Maharashtra Board Solutions Class 12

Maths

Chapter 2 Matrices

Practice set – 2.1

Apply the given elementary transformation on each of the following matrices.

1. $\mathbf{A} = \begin{pmatrix} 1 & 0 \\ -1 & 3 \end{pmatrix}, \mathbf{R_1} \leftrightarrow \mathbf{R_2}$

Ans: The given matrix is,

$$A = \begin{bmatrix} 1 & 3 \\ -1 & 0 \end{bmatrix}$$

By $R_1 \leftrightarrow R_2$, we get,

$$A \sim \begin{bmatrix} -1 & 3 \\ 1 & 0 \end{bmatrix}$$

2. $\mathbf{B} = \begin{bmatrix} 1 & -1 & 3 \\ 2 & 5 & 4 \end{bmatrix}, \mathbf{R_1} \leftrightarrow \mathbf{R_2}$

Ans: The given matrix is,

$$B = \begin{bmatrix} 1 & -1 & 3 \\ 2 & 5 & 4 \end{bmatrix},$$

$R_1 \to R_1 \to R_2$ gives,

$$B \sim \begin{bmatrix} -1 & -6 & -1 \\ 2 & 5 & 4 \end{bmatrix}$$

3. $\mathbf{A} = \begin{bmatrix} 5 & 4 \\ 1 & 3 \end{bmatrix}, \mathbf{C_1} \leftrightarrow \mathbf{C_2}; \mathbf{B} = \begin{bmatrix} 3 & 1 \\ 4 & 5 \end{bmatrix}, \mathbf{R_1} \leftrightarrow \mathbf{R_2}$ **. What do you observe?**

Ans: The given matrix is,

$$A = \begin{bmatrix} 5 & 4 \\ 1 & 3 \end{bmatrix}$$

By $C_1 \leftrightarrow C_2$, we get,

$$A \sim \begin{bmatrix} 4 & 5 \\ 3 & 1 \end{bmatrix} .....(1)$$

$$B = \begin{bmatrix} 3 & 1 \\ 4 & 5 \end{bmatrix}$$

By $R_1 \leftrightarrow R_2$, we get,

$$B \sim \begin{bmatrix} 4 & 5 \\ 3 & 1 \end{bmatrix} .....(2)$$

From (1) and (2), we observe that the new matrices are equal.

4. $\mathbf{A} = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 3 \end{bmatrix}, \mathbf{2C_2}$

$\mathbf{B} = \begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & 5 \end{bmatrix}, \mathbf{-3R_1}$

Find the addition of the two new matrices.

Ans: The given matrix is,

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 3 \end{bmatrix}$$

By 2C2, we get,

$$A \sim \begin{bmatrix} 1 & 4 & -1 \\ 0 & 2 & 3 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 & 2 \\ 2 & 4 & 5 \end{bmatrix}$$

By $-3R_1$, we get,

$$B \sim \begin{bmatrix} 3 & 0 & -6 \\ 2 & 4 & 5 \end{bmatrix}$$

Now, addition of the two new matrices

$$= \begin{pmatrix} 1 & 4 & -1 \\ 0 & 2 & 3 \end{pmatrix} + \begin{pmatrix} -3 & 0 & -6 \\ 2 & 4 & 5 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} -2 & 4 & -7 \\ 2 & 6 & 8 \end{pmatrix}$$

5. $\mathbf{A}=\begin{bmatrix}1 & -1 & 3\\ 2 & 1 & 0\\ 3 & 3 & 1\end{bmatrix}$, $3R_3$ **and then** C_3+2C_2.

Ans:

$$A=\begin{bmatrix}1 & -1 & 3\\ 2 & 1 & 0\\ 3 & 3 & 1\end{bmatrix}$$

By $3R_3$, we get

$$A\sim\begin{bmatrix}1 & -1 & 3\\ 2 & 1 & 0\\ 3 & 3 & 1\end{bmatrix}$$

By C_3+2C_2, we get,

$$A\sim\begin{pmatrix}1 & -1 & 3+2(-1)\\ 2 & 1 & 0+2(1)\\ 9 & 9 & 3+2(9)\end{pmatrix}$$

$$\therefore A\sim\begin{pmatrix}1 & -1 & 1\\ 2 & 1 & 2\\ 9 & 9 & 21\end{pmatrix}$$

