)=0$

$\Rightarrow (x+22)(x-16)=0$

Therefore $-22 \text{ or } x=16$

Value of x cannot be negative

So, $x=16$

18. If the perimeter of a rectangular plot is 68 m and the length of its diagonal is 26 m, find its area.

Ans: Perimeter $=68m$

Diagonal $=29m$

$length+breadth=\frac{perimeter}{2}=\frac{68}{2}=34m$

Let the length of the rectangular plot be x m

Then, breadth $=(34-x)m$

By using Pythagoras Theorem, the diagonal can be given by,

$length^2 + breadth^2 = diagonal^2$

$\Rightarrow x^2 + (34 - x)^2 = 26^2$

$\Rightarrow x^2 + 1156 + x^2 - 68x = 676$

$\Rightarrow 2x^2 - 68x + 480 = 0$

Dividing the equation by 2

$\Rightarrow x^2 - 34x + 240 = 0$

$\Rightarrow x^2 - 24x - 10x + 240 = 0$

$\Rightarrow x(x - 24) - 10(x - 24) = 0$

$\Rightarrow (x - 24)(x - 10) = 0$

Therefore $24 or x = 10$

We are given that length is greater than breadth

So, length $= 24m$ and Breadth $= 34 - 24 = 10m$

And, area of the rectangular plot $= 24 \times 10 = 240m^2$

19. If the sum of two smaller sides of a right-angled triangle is 17cm and the perimeter is 30cm, then find the area of the triangle.

Ans: Perimeter of the triangle $= 30cm$

Let the length of one of the two small sides be x cm

Then, the other side will be $= (17 - x)$ cm

The length of hypotenuse = perimeter – sum of other two sides

$= 30 - 17 = 13cm$

Using Pythagoras Theorem,

$x^2 + (17 - x)^2 = 13^2$

$\Rightarrow x^2 + 289 + x^2 - 34x = 169$

$\Rightarrow 2x^2 - 34x + 120 = 0$

Dividing the equation by 2

$\Rightarrow x^2 - 17x + 60 = 0$

$\Rightarrow x^2 - 12x - 5x + 60 = 0$

$\Rightarrow x(x - 12) - 5(x - 12) = 0$

$\Rightarrow (x - 12)(x - 5) = 0$

Therefore $12 or x = 5$

If,

$x=5$ then

First side is **5cm** and second side will be $(17-5)=12cm$

And if $x=12$ then

First side be **12cm** and second side will be $(17-12)=5cm$

Thus,

Area of the triangle $=\frac{1}{2}(5\times 12)=\frac{60}{2}=30cm^2$

20. The hypotenuse of grassy land in the shape of a right triangle is 1 metre more than twice the shortest side. If the third side is 7 metres more than the shortest side, find the sides of the grassy land.

Ans: Let the shortest side be x cm

Hypotenuse $=2x+1$

And third side $=x+7$

Using the Pythagoras theorem,

$(2x+1)^2=x^2+(x+7)^2$

$\Rightarrow 4x^2+1+4x=x^2+x^2+49+14x$

$\Rightarrow 4x^2-2x^2+4x-14x+1-49=0$

$\Rightarrow 2x^2-10x-48=0$

Dividing the equation by 2

$\Rightarrow x^2-5x-24=0$

$\Rightarrow x^2-8x+3x-24=0$

$\Rightarrow x(x-8)+3(x-8)=0$

$\Rightarrow (x-8)(x+3)=0$

Therefore $8 \text{or} x=-3$

Since the value of cannot be negative

So, the shortest side $=8m$

Third side $=x+7=8+7=15m$

And hypotenuse $=2x+1=2\times 8+1=17m$

21. Mohini wishes to fit three rods together in the shape of a right triangle. If the hypotenuse is 2 cm longer than the base and 4 cm longer than the shortest side, find the lengths of the rods.

Ans: Let the length of hypotenuse be x cm

then base will be $(x-2)$ cm

and shortest side will be $x-4$ cm

Use the data which is given in question, we have

$x^2=(x-2)^2+(x-4)^2$

$\Rightarrow x^2=x^2-4x+4+x^2-8x+16$

$\Rightarrow x^2-12x+20=0$

$\Rightarrow x^2-10x-2x+20=0$

$\Rightarrow x(x-10)-2(x-10)=0$

$\Rightarrow (x-10)(x-2)=0$

Therefore $10 \text{or} x=2$

We know that hypotenuse is the longest side so the value of x cannot be 2

So, the value of x will be 10

So, hypotenuse $=10cm$

Base $=10-2=8cm$

Shortest side $=10-4=6cm$

22. In a P.T. display, 480 students are arranged in rows and columns. If there are 4 more students in each row than the number of rows, find the number of students in each row.

Ans: Let's assume the number of students in each row be x

Total number of students is 480

So, the number of rows will be $=\frac{480}{x}$

Use the data which given in the question, we have

$x=\frac{480}{x}+4$

$\Rightarrow x^2=480+4x$

$\Rightarrow x^2-4x-480=0$

$\Rightarrow x^2-24x+20x-480=0$

$\Rightarrow x(x-24)+20(x-24)=0$

$\Rightarrow (x-24)(x+20)=0$

Therefore $24 \text{or} x=-20$

Since number of students cannot be negative
So, x will be 24 i.e.
Number of students in each row will be 24

23. In an auditorium, the number of rows is equal to the number of seats in each row. If the number of rows is doubled and number of seats in each row is reduced by 5, then the total number of seats is increased by 375. How many rows were there?

Ans: Let's assume the number of rows be x
then no. of seats in each row will also be x
so total number of seats $= x \times x = x^2$
Use the data which is given in question, we have

$2x(x-5) = x^2 + 375$

$\Rightarrow 2x^2 - 10x = x^2 + 375$

$\Rightarrow x^2 - 10x - 375 = 0$

$\Rightarrow x^2 - 25x + 15x - 375 = 0$

$\Rightarrow x(x-25) + 15(x-25) = 0$

$\Rightarrow (x-25)(x+15) = 0$

Therefore $25 \text{or} x = -15$
number of rows cannot be negative
So, the value of x will be 25 i.e.
Number of rows will be 25.

24. At an annual function of a school, each student gives the gift to every other student. If the number of gifts is 1980, find the number of students.

Ans: Let's assume the number of students to be x
then the number of gifts given will be $x-1$
Total number of gifts will be $x(x-1)$
Use the data which is given in the question,

$x(x-1) = 1980$

$\Rightarrow x^2 - x - 1980 = 0$

$\Rightarrow x^2 - 45x + 44x - 1980 = 0$

$\Rightarrow x(x-45) + 44(x-45) = 0$

$\Rightarrow (x-45)(x+44)=0$

Therefore $45 or x=-44$

We know that the number of students cannot be negative

So, the value of x will be 45 i.e.

Total number of students is 45.

25. A bus covers a distance of 240 km at a uniform speed. Due to heavy rain, its speed gets reduced by 10 km/h and as such it takes two hours longer to cover the total distance. Assuming the uniform speed to be 'x' km/h, form an equation and solve it to evaluate x.

Ans: We are given that

Distance $=240km$

Let the speed of a bus be $x\frac{km}{hr}$

Time taken $=\frac{D}{S}=\frac{240}{x} hours$

Due to heavy rain speed of bus has decreased

Therefore $speed=(x-10)\frac{km}{hr}$

Use the data which is given in question,

$\frac{240}{x}=\frac{240}{x-10}-2$

$\Rightarrow \frac{240}{x-10}-\frac{240}{x}=2$

$\Rightarrow \frac{240x-240x+2400}{x(x-10)}=2$

$\Rightarrow \frac{2400}{x^2-10x}=2$

$\Rightarrow 2x^2-20x-2400=0$

Dividing the equation by 2

$\Rightarrow x^2-10x-1200=0$

$\Rightarrow x^2-40x+30x-1200=0$

$\Rightarrow x(x-40)+30(x-40)=0$

$\Rightarrow (x-40)(x+30)=0$

Therefore $40 or x=-30$

The value of x denotes speed which cannot be negative

So, x will be 40

Speed of the bus $=40\frac{km}{hr}$

26. The speed of an express train is x km/hr and the speed of an ordinary train is 12 km/hr less than that of the express train. If the ordinary train takes one hour longer than the express train to cover a distance of 240 km, find the speed of the express train.

Ans: Let the speed of express train be x km

Then the speed of the ordinary train be $(x-12)km$

So the time taken to cover 240 km by the express train $=\frac{240}{x}hours$

Time taken by ordinary train to cover 240km $\frac{240}{x-12}hours$

Using the data given in the question,

$$\frac{240}{x-12}-\frac{240}{x}=1$$

$$\Rightarrow 240\left[\frac{1}{x-12}-\frac{1}{x}\right]=1$$

$$\Rightarrow 240\left[\frac{x-x+12}{x(x-12)}\right]=1$$

$$\Rightarrow 240\left[\frac{12}{x^2-12x}\right]=1$$

$$\Rightarrow 2880=x^2-12x$$

$$\Rightarrow x^2-12x-2880=0$$

$$\Rightarrow x^2-60x+48x-2880=0$$

$$\Rightarrow x(x-60)+48(x-60)=0$$

$$\Rightarrow (x-60)(x+48)=0$$

$x=60 or x=-48$

Therefore

Since the value of speed cannot be negative

So, speed of the express train will be 60 $\frac{km}{hr}$

27. A car covers a distance of 400 km at a certain speed. Had the speed been 12 km/h more, the time taken for the journey would have been 1 hour 40 minutes less. Find the original speed of the car.