6. $\mathbf{A}=\begin{pmatrix}1 & -1 & 3\\ 2 & 1 & 0\\ 3 & 3 & 1\end{pmatrix}$, C_3+2C_2 **and then** $3R_3$. **What do you conclude from Ex. 5 and Ex. 6?**

Ans: The given matrix is,

$$A=\begin{pmatrix}1 & -1 & 3\\ 2 & 1 & 0\\ 3 & 3 & 1\end{pmatrix}$$

By C_3+2C_2, we get,

$$A \sim \begin{pmatrix} 1 & -1 & 3+2(-1) \\ 2 & 1 & 0+2(1) \\ 9 & 9 & 1+2(3) \end{pmatrix}$$

$$\therefore A \sim \begin{pmatrix} 1 & -1 & 1 \\ 2 & 1 & 2 \\ 3 & 3 & 7 \end{pmatrix}$$

By $3R_3$, we get

$$A \sim \begin{pmatrix} 1 & -1 & 1 \\ 2 & 1 & 2 \\ 9 & 9 & 21 \end{pmatrix}$$

We conclude from Ex. 5 and Ex. 6 that the matrix remains same by interchanging the order of the elementary transformations. Hence, the transformations are commutative.

7. Use suitable transformation on $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ into an upper triangular matrix.

Ans: The given matrix is,

Suppose $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

By $R_2 - 3R_1$, we get,

$$A \sim \begin{bmatrix} 1 & 2 \\ 0 & -2 \end{bmatrix}$$

This is an upper triangular matrix.

8. Convert $\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$ into an identity matrix by suitable row transformations.

Ans: The given matrix is,

Suppose $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$

By $R_2 - 2R_1$, we get,

$$A \sim \begin{bmatrix} 1 & -1 \\ 0 & 5 \end{bmatrix}$$

By $\left(\frac{1}{5}\right)R_2$, we get,

$$A \sim \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

By $R_1 + R_2$, we get,

$$A \sim \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

This is an identity matrix.

9. Transform $\begin{bmatrix} 1 & -1 & 2 \\ 2 & 1 & 3 \\ 3 & 2 & 4 \end{bmatrix}$ **into an upper triangular matrix by suitable row transformations.**

Ans: Suppose,

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 1 & 3 \\ 3 & 2 & 4 \end{bmatrix}$$

By $R_2 - 2R_1$ and $R_3 - 3R_1$, we get

$$A \sim \begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & -1 \\ 0 & 5 & -2 \end{bmatrix}$$

By $R_3 - \left(\frac{5}{3}\right)R_2$, we get,

$$A \sim \begin{pmatrix} 1 & -1 & 2 \\ 0 & 3 & -1 \\ 0 & 0 & \frac{-1}{3} \end{pmatrix}$$

This is an upper triangular matrix.

Practice set – 2.2

Find the co-factors of the elements of the following matrices.

1. $\begin{bmatrix} -1 & 2 \\ -3 & 4 \end{bmatrix}$

Ans: Suppose $A = \begin{bmatrix} -1 & 2 \\ -3 & 4 \end{bmatrix}$

Here, $a_{11} = -11,$

$M_{11} = 4$

$\therefore A_{11} = (-1)^{1+1}(4)$

$\Rightarrow 4$

$a_{12} = 2,$

$M_{12} = -3$

$\therefore A_{12} = (-1)^{1+2}(-3)$

$\Rightarrow 3$

$a_{21} = -3,$

$M_{21} = -2$

$\therefore A_{21} = (-1)^{2+1}(2)$

$\Rightarrow -2$

$a_{22} = 4,$

$M_{22} = -1$

$\therefore A_{22} = (-1)^{2+2}(-1)$

$\Rightarrow -1.$

2. $\begin{bmatrix} 1 & -1 & 2 \\ -2 & 3 & 5 \\ -2 & 0 & -1 \end{bmatrix}$

Ans: Suppose $A = \begin{bmatrix} 1 & -1 & 2 \\ -2 & 3 & 5 \\ -2 & 0 & -1 \end{bmatrix}$

The co-factor of a_{ij} is given by $A_{ij} = (-1)^{i+j} M_{ij}$

Now, $M_{11} = \begin{vmatrix} 3 & 5 \\ 0 & -1 \end{vmatrix}$

$\Rightarrow -3 - 0$

$\Rightarrow -3$

$\therefore A_{11} = (-1)^{1+1}(-3)$

$\Rightarrow -3$

$M_{12} = \begin{vmatrix} -2 & 5 \\ -2 & -1 \end{vmatrix}$

$\Rightarrow 2 + 10$

$\Rightarrow 12$

$\therefore A_{12} = (-1)^{1+2}(12)$

$\Rightarrow -12$

$M_{13} = \begin{vmatrix} -2 & 3 \\ -2 & 0 \end{vmatrix}$

$\Rightarrow 0 + 6$

$\Rightarrow 6$

$\therefore A_{13} = (-1)^{1+3}(6)$

$\Rightarrow 6$

$M_{21} = \begin{vmatrix} -1 & 2 \\ 0 & -1 \end{vmatrix}$

$\Rightarrow 1 - 0$

$\Rightarrow 1$

$\therefore A_{21} = (-1)^{2+1}(1)$

$\Rightarrow -1$

$M_{22} = \begin{vmatrix} 1 & 2 \\ -2 & -1 \end{vmatrix}$

$\Rightarrow -1 + 4$

$\Rightarrow 3$

$\therefore A_{22} = (-1)^{2+2}(3)$

$\Rightarrow 3$

$M_{23} = \begin{vmatrix} 1 & -1 \\ -2 & 0 \end{vmatrix}$

$\Rightarrow 0 - 2$

$\Rightarrow -2$

$\therefore A_{23} = (-1)^{2+3}(-2)$

$\Rightarrow 2$

$M_{31} = \begin{vmatrix} -1 & 2 \\ 3 & 5 \end{vmatrix}$

$\Rightarrow -5 - 6$

$\Rightarrow -11$

$\therefore A_{31} = (-1)^{3+1}(-11)$

$\Rightarrow -11$

$M_{32} = \begin{vmatrix} 1 & 2 \\ -2 & 5 \end{vmatrix}$

$\Rightarrow 5 + 4$

$\Rightarrow 9$

$\therefore A_{32} = (-1)^{3+2}(9)$

$\Rightarrow -9$

$M_{33} = \begin{vmatrix} 1 & -1 \\ -2 & 3 \end{vmatrix}$

$\Rightarrow 3 - 2$

$\Rightarrow 1$

$\therefore A_{33} = (-1)^{3+3}(1)$

$\Rightarrow 1$

Find the matrix of co-factors for the following matrices.

3. $\begin{bmatrix} 1 & 3 \\ 4 & -1 \end{bmatrix}$

Ans: Suppose $A = \begin{bmatrix} 1 & 3 \\ 4 & -1 \end{bmatrix}$

Here, $a_{11} = 1,$

$M_{11} = -1$

$\therefore A_{11} = (-1)^{1+1}(-1)$

$\Rightarrow -1$

$a_{12} = 3,$

$M_{12} = 4$

$\therefore A_{12} = (-1)^{1+2}(4)$

$\Rightarrow -4$

$a_{21} = 4,$

$M_{21} = 3$

$\therefore A_{21} = (-1)^{2+1}(3)$

$\Rightarrow -3$

$a_{22} = -1,$

$M_{22} = 1$

$\therefore A_{22} = (-1)^{2+1}(1)$

$\Rightarrow 1$

$\therefore$ the co-factor matrix $= \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$

$\Rightarrow \begin{pmatrix} -1 & -4 \\ -3 & 1 \end{pmatrix}$

4. $\begin{bmatrix} 1 & 0 & 2 \\ -2 & 1 & 3 \\ 0 & 3 & -5 \end{bmatrix}$

Ans: Suppose $A = \begin{bmatrix} 1 & 0 & 2 \\ -2 & 1 & 3 \\ 0 & 3 & 5 \end{bmatrix}$