Ans: Let the original speed of the car be $x\frac{km}{hr}$

It is given that,

Distance covered $=400km$

Time taken will be $=\frac{400}{x}hr$

In the second case speed is increased

So, speed$=\left(x+12\right)\frac{km}{hr}$

New time taken will be $=\frac{400}{x+12}hr$

Using the data given in the question,

$$\frac{400}{x}-\frac{400}{x+12}=1\frac{40}{60}=\frac{5}{3}$$

$$\Rightarrow 400\left[\frac{x+12-x}{x(x+12)}\right]=\frac{5}{3}$$

$$\Rightarrow \frac{400\times 12}{x^2+12x}=\frac{5}{3}$$

$$\Rightarrow 4800(3)=5\left(x^2+12x\right)$$

$$\Rightarrow 5x^2+60x-14400=0$$

Dividing the equation by 5

$$\Rightarrow x^2+12x-2880=0$$

$$\Rightarrow x^2+60x-48x-2880=0$$

$$\Rightarrow x\left(x+60\right)-48\left(x+60\right)=0$$

$$\Rightarrow \left(x+60\right)\left(x-48\right)=0$$

Therefore $-60 or x = 48$

Hence the speed cannot be negative

So, the original speed is $48\frac{km}{hr}$

28. An aeroplane travelled a distance of 400 km at an average speed of x km/hr. On the return journey, the speed was increased by 40 km/hr. Write down an expression for the time taken for

(i) the onward journey

Ans: Distance $= 400 km$

Speed of aeroplane $= x\frac{km}{hr}$

So, the time taken $= \frac{400}{x} hr$

(ii) the return journey.

If the return journey took 30 minutes less than the onward journey, write down an equation in x and find its value.

Ans: In return journey speed is increased by $40\frac{km}{hr}$

So, the time taken$= \frac{400}{x+40} hr$

Using the data given in the question,

$$\frac{400}{x} - \frac{400}{x+40} = 30 min = \frac{1}{2}$$

$$\Rightarrow 400\left[\frac{1}{x} - \frac{1}{x+40}\right] = \frac{1}{2}$$

$$\Rightarrow 400\left[\frac{40+x-x}{x(x+40)}\right] = \frac{1}{2}$$

$$\Rightarrow 400(40)(2) = x^2 + 40x$$

$$\Rightarrow x^2 + 40x - 32000 = 0$$

$$\Rightarrow x^2 + 200x - 160x - 3200 = 0$$

$$\Rightarrow x(x+200) - 160(x+200) = 0$$

$$\Rightarrow (x+200)(x-160) = 0$$

Therefore $-200 or x=160$

Since, speed cannot be negative

So, speed will be $160\dfrac{km}{hr}$

29. The distance by road between two towns A and B, is 216 km, and by rail it is 208 km. A car travels at a speed of x km/hr, and the train travels at a speed which is 16 km/hr faster than the car. Calculate:

(i) The time taken by the car, to reach town B from A, in terms of x;

Ans: It is given that

the distance by road between A and B $=216km$

and the distance by rail $=208km$

let the speed of car be $x\dfrac{km}{hr}$

time taken by car $=\dfrac{216}{x}hr$

(ii) The time taken by the train, to reach town B from A, in terms of x;

Ans: Speed of the train $=\left(x+16\right)\dfrac{km}{hr}$

So, the time taken by train $=\dfrac{208}{x+16}hr$

(iii) If the train takes 2 hours less than the car, to reach town B, obtain an equation in x and solve it.

Ans: Using the data given in the question,

$$\frac{216}{x}-\frac{208}{x+16}=2$$

$$\Rightarrow \frac{216x+216\times 16-208x}{x(x+16)}=2$$

$$\Rightarrow \frac{8x+3456}{x^2+16x}=2$$

$$\Rightarrow 8x+3456=2x^2+32x$$

$$\Rightarrow 2x^2+24x-3456=0$$

Dividing the equation by 2

$\Rightarrow x^2 + 12x - 1728 = 0$

$\Rightarrow x^2 + 48x - 36x - 1728 = 0$

$\Rightarrow x(x+48) - 36(x+1728) = 0$

$\Rightarrow (x+48)(x-36) = 0$

Therefore $-48 or x = 36$

(iv) Hence find the speed of the train.

Ans: We know that the speed cannot be negative

So, value of x will be 36

Speed of the train

$= x + 16 = 36 + 16 = 52\frac{km}{hr}$

30. An aeroplane flying with a wind of 30 km/hr takes 40 minutes less to fly 3600 km, than what it would have taken to fly against the same wind. Find the planes speed of flying in still air.

Ans: Let's assume the speed of the plane in still air be $x\frac{km}{hr}$

It is given that

Speed of wind $= 30\frac{km}{hr}$

Distance $= 300 km$

So, time taken with the wind $= \frac{3600}{x+30}$

And against the wind will be $\frac{3600}{x-30}$

Using the data in the question,

$\frac{3600}{x-30} - \frac{3600}{x+30} = 40 min = \frac{2}{3} hr$

$\Rightarrow 3600\left[\frac{1}{x-30} - \frac{1}{x+30}\right] = \frac{2}{3}$

$\Rightarrow 3600\left[\frac{x+30-x+30}{(x-30)(x+30)}\right] = \frac{2}{3}$

$$\Rightarrow \frac{3600 \times 60}{x^2 - 900} = \frac{2}{3}$$

$$\Rightarrow 2x^2 - 1800 = 3600 \times 60 \times 3$$

$$\Rightarrow 2x^2 = 646200$$

$$\Rightarrow x^2 = 323100$$

$$\Rightarrow x \approx 568 \frac{km}{hr}$$

Therefore, the speed of plane in still air is $568 \frac{km}{hr}$

31. A school bus transported an excursion party to a picnic spot 150 km away. While returning, it was raining and the bus had to reduce its speed by 5 km/hr, and it took one hour longer to make the return trip. Find the time taken to return.

Ans: Given that

Distance $= 150 km$

Let the speed of bus $= x \frac{km}{hr}$

So, time taken is $\frac{150}{x} hr$

Returning speed is decreased

So, the speed is $(x - 5) \frac{km}{hr}$

So now the time will be $= \frac{150}{x-5} hr$

Using the data given in the question,

$$\frac{150}{x-5} - \frac{150}{x} = 1$$

$$\Rightarrow 150\left[\frac{1}{x-5} - \frac{1}{x}\right] = 1$$

$$\Rightarrow 150\left[\frac{x - x + 5}{x(x-5)}\right] = 1$$

$\Rightarrow \dfrac{150\times5}{x^2-5x}=1$

$\Rightarrow x^2-5x-750=0$

$\Rightarrow x^2-30x+25x-750=0$

$\Rightarrow x(x-30)+25(x-30)=0$

$\Rightarrow (x-30)(x+25)=0$

Therefore $30 or x=-25$

Since speed cannot be negative

So, speed

$=30\dfrac{km}{hr}$

Time taken while returning $=\dfrac{150}{x-5}=\dfrac{150}{30-5}=6hr$

32. A boat can cover 10 km up the stream and 5 km down the stream in 6 hours. If the speed of the stream is 1.5 km/hr. find the speed of the boat in still water.

Ans: It is given that

Distance up stream $=10km$

and downstream $=5km$

Total time taken is $=6hours$

Speed of stream = $1.5\dfrac{km}{hr}$

Let's assume the speed of a boat in still water be $x\dfrac{km}{hr}$

Using the data given in question,

$\dfrac{10}{x-1.5}+\dfrac{5}{x+1.5}=6$

$\Rightarrow 10x+1.5+5x+5x-7.5=6(x-1.5)(x+1.5)$

$\Rightarrow 15x+7.5=6x^2-13.5$

$\Rightarrow 6x^2-15x-21=0$

Dividing the equation by 3

$\Rightarrow 2x^2-5x-7=0$

$\Rightarrow 2x^2-7x+2x-7=0$

$\Rightarrow x(2x-7)+1(2x-7)=0$

$\Rightarrow (x+1)(2x-7)=0$

Therefore $-1 or x=\frac{7}{2}=3.5$

Since the speed cannot be negative

So, the speed of the boat will be

$3.5\frac{km}{hr}$

33. Two pipes running together can fill a tank in $11\frac{1}{9}$ minutes. If one pipe takes 5 minutes more than the other to fill the tank, find the time in which each pipe would/fill the tank.

Ans: Let the time taken by one pipe be x minutes

Then time taken by second pipe will be $(x+5)$ minutes

Time taken by both pipes = $11\frac{1}{9}min$

Using the data given in the question,

$\frac{1}{x}+\frac{1}{x+5}=\frac{9}{100}$

$\Rightarrow \frac{(x+5)+x}{x(x+5)}=\frac{9}{100}$

$\Rightarrow \frac{2x+5}{x^2+5x}=\frac{9}{100}$

$\Rightarrow 9x^2+45x=200x+500$

$\Rightarrow 9x^2-155x-500=0$

$\Rightarrow 9x^2-180x+25x-500=0$

$\Rightarrow 9x(x-20)+25(x-20)=0$

$\Rightarrow (x-20)(9x+25)=0$

Therefore $20 or x=\left(\frac{-25}{9}\right)$

Since the time cannot be negative

So, the first pipe will fill tank in $20\min$

And the second pipe will fill tank in $20+5=25\text{min}$

34. (i) Rs. 480 is divided equally among 'x' children. If the number of children was 20 more than each would have got Rs. 12 less. Find 'x'.

Ans: Share of each child will be Rs $\frac{480}{x}$

Using the data given in the question,

$$\frac{480}{x+20}=\frac{480}{x}-12$$

$$\Rightarrow \frac{480}{x+20}=\frac{480-12x}{x}$$

$$\Rightarrow \frac{480}{x+20}=\frac{12(40-x)}{x}$$

$$\Rightarrow (x+20)(40-x)12=480x$$

$$\Rightarrow x^2+20x-800=0$$

$$\Rightarrow x^2+40x-20x-800=0$$

$$\Rightarrow x(x+40)-20(x+40)=0$$

$$\Rightarrow (x+40)(x-20)=0$$

Therefore $-40 or x=20$

Since number of children cannot be negative

So, number of children is 20

(ii) Rs. 7500 is divided equally among a certain number of persons. Had there been 20 more persons, each would have received 100 more. Find the original number of persons.

Ans: Let the original number of person be x, then 7500 divided equally between x person, each one gets $=\frac{7500}{x}$

7500 divided equally between $x-20$ children

each one gets $75=\frac{7500}{x-20}$

According to the question

$$\frac{7500}{x-20}=\frac{7500}{x}+\frac{100}{1}$$

$$\frac{7500}{x-20}=\frac{7500+100x}{x}$$
$$7500x=(x-20)(7500+100x)$$
$$75x=(x-20)(75+x)$$
$$75x=75x+x^2-1500-20x$$
$$x^2-20x-1500=0$$
$$x=\frac{20\pm\sqrt{400-4(-1500)}}{2}$$
$$x=\frac{20\pm\sqrt{400+6000}}{2}$$
$$x=\frac{20\pm 80}{2}$$
$$x=\frac{20+80}{2} \text{ or } x=\frac{20-80}{2}$$
$x=50$ or $x=-30$ (not possible)

$\therefore$ original number of children =50

So, the number of persons in the beginning will be 50

35. 2x articles cost Rs. (5x + 54) and (x + 2) similar articles cost Rs. (10x – 4), find x.

Ans: We are given that

Cost of **2x** articles $=5x+54$

So, the cost of 1 article will be $\frac{5x+54}{2x}$ …(1)

Again, the cost of $x+2$ articles $=10x-4$

So, the cost of 1 article will be $\frac{10x-4}{x+2}$ …(2)

Equation 1 and 2 are equal so
$$\frac{5x+54}{2x}=\frac{10x-4}{x+2}$$
$$\Rightarrow 5x+54(x+2)=10x-4(2x)$$
$$\Rightarrow 5x^2+10x+54x+108=20x^2-8x$$
$$\Rightarrow 15x^2-72x-108=0$$

Dividing the equation by 3

$\Rightarrow 5x^2 - 24x - 36 = 0$

$\Rightarrow 5x^2 - 30x + 6x - 36 = 0$

$\Rightarrow 5x(x-6) + 6(x-6) = 0$

$\Rightarrow (5x+6)(x-6) = 0$

Therefore $6 or x = -\frac{6}{5}$

Since the number cannot be negative

So, the value of x will be 6

36. A trader buys x articles for a total cost of Rs. 600.
(i) Write down the cost of one article in terms of x. If the cost per article were Rs. 5 more, the number of articles that can be bought for Rs. 600 would be four less.

Ans: It is given that

Total cost of articles $= Rs.600$

Total number of articles $= x$

So, the value of one article will be $\frac{600}{x}$

In the next case the value of the article is given as

$\frac{600}{x} + 5 = \frac{600}{x-4}$

(ii) Write down the equation in x for the above situation and solve it to find x.

Ans: The above equation is

$\frac{600}{x} + 5 = \frac{600}{x-4}$

$\Rightarrow \frac{600+5x}{x} = \frac{600}{x-4}$

$\Rightarrow 600 + 5x(x-4) = 600x$

$\Rightarrow 600x - 2400 + 5x^2 - 20x = 600x$

$\Rightarrow 5x^2 - 20x - 2400 = 0$

Dividing the equation by 5

$\Rightarrow x^2 - 4x - 480 = 0$

$\Rightarrow x^2 - 24x + 20x - 480 = 0$
$\Rightarrow x(x-24) + 20(x-24) = 0$
$\Rightarrow (x-24)(x+20) = 0$

Therefore $24 or x = -20$

We know that the number of articles cannot be negative

So total number of articles is 24

37. A shopkeeper buys a certain number of books for Rs 960. If the cost per book was Rs 8 less, the number of books that could be bought for Rs 960 would be 4 more. Taking the original cost of each book to be Rs x, write an equation in x and solve it to find the original cost of each book.

Ans: Let the original cost be **Rs.x**

Total No. of books bought will be

$\frac{960}{x}$

New cost of books $= Rs.(x-8)$

Then no. of books bought will be $\frac{960}{x-8}$

If 4 more books are bought then the cost will be $\frac{960}{x} + 4$

Using the data given in the question,

$$\frac{960}{x-8} - \frac{960}{x} = 4$$

$$\Rightarrow 960\left[\frac{1}{x-8} - \frac{1}{x}\right] = 4$$

$$\Rightarrow 960\left[\frac{x - x + 8}{x(x-8)}\right] = 4$$

$$\Rightarrow \frac{960 \times 8}{x^2 - 8x} = 4$$

$$\Rightarrow 960 \times 8 = 4\left(x^2 - 8x\right)$$

$$\Rightarrow x^2 - 8x - 1920 = 0$$

$$\Rightarrow x=\frac{-(-8)\pm\sqrt{(-8)^2-4(1)(-1920)}}{2}$$

$$\Rightarrow x=\frac{8\pm\sqrt{64+7680}}{2}=\frac{8\pm\sqrt{7744}}{2}$$

$$\Rightarrow=\frac{8\pm 88}{2}=\frac{8+88}{2},\frac{8-88}{2}$$

$$\Rightarrow=48,-40$$

Since the cost cannot have negative value

So, the cost of book is **Rs.48**

38. A piece of cloth costs Rs. 300. If the piece was 5 metres longer and each metre of cloth costs Rs. 2 less, the cost of the piece would have remained unchanged. How long is the original piece of cloth and what is the rate per metre?

Ans: Given that

The total cost of cloth piece

$=Rs.300$

Let the length of the piece of cloth in the beginning = x m

Then the cost of 1 metre $=Rs.\frac{300}{x}$

In the second case, the length of cloth $=x+5$

Now the cost of 1 metre will be $=Rs.\frac{300}{x+5}$

Using the data given in the question,

$$\frac{300}{x}-\frac{300}{x+5}=2$$

$$\Rightarrow 300\left[\frac{1}{x}-\frac{1}{x+5}\right]=2$$

$$\Rightarrow 300\left[\frac{x+5-x}{x(x+5)}\right]=2$$

$$\Rightarrow \frac{300\times 5}{x^2+5x}=2$$

$$\Rightarrow x^2+5x-750=0$$

$\Rightarrow x^2+30x-25x-750=0$

$\Rightarrow x(x+30)-25(x+30)=0$

$\Rightarrow (x-25)(x+30)=0$

Therefore $25 or x=-30$

Since the length cannot be negative

So, the length of the cloth is **25m** And rate will be $=Rs.\frac{300}{25}=12$

39. The hotel bill for a number of people for an overnight stay is Rs. 4800. If there were 4 more, the bill each person had to pay would have reduced by Rs. 200. Find the number of people staying overnight.

Ans: Let the number of people be x

Given that Amount of bill $=Rs.4800$

So, bill for each person $=Rs.\frac{4800}{x}$

In the second case,

It is given that

The number of people $=x+4$

So, now the bill of each person $=\frac{4800}{x+4}$

Using the data given in the question,

$$\frac{480}{x}-\frac{480}{x+4}=200$$

$$\Rightarrow 4800\left[\frac{1}{x}-\frac{1}{x+4}\right]=200$$

$$\Rightarrow 4800\left[\frac{x+4-x}{x\left(x+4\right)}\right]=200$$

$$\Rightarrow \frac{4800\times 4}{x^2+4x}=200$$

$\Rightarrow 200x^2+800x-19200=0$

Dividing the equation by 200

$\Rightarrow x^2+4x-96=0$

$\Rightarrow x^2 + 12x - 8x - 96 = 0$

$\Rightarrow x(x+12) - 8(x+12) = 0$

$\Rightarrow (x+12)(x-8) = 0$

Therefore $-12 or x = 8$

We know that the number of people cannot be negative

So, the number of people is 8

40. A person was given Rs. 3000 for a tour. If he extends his tour programme by 5 days, he must cut down his daily expenses by Rs. 20. Find the number of days of his tour programme.