Here, $a_{11} = 1,$

$A_{11} = \begin{vmatrix} 1 & 3 \\ 3 & -5 \end{vmatrix}$

$\Rightarrow -14$

$a_{12} = 0,$

$A_{12} = \begin{vmatrix} -2 & 0 \\ 3 & -5 \end{vmatrix}$

$\Rightarrow -10$

$a_{13} = 2,$

$A_{13} = \begin{vmatrix} -2 & 1 \\ 0 & 3 \end{vmatrix}$

$\Rightarrow -6$

$a_{21} = -2$

$A_{21} = \begin{vmatrix} 0 & 2 \\ 3 & -5 \end{vmatrix}$

$\Rightarrow 6$

$a_{22} = 1$

$A_{22} = \begin{vmatrix} 1 & 0 \\ 2 & -5 \end{vmatrix}$

$\Rightarrow -5$

$a_{23} = 3$

$A_{23} = \begin{vmatrix} 1 & 0 \\ 0 & 3 \end{vmatrix}$

$\Rightarrow -3$

$a_{31} = 0$

$A_{31} = \begin{vmatrix} 0 & 2 \\ 1 & 3 \end{vmatrix}$

$\Rightarrow -2$

$a_{32} = 3$

$A_{32} = \begin{vmatrix} 1 & 2 \\ -2 & 3 \end{vmatrix}$

$\Rightarrow -7$

$a_{33} = -5$

$A_{33} = \begin{vmatrix} 1 & 0 \\ -2 & 1 \end{vmatrix}$

$\Rightarrow 1$

$A_{11} = -14,$

$A_{12} = -10,$

$A_{13} = -6,$

$A_{21} = 6,$

$A_{22} = -5,$

$A_{23} = -3,$

$A_{31} = -2,$

$A_{32} = -7,$

$A_{33} = 1.$

∴ the co-factor matrix

$$\Rightarrow \begin{bmatrix} A_{11} A_{12} A_{13} \\ A_{21} A_{22} A_{23} \\ A_{31} A_{32} A_{33} \end{bmatrix} = \begin{bmatrix} -14 & -10 & -6 \\ 6 & -5 & -3 \\ -2 & -7 & 1 \end{bmatrix}$$

Find the adjoint of the following matrices.

5. $\begin{bmatrix} 2 & -3 \\ 3 & 5 \end{bmatrix}$

Ans: Suppose $A = \begin{bmatrix} 2 & -3 \\ 3 & 5 \end{bmatrix}$

Here, $a_{11} = 2,$

$M_{11} = 5$

$\therefore A_{11} = (-1)^{1+1}(5)$

$\Rightarrow 5$

$a_{12} = -3,$

$M_{12} = 3$

$\therefore A_{12} = (-1)^{1+2}(3)$

$\Rightarrow -3$

$a_{21} = 3,$

$M_{21} = -3$

$\therefore A_{21} = (-1)^{2+1}(-3)$

$\Rightarrow 3$

$a_{22} = 5,$

$M_{22} = 2$

$\therefore A_{22} - (-1)^{2+1}$

$\Rightarrow 2$

$= \begin{bmatrix} A_{11} A_{12} \\ A_{21} A_{22} \end{bmatrix}$

$\therefore$ the co-factor matrix

$\Rightarrow \begin{bmatrix} 5 & -3 \\ 3 & 2 \end{bmatrix}$

$\therefore adj\ A = \begin{pmatrix} 5 & 3 \\ -3 & 2 \end{pmatrix}$

6. $\begin{bmatrix} \mathbf{1} & \mathbf{-1} & \mathbf{2} \\ \mathbf{-2} & \mathbf{3} & \mathbf{5} \\ \mathbf{-2} & \mathbf{0} & \mathbf{-1} \end{bmatrix}$