Ans: Let the number of days of tour programme be x

Given that

Amount =**Rs.3000**

So, expenses for each day $= \frac{3000}{x}$

In other case no. of days $= x + 5$

Then the expenses of each day $= \frac{3000}{x+5}$

Using the data given in the question,

$$\frac{3000}{x} - \frac{300}{x+5} = 20$$

$$\Rightarrow 3000\left[\frac{1}{x} - \frac{1}{x+5}\right] = 20$$

$$\Rightarrow 3000\left[\frac{x+5-x}{x^2+5x}\right] = 20$$

$\Rightarrow 3000 \times 5 = 20x^2 + 100x$

$\Rightarrow 20x^2 + 100x - 15000 = 0$

Dividing the equation by 20

$\Rightarrow x^2 + 5x - 750 = 0$

$\Rightarrow x^2 + 30x - 25x - 750 = 0$

$\Rightarrow x(x+30) - 25(x+30) = 0$

$\Rightarrow (x+30)(x-25) = 0$

Therefore $-30 or x = 25$

Since the number of days cannot have negative value
So, the number of days is 25

41. Ritu bought a saree for Rs. 60 x and sold it for Rs. (500 + 4x) at a loss of x%. Find the cost price.

Ans: It is given that

The cost price of saree $= Rs.60x$

And selling price $= Rs.(500+4x)$

Loss = $x\%$

Using the data given in the question,

$$S.P. = C.P. \times \frac{100 - LOSS\%}{100}$$

$$\Rightarrow 500 + 4x = \frac{60x(100-x)}{100}$$

$$\Rightarrow 50000 + 400x = 6000x - 60x^2$$

$$\Rightarrow 60x^2 - 5600x + 50000 = 0$$

Dividing the equation by 20

$$\Rightarrow 3x^2 - 280x + 2500 = 0$$

$$\Rightarrow 3x^2 - 30x - 250x + 2500 = 0$$

$$\Rightarrow 3x(x-10) - 250(x-10) = 0$$

$$\Rightarrow (x-10)(3x-250) = 0$$

Therefore $10 or x = \frac{250}{3}$

It is given that loss is x%

Therefore, the value of x will be **10**

Cost price $= 60x = 60 \times 10 = 600Rs$

42. (i) The sum of the ages of Vivek and his younger brother Amit is 47 years. The product of their ages in years is 550. Find their ages.

Ans: Let Vivek's present age be x years.

His brother's age = $(47-x)$ years

Using the data given in the question,

$$x(47-x) = 550$$

$\Rightarrow x^2 - 47x + 550 = 0$

$\Rightarrow x^2 - 25x - 22x + 550 = 0$

$\Rightarrow x(x-25) - 22(x-25) = 0$

$\Rightarrow (x-25)(x-22) = 0$

$x = 25$

Therefore $25 or x = 22$

If then $47 - 25 = 22$

And if $x = 22$ then $47 - 22 = 25$

Which does not satisfy the given condition

So, the age of Vivek will be **25years**

And his younger brother will be

(ii) Paul is x years old and his father's age is twice the square of Paul's age. Ten years hence, the father's age will be four times Paul's age. Find their present ages.

Ans: Let the age of Paul be x years

Then his father's age will be $2x^2$

After 10 years,

Age of Paul will be $x + 10$

And his father's age will be $2x^2 + 10$

Using the information given in the question,

$2x^2 + 10 = 4(x + 10)$

$\Rightarrow 2x^2 + 10 = 4x + 40$

$\Rightarrow 2x^2 - 4x - 30 = 0$

Dividing the equation by 2

$\Rightarrow x^2 - 2x - 15 = 0$

$\Rightarrow x^2 - 5x + 3x - 15 = 0$

$\Rightarrow x(x-5) + 3(x-5) = 0$

$\Rightarrow (x-5)(x+3) = 0$

Therefore $x = 5 or x = -3$

Since the age cannot have negative value

$2x^2 = 2 \times 25 = 50 years$

So, age of Paul will be 5years

And his father's age will be 50 years.

43. The age of a man is twice the square of the age of his son. Eight years hence, the age of the man will be 4 years more than three times the age of his son. Find the present age.

Ans: Let the present age of the son $= x$ years then, the present age of the man $= 2x^2$ years. 8 years hence,

The age of son will be $= (x+8)$ years and the age of man $= \left(2x^2+8\right)$ years

According to the problem,

$2x^2+8=3(x+8)+4$

$\Rightarrow 2x^2+8=3x+24+4$

$\Rightarrow 2x^2-3x-24-4+8=0$

$\Rightarrow 2x^2-3x-20=0$

$\Rightarrow 2x^2-8x+5x-20=04$

$\Rightarrow 2x(x-4)+5(x-4)=0$

$\Rightarrow (x-4)(2x+5)=0$

Either $x-4=0$,

then $x=4$

or

$2x+5=0$

then $2x=-5$

$\Rightarrow x=-\dfrac{5}{2}$

But, it is not possible.

The present age of the son $=4$ years

And present age of the man $=2x^2 \;=2(4)^2$ years $=32$ years.

44. Two years ago, a man's age was three times the square of his daughter's age. Three years hence, his age will be four times his daughter's age. Find their present ages.

Ans: 2 years ago,

Let the age of daughter $= x$

Age of man = $3x^2$

Then present age of daughter $= x+2$

And mean = $3x^2+2$

After 3 years, the age of the daughter $= x+2+3 = x+5$

Man $= 3x^2+2+3 = 3x^2+5$

Using the data information in the question, we have

$3x^2+5 = 4(x+5)$

$\Rightarrow 3x^2+5 = 4x+20$

$\Rightarrow 3x^2-4x-15 = 0$

$\Rightarrow 3x^2-9x+5x-15 = 0$

$\Rightarrow 3x(x-3)+5(x-3) = 0$

$(x-3)(3x+5) = 0$

So, $x = 3 or x = \frac{-5}{3}$

Since, negative value is not possible, as age can't be negative

If $x=3$, then Present age of man $= 3x^2+2$

$= 3(3)^2+2 = 27+2 = 29 years$

Age of daughter $= x+2 = 3+2 = 5 years$

45.The length (in cm) of the hypotenuse of a right-angled triangle exceeds the length of one side by 2 cm and exceeds twice the length of another side by 1 cm. Find the length of each side. Also, find the perimeter and the area of the triangle.

Ans: Let the length of one side = x cm

and other side = y cm.

then hypotenuse is $x+2$, and $2y+1$

$x+2 = 2y+1$

$\Rightarrow x-2y = 1-2$

$\Rightarrow x-2y = -1$

$\Rightarrow x = 2y-1 \qquad \ldots(1)$

and by using Pythagoras theorem,

$\Rightarrow x^2+y^2=(2y+1)^2$

$\Rightarrow x^2+y^2=4y^2+4y+1$

Using the value of x from eq.1,

$\Rightarrow (2y+1)^2+y^2=4y^2+4y+1$

$\Rightarrow 4y^2-4y+1+y^2=4y^2+4y+1$

$\Rightarrow 4y^2-4y+1+y^2-4y^2-4y-1=0$

$\Rightarrow y^2-8y=0$

$\Rightarrow y(y-8)=0$

Therefore, $y=0 or y=8$

Since the value cannot be zero

So, $y=8$

Substituting the value of y in (i)

$x=2(8)-1=15$

Length of one side = 15 cm and

Length of another side = 8 cm and hypotenuse $=x+2=15+2=17cm$

:. Perimeter = $15+8+17=40cm$

And Area = $\frac{1}{2}\times$one side x other side $\Rightarrow \frac{1}{2}\times 15\times 8=60\,\text{cm}^2$

46. If twice the area of a smaller square is subtracted from the area of a larger square, the result is 14 cm². However, if twice the area of the larger square is added to three times the area of the smaller square, the result is 203 cm². Determine the sides of the two squares.

Ans: Let the side of smaller square = x cm

and side of bigger square = y cm

Using the data given the question, we have

$y^2+2x^2=14$ …. (i)

$2y^2+3x^2=203$ …. (ii)

Multiply eq.(i) by 2 and eq.(ii) by 1

$\Rightarrow 2y^2-4x^2=28$

$\Rightarrow 2y^2 + 3x^2 = 203$

Subtracting, we get,

$\Rightarrow -7x^2 = -175$

$\Rightarrow x^2 = 25$

$\Rightarrow x^2 - 25 = 0$

$\Rightarrow (x-5)(x+5) = 0$

Therefore, $x = 5$ or $x = -5$

Since, negative value is not possible

So, $x = 5$

Substitute the value of x in (i)

$\Rightarrow y^2 - 2(5)^2 = 14$

$\Rightarrow y^2 = 14 + 2 \times 25$

So, the side of smaller square = 5 cm and side of bigger square = 8 cm

MCQ

1. Which of the following is not a quadratic equation?

(a) $(x+2)^2 = 2(x+3)$

(b) $x^2 + 3x = (-1)(1-3x)$

(c) $(x+2)(x-1) = x^2 - 2x - 3$

(d) $x^3 - x^2 + 2x + 1 = (x+1)^3$

Ans: Here, we'll determine whether a quadratic equation exists for each option individually.

(a) $(x+2)^2 = 2(x+3)$

$\Rightarrow x^2 + 4x + 4 = 2x + 6$

$\Rightarrow x^2 + 4x - 2x + 4 - 6 = 0$

$\Rightarrow x^2 + 2x - 2 = 0$

Option (a) is a quadratic equation.

(b) $x^2 + 3x = (-1)(1-3x)$

$\Rightarrow x^2 + 3x = -1 + 3x$

$\Rightarrow x^2 + 1 = 0$

Option (b) is a quadratic equation.

(c) $(x+2)(x-1)=x^2-2x-3$

$\Rightarrow x^2-x+2x-2=x^2-2x-3$

$\Rightarrow x^2-x^2+x+2x-2+3=0$

$\Rightarrow 3x+1=0$

Option (c) is not a quadratic equation.

(d) $x^3-x^2+2x+1=(x+1)^3$

$\Rightarrow x^3-x^2+2x+1=x^3+3x^2+3x+1$

$\Rightarrow -x^2+2x=3x^2+3x$

$\Rightarrow 4x^2+x=0$

Option (d) is a quadratic equation.

Therefore, option (c) is correct.

2. If $\frac{1}{2}$ is a root of the quadratic equation $4x^2-4kx+k+5=0$, then the value of k is

(a) -6

(b) -3

(c) 3

(d) 6

Ans: $\frac{1}{2}$ is a root of the equation.

$\Rightarrow 4x^2-4kx+k+5=0$

Substituting the value of $x=\frac{1}{2}$ in the equation

$\Rightarrow 4(\frac{1}{2})^2-4\times k\times\frac{1}{2}+k+5=0$

$\Rightarrow 1-2k+k+5=0$

$\Rightarrow -k+6=0$

$\Rightarrow k=6$

Hence, option (d) is correct.

3. The roots of the equation $x^2-3x-10=0$ are

(a) **2, -5**
(b) **-2, 5**
(c) **2, 5**
(d) **-2, -5**

Ans: We shall utilize the quadratic formula $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ to find the equation's roots.

Here, in the quadratic equation $x^2 - 3x - 10 = 0$,
$a = 1, b = -3$ and $c = -10$

$$\Rightarrow x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4 \times 1 \times (-10)}}{2 \times 1}$$

$$\Rightarrow x = \frac{3 \pm \sqrt{9 + 40}}{2}$$

$$\Rightarrow x = \frac{3 \pm \sqrt{49}}{2}$$

$$\Rightarrow x = \frac{3 \pm 7}{2}$$

When the positive sign is taken in this equation,

$$\Rightarrow x = \frac{3 + 7}{2}$$

$$\Rightarrow x = \frac{10}{2}$$

$$\therefore x = 5$$

If a negative sign is taken in this equation,

$$\Rightarrow x = \frac{3 - 7}{2}$$

$$\Rightarrow x = \frac{-4}{2}$$

$$\therefore x = -2$$

Hence, $x = -2, 5$

Hence, option (b) is correct.

4. If one root of a quadratic equation with a rational coefficient is $\frac{3-\sqrt{5}}{2}$, then the other root is

(a) $\frac{-3-\sqrt{5}}{2}$

(b) $\frac{-3+\sqrt{5}}{2}$

(c) $\frac{3+\sqrt{5}}{2}$

(d) $\frac{\sqrt{3}+5}{2}$

Ans: One root of the quadratic equation is $\frac{3-\sqrt{5}}{2}$.

When this root is compared to the simpler form of roots of quadratic equation i.e. $\frac{b\pm\sqrt{d}}{2a}$, we can conclude that the other root will be $\frac{3+\sqrt{5}}{2}$.

Hence, option (c) is correct.

5. If the equation $2x^2-5x+(k+3)=0$ has equal roots then the value of k is

(a) $\frac{9}{8}$

(b) $-\frac{9}{8}$

(c) $\frac{1}{8}$

(d) $-\frac{1}{8}$

Ans: The equation we have is $2x^2-5x+(k+3)=0$, which when compared to the general form of a quadratic equation ax^2+bx+c, we get

$a=2, b=-5$ and $c=k+3$

Now, for the condition of equal roots

$b^2-4ac=0$

The values of a, b, and c are substituted in $b^2 - 4ac = 0$.

$\Rightarrow (-5)^2 - 4\times 2\times (k+3)$

$\Rightarrow 25 - 8(k+3)$

$\Rightarrow 25 - 8k - 24 = 0$

$\Rightarrow 8k = 25 - 24$

$\Rightarrow 8k = 1$

$\therefore k = \frac{1}{8}$.

Hence, option (c) is correct.

6. The value(s) of k for which the quadratic equation $2x^2 - kx + k = 0$ has equal roots is (are)

(a) 0 only

(b) 4

(c) 8 only

(d) 0, 8

Ans: The equation we have is $2x^2 - kx + k = 0$, which when compared to the general form of a quadratic equation $ax^2 + bx + c$, we get

$a = 2, b = -k$ and $c = k$

Now, for the condition of equal roots

$b^2 - 4ac = 0$

The values of a, b, and c are substituted in $b^2 - 4ac = 0$.

$\Rightarrow (-k)^2 - 4\times 2\times k = 0$

$\Rightarrow k^2 - 8k = 0$

$\Rightarrow k(k-8) = 0$

This provides us with two values of k.

$\Rightarrow k = 0$ or $(k-8) = 0$

$\Rightarrow k = 0$ or $k = 8$

$\therefore k = 0, 8$

Hence, option (d) is correct.

7. If the equation $3x^2 - kx + 2k = 0$ has equal roots, then the values of k is (are)

(a) 6
(b) 0 only
(c) 24 only
(d) 0 or 24

Ans: The equation we have is $3x^2 - kx + 2k = 0$, which when compared to the general form of a quadratic equation $ax^2 + bx + c$, we get

$a = 3, b = -k$ and $c = 2k$

Now, for the condition of equal roots

$b^2 - 4ac = 0$

The values of a, b, and c are substituted in $b^2 - 4ac = 0$.

$\Rightarrow (-k)^2 - 4 \times 3 \times 2k = 0$

$\Rightarrow k^2 - 12 \times 2k = 0$

$\Rightarrow k^2 - 24k = 0$

$\Rightarrow k(k - 24) = 0$

This provides us with two values of k.

$\Rightarrow k = 0$ or $(k - 24) = 0$

$\Rightarrow k = 0$ or $k = 24$

$\therefore k = 0$ or **24 Hence, option (d) is correct.**

8.If the equation $(k+1)x^2 - 2(k+1)x + 1 = 0$ **has equal roots, then the values of kare**

(a)1,3(b)0,3(c)0,1(d) $0, \frac{3}{4}$

Ans: The equation we have is $(k+1)x^2 - 2(k+1)x + 1 = 0$, which when compared to the general form of a quadratic equation $ax^2 + bx + c$, we get

$a = k+1, b = -2(k+1)$ and $c = 1$

Now, for the condition of equal roots

$b^2 - 4ac = 0$

The values of a, b, and c are substituted in $b^2 - 4ac = 0$.

$\Rightarrow (-2(k+1))^2 - 4 \times (k+1) \times 1 = 0$

$\Rightarrow (-2(k+1))^2 - (4k+4) = 0$

$\Rightarrow 4(k^2 - 2k + 1) - (4k+4) = 0$

$\Rightarrow 4k^2 + 4 - 8k - 4k - 4 = 0$
$\Rightarrow 4k^2 - 12k = 0$
$\Rightarrow 4k(k-3) = 0$
This provides us with two values of k.
$\Rightarrow 4k = 0$ or $k - 3 = 0$
$\Rightarrow k = 0$ or $k = 3$
Hence, option (b) is correct.

9.If the equation $2x^2 - 6x + p = 0$ has real and different roots, then the values of p are given by

(a). $p < \frac{9}{2}$

(b). $p \leqslant \frac{9}{2}$

(c). $p > \frac{9}{2}$

(d). $p \geqslant \frac{9}{2}$

Ans: When we compare the equation $2x^2 - 6x + p = 0$ we have to the quadratic equation's basic form $ax^2 + bx + c$, we get
$a = 2, b = -6$ and $c = p$
Regarding the condition of equal roots, now
$b^2 - 4ac > 0$
The values of a, b, and c are substituted in $b^2 - 4ac > 0$.
$\Rightarrow (-6)^2 - 4 \times 2 \times p > 0$
$\Rightarrow 36 - 8p > 0$
$\Rightarrow 8p < 36$
$\Rightarrow p < \frac{36}{8}$
$\Rightarrow p < \frac{9}{2}$
Hence, option (a) is correct.

10. The quadratic equation $2x^2-\sqrt{5}x+1=0$ has
(a). Two distinct real roots
(b). Two equal real roots
(c). No real roots
(d). More than two real roots

Ans. When the given equation $2x^2-\sqrt{5}x+1=0$ is compared to the general form of the quadratic equation ax^2+bx+c,

We get, $a=2, b=-\sqrt{5}$ and $c=1$

Substituting the values a, b and c in $d=b^2-4ac$, we get $(-\sqrt{5})^2-4\times 2\times 1$

$\Rightarrow d=5-8$

$\Rightarrow d=-4$

Since, the value of d is less than zero, it indicates that the quadratic equation has no real roots.

Hence, option (c) is correct.

CHAPTER TEST

Solve the following equations (1 to 4) by factorisation:

1.(i) $x^2+6x-16=0$

Ans: Given equation is: $x^2+6x-16=0$

On factorisation, we have:

$\Rightarrow x^2+6x+16=0$

$\Rightarrow x^2+8x-2x+16=0$

$\Rightarrow x(x+8)-2(x+8)=0$

$\Rightarrow (x-2)(x+8)=0$

$\therefore x=2$ or $x=-8$

Hence, the value of x is 2 and -8.

(ii) $3x^2+11x+10=0$

Ans: Given equation is: $3x^2+11x+10=0$

On factorisation, we have:

$\Rightarrow 3x^2+11x+10=0$

$\Rightarrow 3x^2+6x+5x+10=0$

$\Rightarrow 3x(x+2)+5(x+2)=0$

$\Rightarrow (3x+5)(x+2)=0$

$\therefore x=\frac{-5}{3}$ and $x=-2$

Hence, the value of x = $\frac{-5}{3}$ and -2.

2. (i) $2x^2+ax-a^2=0$

Ans: Given equation is: $2x^2+ax-a^2=0$

On factorisation we have:

$\Rightarrow 2x^2+ax-a^2=0$

$\Rightarrow 2x^2+2ax-ax-a^2=0$

$\Rightarrow 2x(x+a)-a(x+a)=0$

$\Rightarrow (2x-a)(x+a)=0$

$\therefore x=\frac{a}{2}$ and $x=-a$

Hence, value of x = $\frac{a}{2}$ and -a.