Ans: Now, $M_{11} = \begin{vmatrix} 3 & 5 \\ 0 & -1 \end{vmatrix}$

$\Rightarrow -3 - 0$

$\Rightarrow -3$

$A_{11} - (-1)^{1+1}(-3)$

$\Rightarrow -3$

$M_{12} = \begin{vmatrix} -2 & 5 \\ -2 & -1 \end{vmatrix}$

$\Rightarrow 2 + 10$

$\Rightarrow 12$

$A_{12} - (-1)^{1+2}(12)$

$\Rightarrow -12$

$$M_{13} = \begin{vmatrix} -2 & 3 \\ -2 & 0 \end{vmatrix}$$

$\Rightarrow 0 + 6$

$\Rightarrow 6$

$A_{13} - (-1)^{1+3}(6)$

$\Rightarrow 6$

$$M_{21} = \begin{vmatrix} -1 & 2 \\ 0 & -1 \end{vmatrix}$$

$\Rightarrow 1 - 0$

$\Rightarrow 1$

$A_{21} - (-1)^{2-1}(1)$

$\Rightarrow -1$

$$M_{22} = \begin{vmatrix} 1 & 2 \\ -2 & -1 \end{vmatrix}$$

$\Rightarrow -1 + 4$

$\Rightarrow 3$

$A_{22} = (-1)^{2+2}(3)$

$\Rightarrow 3$

$$M_{23} = \begin{vmatrix} 1 & -1 \\ -2 & 0 \end{vmatrix}$$

$\Rightarrow 0 - 2$

$\Rightarrow -2$

$A_{23} = (-1)^{2+3}(-2)$

$\Rightarrow 2$

$$M_{31} = \begin{vmatrix} -1 & 2 \\ 3 & 5 \end{vmatrix}$$

$\Rightarrow -5 - 6$

$\Rightarrow -11$

$A_{31} = (-1)^{3+1}(-11)$

$\Rightarrow -11$

$M_{32} = \begin{vmatrix} 1 & 2 \\ -2 & 5 \end{vmatrix}$

$\Rightarrow 5 + 4$

$\Rightarrow 9$

$A_{32} - (-1)^{3+2}(9)$

$\Rightarrow -9$

$M_{33} = \begin{vmatrix} 1 & -1 \\ -2 & 3 \end{vmatrix}$

$\Rightarrow 3 - 2$

$\Rightarrow 1$

$A_{33} - (-1)^{3+3}(1)$

$\Rightarrow 1$

$A_{11} = -3,\ A_{12} = -12,\ A_{13} = 6,$

$A_{21} = -1,\ A_{22} = 3,\ A_{23} = 2,$

$A_{31} = -11,\ A_{32} = -9,\ A_{33} = 1$

∴ the co-factor matrix

$$\begin{bmatrix} A_{11}A_{12}A_{13} \\ A_{21}A_{22}A_{23} \\ A_{31}A_{32}A_{33} \end{bmatrix} = \begin{bmatrix} -3 & -12 & 6 \\ -1 & 3 & 2 \\ -11 & -9 & 1 \end{bmatrix}$$

$$\therefore adj\ A = \begin{bmatrix} -3 & -1 & -11 \\ -12 & 3 & -9 \\ 6 & 2 & 1 \end{bmatrix}$$

7. If $\mathbf{A}=\begin{bmatrix}1 & -1 & 2\\ 3 & 0 & -2\\ 1 & 0 & 3\end{bmatrix}$ **verify that** $\mathbf{A}\,(\mathbf{adj}\,\mathbf{A}) = (\mathbf{adj}\,\mathbf{A})\,\mathbf{A} = |\mathbf{A}|\cdot\mathbf{I}$

Ans: The given matrix is,

$$A=\begin{bmatrix}1 & -1 & 2\\ 3 & 0 & -2\\ 1 & 0 & 3\end{bmatrix}$$

$$\therefore |A| = \begin{vmatrix}1 & -1 & 2\\ 3 & 0 & -2\\ 1 & 0 & 3\end{vmatrix}$$