(ii) $\sqrt{3}x^2+10x+7\sqrt{3}=0$

Ans: Given equation is: $\sqrt{3}x^2+10x+7\sqrt{3}=0$

On factorisation, we have:

$\Rightarrow \sqrt{3}x^2+10x+7\sqrt{3}=0$

$\Rightarrow \sqrt{3}x^2+3x+7x+7\sqrt{3}=0$

$\Rightarrow \sqrt{3}x(x+\sqrt{3})+7(x+\sqrt{3})=0$

$\Rightarrow (\sqrt{3}x+7)(x+\sqrt{3})=0$

$\therefore x=\frac{-7}{\sqrt{3}}$ and $x=-\sqrt{3}$

Hence, roots of the equation are $\frac{-7}{\sqrt{3}}$ and $-\sqrt{3}$.

3. (i) $\mathbf{x(x+1)+(x+2)(x+3)=42}$

Ans: Given equation is: $x(x+1)+(x+2)(x+3)=42$

On factorisation we have:

$\Rightarrow x(x+1)+(x+2)(x+3)=42$

$\Rightarrow x^2+x+\left(x^2+3x+2x+6\right)=42$

$\Rightarrow 2x^2+6x-36=0$

$\Rightarrow x^2+3x-18=0$

$\Rightarrow x^2+6x-3x-18=0$

$\Rightarrow x(x+6)-3(x+6)=0$

$\Rightarrow (x-3)(x+6)=0$

$\therefore x=3$ and $x=-6$

Hence, the roots are 3 and -6.

(ii) $\dfrac{6}{x}-\dfrac{2}{(x-1)}=\dfrac{1}{(x-2)}$

Ans: Given equation is: $\dfrac{6}{x}-\dfrac{2}{(x-1)}=\dfrac{1}{(x-2)}$

Given equation is: $\dfrac{6}{x}-\dfrac{2}{(x-1)}=\dfrac{1}{(x-2)}$

On factorisation we have:

$\Rightarrow \dfrac{6}{x}-\dfrac{2}{(x-1)}=\dfrac{1}{(x-2)}$

$\Rightarrow \dfrac{(6x-6-2x)}{\left(x^2-x\right)}=\dfrac{1}{(x-2)}$

$\Rightarrow 4x^2-6x-8x+12=x^2-x$

$\Rightarrow 3x^2-13x+12=0$

$\Rightarrow 3x^2-9x-4x+12=0$

$\Rightarrow 3x(x-3)-4(x-3)=0$

$\Rightarrow (3x-4)(x-3)=0$

$\therefore x=\dfrac{4}{3}$ and $x=3$

Hence, the roots are $\frac{4}{3}$ and 3.

4. (i) $\mathbf{\sqrt{(x+15)}=x+3}$

Ans: Given equation is: $\sqrt{(x+15)}= x+3$

Expand the given equation:

$\Rightarrow x+15=(x+3)^2$

$\Rightarrow x+15=x^2+6x+9$

$\Rightarrow x^2+5x-6=0$

On factorisation we have:

$\Rightarrow x^2+5x-6=0$

$\Rightarrow x^2+6x-x-6=0$

$\Rightarrow x(x+6)-1(x+6)=0$

$\Rightarrow (x-1)(x+6)=0$

$\therefore x=1$ and $x=-6$

Hence, the roots are 1 and -6.

(ii) $\mathbf{\sqrt{(3x^2-2x-1)}=2x-2}$

Ans: $\sqrt{(3x^2-2x-1)}=2x-2$

Expand the equation:

$\Rightarrow \sqrt{(3x^2-2x-1)}=2x-2$

$\Rightarrow 3x^2-2x-1=(2x-2)^2$

$\Rightarrow 3x^2-2x-1=4x^2-8x+4$

$\Rightarrow x^2-6x+5=0$

On factorisation we have:

$\Rightarrow x^2-6x+5=0$

$\Rightarrow x^2-5x-x+5=0$

$\Rightarrow x(x-5)-1(x-5)=0$

$\Rightarrow (x-1)(x-5)=0$

$\therefore x=1$ and $x=5$

Hence, the roots are 1 and 5.

Solve the following equations (5 to 8) by using formula:

5. (i) $\mathbf{2x^2-3x-1=0}$

Ans: Given equation is: $2x^2-3x-1=0$

$\Rightarrow 2x^2-3x-1=0$

$\Rightarrow a=2, b=-3, c=-1$

By quadratic formula we have:

$$\Rightarrow x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$\Rightarrow x=\frac{-(-3)\pm\sqrt{(-3)^2-4(2)(-1)}}{2(2)}$$

$$\Rightarrow \frac{3\pm\sqrt{9+8}}{4}=\frac{3\pm\sqrt{17}}{4}$$

$$\therefore x=\frac{3+\sqrt{17}}{4} \text{ and } x=\frac{3-\sqrt{17}}{4}$$

Hence, roots are $\frac{3+\sqrt{17}}{4}$ and $x=\frac{3-\sqrt{17}}{4}$.

(ii) $\mathbf{x\left(3x+\frac{1}{2}\right)=6}$

Ans: Given equation is: $x\left(3x+\frac{1}{2}\right)=6$

$$\Rightarrow x\left(3x+\frac{1}{2}\right)=6$$

$$\Rightarrow 3x^2+\frac{x}{2}=6$$

$\Rightarrow 6x^2+x-12=0$

$\Rightarrow a=6, b=1, c=-12$

By quadratic formula we have:

$$\Rightarrow x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$\Rightarrow x=\frac{-(1)\pm\sqrt{(1)^2-4(6)(-12)}}{2(6)}$$

$$\Rightarrow \frac{-1\pm\sqrt{289}}{12}=\frac{-1\pm17}{12}$$

$$\therefore x=\frac{-1+17}{12}=\frac{16}{12}=\frac{4}{3} \text{ and } x=\frac{-1-17}{12}=\frac{-18}{12}=\frac{-3}{2}$$

Hence, roots are $\frac{4}{3}$ and $\frac{-3}{2}$.

6. (i) $\frac{(\mathbf{2x+5})}{(\mathbf{3x+4})}=\frac{(\mathbf{x+1})}{(\mathbf{x+3})}$

Ans: Given equation is: $\frac{(2x+5)}{(3x+4)}=\frac{(x+1)}{(x+3)}$

On simplifying:

$$\Rightarrow \frac{(2x+5)}{(3x+4)}=\frac{(x+1)}{(x+3)}$$

$$\Rightarrow 2x^2+6x+5x+15=3x^2+3x+4x+4$$

$$\Rightarrow 2x^2+11x+15=3x^2+7x+4$$

$$\Rightarrow x^2-4x-11=0$$

$$\Rightarrow a=1, b=-4, c=-11$$

By quadratic formula we have:

$$\Rightarrow x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$\Rightarrow x=\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(-11)}}{2(1)}$$

$$\Rightarrow \frac{4\pm\sqrt{16+44}}{2}=\frac{4\pm\sqrt{60}}{2}$$

$\therefore x=\frac{4+2\sqrt{15}}{2}=2+\sqrt{15}$ and $x=\frac{4-2\sqrt{15}}{2}=2-\sqrt{15}$

Hence, roots are $2+\sqrt{5}$ and $2-\sqrt{5}$.

(ii) $\mathbf{\frac{2}{(x+2)}-\frac{1}{(x+1)}=\frac{4}{(x+4)}-\frac{3}{(x+3)}}$

Ans: Given equation is: $\frac{2}{(x+2)}-\frac{1}{(x+1)}=\frac{4}{(x+4)}-\frac{3}{(x+3)}$

$\Rightarrow \frac{2}{(x+2)}-\frac{1}{(x+1)}=\frac{4}{(x+4)}-\frac{3}{(x+3)}$

$\Rightarrow x^2+7x+12-x^2-3x-2 \ =0$

$\Rightarrow 4x+10 \ =0$

$\therefore x=\frac{-10}{4}=\frac{-5}{2}$

Hence, roots are 0 and $\frac{-5}{2}$

7. (i) $\frac{(3x-4)}{7}+\frac{7}{(3x-4)}=\frac{5}{2}, \mathbf{x\neq\frac{4}{3}}$

Ans: Given equation is: $\frac{(3x-4)}{7}+\frac{7}{(3x-4)}=\frac{5}{2}$

On simplifying:

$\Rightarrow \frac{(3x-4)}{7}+\frac{7}{(3x-4)}=\frac{5}{2}$

$\Rightarrow \frac{\left[(3x-4)^2+(7)^2\right]}{7(3x-4)}=\frac{5}{2}$

$\Rightarrow 2\left(9x^2+16-24x+49\right)=35(3x \ -4)$

$\Rightarrow 18x^2-48x+130=105x-140$

$\Rightarrow 18x^2-153x+270 \ =0$

$\Rightarrow 2x^2-17x+30=0$

Substituting the values in formula:

$$\Rightarrow a = 2, b = -17, c = 30$$

$$\Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow x = \frac{-(-17) \pm \sqrt{(-17)^2 - 4(2)(30)}}{2(2)}$$

$$\Rightarrow \frac{17 \pm \sqrt{289 - 240}}{4} = \frac{17 \pm \sqrt{49}}{4}$$

$$\therefore x = \frac{17+7}{4} = 6 \text{ and } x = \frac{17-7}{4} = \frac{10}{4} = \frac{5}{2}$$

Hence, the roots are 6 and $\frac{5}{2}$.

(ii) $\frac{(4-3x)}{x} = \frac{5}{(2x+3)}$, $\mathbf{x \neq 0, \frac{-3}{2}}$

Ans: Given equation is: $\frac{(4-3x)}{x} = \frac{5}{(2x+3)}$

On simplifying the equation:

$$\Rightarrow \frac{(4-3x)}{x} = \frac{5}{(2x+3)}$$

$$\Rightarrow (4-3x)(2x+3) = 5x$$

$$\Rightarrow 6x^2 + 6x - 12 = 0$$

$$\Rightarrow x^2 + x - 2 = 0$$

Substituting the values in formula:

$$\Rightarrow a = 1, b = 1, c = -2$$

$$\Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow x = \frac{-(1) \pm \sqrt{(1)^2 - 4(1)(-2)}}{2(1)}$$

$$\Rightarrow \frac{-1\pm\sqrt{1+8}}{2}=\frac{-1\pm\sqrt{9}}{2}=\frac{-1\pm3}{2}$$

$$\therefore x=\frac{-1+3}{2}=1 \text{ and } x=\frac{-1-3}{2}=-2$$

Hence, the roots are 1 and -2.

8. (i) $\mathbf{x^2+(4-3a)x-12a=0}$

Ans: $x^2+(4-3a)x-12a=0$

Here $a=1, b=4-3a, c=-12a$

$$\therefore D=b^2-4ac$$
$$=(4-3a)^2-4\times1\times(-12a)$$
$$=16-24a+9a^2+48a$$
$$=16+24a+9a^2=(4+3a)$$
$$\therefore x=\frac{-b\pm\sqrt{D}}{2a}$$
$$=\frac{-(4-3a)\pm\sqrt{4+3a^2}}{2\times1}$$
$$=\frac{3a-4\pm3a+4}{2}$$
$$\therefore x_1=\frac{3a-4+3a+4}{2}$$
$$=\frac{6a}{2}$$
$$=3a$$

And $x_2=\dfrac{3a-4-3a-4}{2}$

$$=\frac{-8}{2}$$
$$=-4$$

Hence, roots are 3a and -4.

(ii) $\mathbf{10ax^2-6x+15ax-9=0, a\neq0}$

Ans: $10ax^2-6x+15ax-9=0$

Here a=10 a, b=-(6-15 a), c=-9

$D=b^2-4ac$

$=[-(6-15a)]^2-4\times 10a(-9)$

$=36-180a+225a^2+360a$

$=36+180a+225a^2=(6+15a)^2$

$\therefore \text{x}=\frac{-b\pm\sqrt{D}}{2a}$

$=\frac{-[-(6-15a)]\pm\sqrt{(6+15a)^2}}{2\times 10a}$

$=\frac{(6-15a)\pm(6+15a)}{20a}$

$\therefore \text{x}_1=\frac{6-15a+6+15a}{20a}$

$=\frac{12}{20a}$

$=\frac{3}{5a}$

$\text{x}_2=\frac{6-15a-6-15a}{20a}$

$=\frac{-30a}{20a}$

$=\frac{-3}{2}$

Hence $x=\frac{3}{5}a,\frac{-3}{2}.$

9. Solve for x using the quadratic formula. Write your answer correct to two significant figures: $(x-1)^2-3x+4=0$

Ans: $(x-1)^2-3x+4=0$

$x^2+1-2x-3x+4=0$

$x^2 - 5x + 5 = 0$

Comparing it with

$ax^2 + bx + c = 0$, we get

a=1, b=-5, c=5

By using the formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{5 \pm \sqrt{25 - 20}}{2}$$

$$= \frac{5 \pm \sqrt{5}}{2}$$

$$x = \frac{5 \pm 2 \cdot 24}{2}$$

Taking + ve sign

$$x = \frac{5 + 2.24}{2}$$

$x = 3.62$

Taking - ve sign

$$x = \frac{5 - 2.25}{2}$$

$$= \frac{2.76}{2} = 1.38$$

Thus required values are 3.62 and 1.38

.

10. Discuss the nature of roots of the following equations:

In case the real roots exist, then find them.

(i) $\mathbf{3x^2 - 7x + 8 = 0}$

Ans: 1. A quadratic equation has no real roots if D < 0

2. A quadratic equation has equal and real roots if D = 0

3. A quadratic equation has real and distinct roots if D > 0

Given equation is: $3x^2 - 7x + 8 = 0$

We know that D = $b^2 - 4ac$

$\Rightarrow D = (-7)^2 - 4(3)(8)$

$\Rightarrow 49 - 12(8)$

$\Rightarrow 49 - 96 = -47$

$\therefore D < 0$

Hence, equation has no real roots.

(ii) $\mathbf{x^2 - \frac{1}{2}x - 4 = 0}$

Ans: Given equation is: $x^2 - \frac{1}{2}x - 4 = 0$

We know that D = $b^2 - 4ac$

$\Rightarrow D = \left(-\frac{1}{2}\right)^2 - 4(1)(-4)$

$\Rightarrow \frac{1}{4} + 16 = \frac{65}{16}$

$\therefore D > 0$

Therefore, equation has real and distinct roots.

Finding the roots of the equation:

$\Rightarrow x^2 - \frac{1}{2}x - 4 = 0$

$\Rightarrow 2x^2 - x - 8 = 0$

$\Rightarrow a = 2, b = -1, c = -8$

$\therefore \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(-8)}}{2(2)}$

$\Rightarrow \frac{1 \pm \sqrt{1 - 4(-16)}}{4} = \frac{1 \pm \sqrt{1 + 64}}{4} = \frac{1 \pm \sqrt{65}}{4}$

$\therefore x = \frac{1 + \sqrt{65}}{4}, \frac{1 - \sqrt{65}}{4}$

Hence, roots of the equation are $\frac{1+\sqrt{65}}{4}$ and $\frac{1-\sqrt{65}}{4}$.

(iii) $\mathbf{5x^2 - 6\sqrt{5}x + 9 = 0}$

Ans: Given equation is: $5x^2 - 6\sqrt{5}x + 9 = 0$

We know that D = $b^2 - 4ac$

$\Rightarrow 5x^2 - 6\sqrt{5}x + 9 = 0$

$\Rightarrow a = 5, b = -6\sqrt{5}, c = 9$

$\Rightarrow \left(-6\sqrt{5}\right)^2 - 4(5)(9)$

$\Rightarrow 180 - 180 = 0$

$\therefore D = 0$

Therefore, equation has equal and real roots.

Finding the roots:

$\Rightarrow x = \frac{-b}{2a} = \frac{-\left(-6\sqrt{5}\right)}{2(5)} = \frac{6\sqrt{5}}{10} = \frac{3\sqrt{5}}{5}$

$\therefore x = \frac{3\sqrt{5}}{5}, \frac{3\sqrt{5}}{5}$

Hence, the roots of the equation are $\frac{3\sqrt{5}}{5}$ and $\frac{3\sqrt{5}}{5}$.

(iv) $\mathbf{\sqrt{3}x^2 - 2x - \sqrt{3} = 0}$

Ans: Given equation is: $\sqrt{3}x^2 - 2x - \sqrt{3} = 0$

We know that D = $b^2 - 4ac$

$\Rightarrow \sqrt{3}x^2 - 2x - \sqrt{3} = 0$

$\Rightarrow a = \sqrt{3}, b = -2, c = \sqrt{3}$

$\Rightarrow (-2)^2 - 4\left(\sqrt{3}\right)\left(\sqrt{3}\right)$

$\Rightarrow 4 - 12 = -8$

$\therefore D < 0$

Hence, equation has no real roots.

11. Find the values of k so that the quadratic equation has equal roots.

Ans: A quadratic equation is said to have two equal roots if D = 0.

We know that, D = b^2-4ac

$\Rightarrow (4-k)x^2+2(k+2)x+(8k+1)=0$

$\Rightarrow D=b^2-4ac$

$\Rightarrow (2k+4)^2-4(4-k)(8k+1)=0$

$\Rightarrow (4k^2+16+16k)-4(32k+4-8k^2-k)=0$

$\Rightarrow (4k^2+16+16k)-(128k-32k^2+16-4k)=0$

$\Rightarrow 4k^2+16+16k-128k+32k^2-16+4k=0$

$\Rightarrow 36k^2-108k=0$

$\Rightarrow 4k^2-12k=0$

$\Rightarrow k^2-3k=0$

$\Rightarrow k(k-3)=0$

$\therefore k=0, k=3$

Hence, for k = 0 and k = 3 equation has equal roots.

12. Find the values of m so that the quadratic equation $3x^2-5x-2m=0$ has two distinct real roots.

Ans: A quadratic equation is said to have two distinct real roots if D > 0.

We know that, D = b^2-4ac

$\Rightarrow (-5)^2-4(3)(-2m)>0$

$\Rightarrow 25-12(-2m)>0$

$\Rightarrow 25+24m>0$

$\Rightarrow m>-\frac{25}{24}$

So, $D<0$.

Hence, value of m is not greater than 0.

13. Find the value(s) of k for which each of the following quadratic equation has equal roots:

(i) $3kx^2=4(kx-1)$

Ans: For the equation to have equal roots Discriminant (D = b^2-4ac) = 0

Given equation is: $3kx^2 = 4(kx-1)$

Solving the equation:

$\Rightarrow 3kx^2 = 4(kx-1)$

$\Rightarrow 3kx^2 = 4kx - 4$

$\Rightarrow 3kx^2 - 4kx + 4$

$\Rightarrow \text{D}=b^2 - 4ac = (-4k)^2 - 4(3k)(4)$

$\Rightarrow 16k^2 - 48k = 0$

$\Rightarrow 16k(k-3) = 0$

$\therefore k=0 or k=3$

Now, D = 0, so finding roots by quadratic equation formula:

$$\Rightarrow x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} or = \frac{-b \pm \sqrt{D}}{2a}$$

Therefore,

$$\Rightarrow x = \frac{-b \pm 0}{2a} = \frac{-b}{2a}$$

$$\Rightarrow x = \frac{-(-4k)}{2(3k)} = \frac{2}{3}$$

Hence, roots are $\frac{2}{3}$ and $\frac{2}{3}$.

(ii) $\mathbf{(k+4)x^2 + (k+1)x + 1 = 0}$

Ans: Given equation is: $(k+4)x^2 + (k+1)x + 1 = 0$

Solving the equation:

$\Rightarrow (k+4)x^2 + (k+1)x + 1 = 0$

$\Rightarrow D = b^2 - 4ac = 0$

$\therefore D = (k+1)^2 - 4(k+4)(1) = 0$

$\Rightarrow k^2 + 2k + 1^2 - 4k - 16 = 0$

$\Rightarrow k^2 - 2k - 15 = 0$

$\Rightarrow k^2 - 5k + 3k - 15 = 0$

$\Rightarrow k(k-5) + 3(k-5) = 0$

$\therefore k = 5, k = -3$

Now, D = 0, so finding roots by quadratic equation formula:

$$\Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} or = \frac{-b \pm \sqrt{D}}{2a}$$

Therefore,

$$\Rightarrow x = \frac{-b \pm 0}{2a} = \frac{-b}{2a} = \frac{-(k+1)}{2(k+4)} = \frac{-k-1}{2k+8}$$

If k = 5

$$\Rightarrow x = \frac{-k-1}{2k+8} = \frac{-5-1}{2(5)+8} = \frac{-6}{18} = -\frac{1}{3}$$

So, roots will be $\frac{-1}{3}$ and $\frac{-1}{3}$.

if k = - 3

$$\Rightarrow x = \frac{-k-1}{2k+8} = \frac{-(-3)-1}{2(-3)+8} = \frac{2}{2} = 1$$

So, roots will be 1 and 1.

14. Find two natural numbers which differ by 3 and whose squares have the sum 117.

Ans: Let the two natural number be a and (a +3) then;

$$\Rightarrow a^2 + (a+3)^2 = 117$$

$$\Rightarrow a^2 + a^2 + 9 + 6a = 117$$

$$\Rightarrow 2a^2 + 6a = 117 - 9$$

$$\Rightarrow 2a^2 + 6a - 108 = 0$$

$$\Rightarrow a^2 + 3a - 54 = 0$$

Solving for value of a:

$$\Rightarrow a^2 + 3a - 54 = 0$$

$$\Rightarrow a^2 + 9a - 6a - 54 = 0$$

$$\Rightarrow a(a+9) - 6(a+9) = 0$$

$$\Rightarrow (a-6)(a+9) = 0$$

$$\therefore a = 6$$

Because value of a can't be negative.
Hence, natural numbers are a = 6
And (a+3) = 6+3= 9.

15. Divide 16 into two parts such that the twice the square of the larger part exceeds the square of the smaller part by 164.

Ans: If larger part is = a
And smaller part = (16 – a)
Therefore, according to the question:

$\Rightarrow 2a^2 - (16-a)^2 = 164$

$\Rightarrow 2a^2 - \left(256 + a^2 - 32a\right) = 164$

$\Rightarrow 2a^2 - 256 - a^2 + 32a = 164$

$\Rightarrow a^2 + 32a - 420 = 0$

Solving for value of a:

$\Rightarrow a^2 + 32a - 420 = 0$

$\Rightarrow a^2 + 42a - 10a - 420 = 0$

$\Rightarrow a(a+42) - 10(a+42) = 0$

$\Rightarrow (a-10)(a+42) = 0$

Value of x can't be negative. So, $x=10$
Hence, value of larger part is = 10
Smaller part = 16 – 10 = 6.

16. Two natural numbers are in the ratio 3:4. Find the numbers if the difference between their squares is 175.

Ans: Two natural numbers in ratio are given as = 3:4
If the numbers be 3x and 4x then, according to the question:

$\Rightarrow (4x)^2 - (3x)^2 = 175$

$\Rightarrow 16x^2 - 9x^2 = 175$

$\Rightarrow 7x^2 = 175$

$\Rightarrow x^2 = 25$

$\therefore x = \pm 5$

Taking value of x = 5 as natural numbers are not negative.
Hence, two numbers are
$\Rightarrow 3(5)=15$ and $4(5)=20$.

17. Two squares have sides x cm and (x + 4) cm. The sum of their areas is 656 sq. cm. Express this as an algebraic equation and solve it to find the sides of the squares.

Ans: Let the side of square 1 = x cm
Sides of square 2 = (x + 4) cm
We know that area of square = $x \times x cm^2$
Therefore,
Area of square 1 = $x \times x = x^2 cm^2$
Area of square 2 = $(x+4)\times(x+4)$
$\Rightarrow (x+4)^2 = x^2 + 8x + 16$
According to the question:
$\Rightarrow x^2 + (x^2 + 8x + 16) = 656$
$\Rightarrow 2x^2 + 8x + 16 = 656$
$\Rightarrow 2x^2 + 8x = 656 - 16$
$\Rightarrow 2x^2 + 8x - 640 = 0$
$\Rightarrow x^2 + 4x - 320 = 0$
Solving for the value of x:
$\Rightarrow x^2 + 4x - 320 = 0$
$\Rightarrow x^2 + 20x - 16x - 320 = 0$
$\Rightarrow x(x+20) - 16(x+20) = 0$
$\Rightarrow (x-16)(x+20) = 0$
$\Rightarrow x = 16 or -20$
$\because x \neq -20 \therefore x = 16$
Hence, sides of square 1= x cm = 16 cm
Sides of square 2 = (x + 4) cm = 20 cm.

18. The length of a rectangular garden is 12 m more than its breadth. The numerical value of its area is equal to 4 times the numerical value of its perimeter. Find the dimensions of the garden.

Ans: Let the breadth of the rectangle = x m

Length of the rectangle = (12 + x) m

We know that, perimeter of rectangle = $2\times(l+b)$

$\Rightarrow 2((12+x)+x)=24+4x$

Also, given that the area of rectangle = 4 x perimeter of the rectangle

$\Rightarrow 4\times(24+4x)=96+16x$

Now, according to the question:

$\Rightarrow$ Area of rectangle $=$ $(l\times b)$

$\Rightarrow (12+x)(x)=96+16x$

$\Rightarrow 12x+x^2=96+16x$

$\Rightarrow x^2-4x-96=0$

Finding the value of x:

$\Rightarrow x^2-4x-96=0$

$\Rightarrow x^2-12x+8x-96=0$

$\Rightarrow x(x-12)+8(x-12)=0$

$\therefore x=12$ and -8

Because value of x can't be negative therefore, x = 12m

Hence, breadth = 12 m

Length = 12 + x = 12 + 12 = 24 m.

19. A farmer wishes to grow a 100 m^2 rectangular vegetable garden. Since he has with him only 30 m barbed wire, he fences three sides of the rectangular garden letting compound wall of his house act as the fourth side fence. Find the dimensions of his garden.

Ans: As the garden is in rectangular shape therefore area of rectangle = 100 m^2

Given length of the wire = 30 m

Taking length of the opposite wall as x m

Then,

The length of the other side becomes = $\frac{30-x}{2}$

So, the area of rectangle = length x breadth

$\Rightarrow\left(\frac{30-x}{2}\right)\times x=100$

$\Rightarrow\frac{x(30-x)}{2}=100$

$\Rightarrow 30x-x^2=200$

$\Rightarrow x^2-30x+200=0$

On solving the above equation:

$\Rightarrow x^2-30x+200=0$

$\Rightarrow x^2-20x-10x+200=0$

$\Rightarrow x(x-20)-10(x-20)=0$

$\Rightarrow (x-10)(x-20)=0$

$\therefore x=10$ and $x=20$

If x = 10 m

Length of the opposite wall = 10 m

Length of the other side = $\frac{30-x}{2}=\frac{30-10}{2}=\frac{20}{2}=10m$

If x = 20 m

Length of the opposite wall = 20 m

Length of the other side = $\frac{30-x}{2}=\frac{30-20}{2}=\frac{10}{2}=5m$

20. The hypotenuse of a right-angled triangle is 1 m less than twice the shortest side. If the third side is 1 m more than the shortest side, find the sides of the triangle.

Ans: Assuming the length of the shortest side be xm

And length of hypotenuse 1 m less than twice the shortest side, therefore

$=2x-1$

Also, third side is 1 m more than the shortest side so,

$\Rightarrow x+1$

According to the Pythagoras theorem:

$\Rightarrow P^2 + B^2 = H^2$

$\Rightarrow x^2 + (x+1)^2 = (2x-1)^2$

$\Rightarrow x^2 + (x^2 + 2x + 1) = 4x^2 + 1 - 4x$

$\Rightarrow 4x^2 - 2x^2 - 4x - 2x + 1 - 1 = 0$

$\Rightarrow 2x^2 - 6x = 0$

$\Rightarrow 2x^2 = 6x$

$\therefore x = 3, 0$

Because x = 0 is not possible, therefore the value of x = 3.

So, the length of the shortest side is $3m$.

Length of hypotenuse is $= (2x-1) = 2(3) - 1 = 6 - 1 = 5m$

Length of third side of a triangle $= (x+1) = 3 + 1 = 4m$.