$$\Rightarrow 1\,(0+0) + 1\,(9+2) + 2\,(0-0)$$

$$\Rightarrow 0 + 11 + 0$$

$$\Rightarrow 11$$

First we have to find the co-factor matrix $= \left[A_{ij}\right]_{3\times 3}$,

Where, $A_{ij} - (-1)^{i+j} M_{ij}$

Now, $A_{11} = (-1)^{1+1} M_{11} = \begin{vmatrix}0 & -2\\ 0 & 3\end{vmatrix}$

$$\Rightarrow 0 + 0$$

$$\Rightarrow 0$$

$$A_{12} = (-1)^{1+2} M_{12} = -\begin{vmatrix}3 & -2\\ 1 & 3\end{vmatrix}$$

$$\Rightarrow -(9+2)$$

$$\Rightarrow -11$$

$$A_{13} = (-1)^{1+3} M_{13} = \begin{vmatrix}3 & 0\\ 1 & 0\end{vmatrix}$$

$$\Rightarrow 0 - 0$$

$$\Rightarrow 0$$

$$A_{21} = (-1)^{2+1} M_{21} = - \begin{vmatrix} -1 & 2 \\ 0 & 3 \end{vmatrix}$$

$$\Rightarrow - (-3 - 0)$$

$$\Rightarrow 3$$

$$A_{22} = (-1)^{2+2} M_{22} = \begin{vmatrix} 1 & 2 \\ 1 & 3 \end{vmatrix}$$

$$\Rightarrow 3 - 2$$

$$\Rightarrow 1$$

$$A_{23} = (-1)^{2+3} M_{23} = - \begin{vmatrix} 1 & -1 \\ 1 & 0 \end{vmatrix}$$

$$\Rightarrow - (0 + 1)$$

$$\Rightarrow -1$$

$$A_{31} = (-1)^{3+1} M_{31} = \begin{vmatrix} -1 & 2 \\ 0 & -2 \end{vmatrix}$$

$$\Rightarrow 2 - 0$$

$$\Rightarrow 2$$

$$A_{32} = (-1)^{3+2} M_{32} = - \begin{vmatrix} 1 & 2 \\ 3 & -2 \end{vmatrix}$$

$$\Rightarrow - (-2 - 6)$$

$$\Rightarrow 8$$

$$A_{33} = (-1)^{3+3} M_{33} = \begin{vmatrix} 1 & -1 \\ 3 & 0 \end{vmatrix}$$

$$\Rightarrow 0 + 3$$

$$\Rightarrow 3$$

Hence, the co – factor matrix

$$\begin{bmatrix} A_{11}A_{12}A_{13} \\ A_{21}A_{22}A_{23} \\ A_{31}A_{32}A_{33} \end{bmatrix} = \begin{bmatrix} 0 & -11 & 0 \\ 3 & 1 & -1 \\ 2 & 8 & 3 \end{bmatrix}$$

$$\therefore adj\ A = \begin{pmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{pmatrix}\begin{pmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 0+11+0 & 3-1-2 & 2-8+6 \\ 0+0-0 & 9+0+2 & 6+0-6 \\ 0+0+0 & 3+0-3 & 2+0+9 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{pmatrix} \ldots\ldots(1)$$

$$(adj\ A)\,A$$

$$\Rightarrow \begin{pmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{pmatrix}\begin{pmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 0+9+2 & 0+0+0 & 0-6+6 \\ -11+3+8 & 11+0+0 & -22-2+24 \\ 0-3+3 & 0-0+0 & 0+2+9 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{pmatrix} \ldots\ldots(2)$$

$$|A|.I = 11\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix} = \begin{pmatrix}11 & 0 & 0\\ 0 & 11 & 0\\ 0 & 0 & 11\end{pmatrix} .....(3)$$

From (1), (2) and (3), we get,

$$A(adj\ A) = (adj\ A)A = |A| \cdot I.$$

Find the inverse of the following matrices by the adjoint method.

8. $\begin{bmatrix}-1 & 5\\ -3 & 2\end{bmatrix}$

Ans: Suppose $A = \begin{bmatrix}-1 & 5\\ -3 & 2\end{bmatrix}$

$$\therefore |A| = \begin{vmatrix}-1 & 5\\ -3 & 2\end{vmatrix}$$

$$\Rightarrow -2 + 15$$

$$\Rightarrow 13 \neq 0$$

$\therefore A^{-1}$ exists.

First we have to find the co-factor matrix

$= [A_{ij}]_{2\times 2}$, where $A_{ij} = (-1)^{i+j} M_{ij}$

Now, $A_{11} = (-1)^{1+1} M_{11}$

$$\Rightarrow 2$$

$$A_{12} = (-1)^{1+2} M_{12}$$

$$\Rightarrow -(-3)$$

$$\Rightarrow 3$$

$$A_{21} = (-1)^{2+1} M_{21}$$

$$\Rightarrow -5$$

$$A_{22} = (-1)^{2+2} M_{22}$$

$$\Rightarrow -1$$

Hence, the co-factor matrix

$$\Rightarrow \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} 2 & 3 \\ -5 & -1 \end{pmatrix}$$

$$\therefore adj\ A = \begin{pmatrix} 2 & -5 \\ 3 & -1 \end{pmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|}(adj\ A)$$

$$\Rightarrow \frac{1}{13}\begin{pmatrix} 2 & -5 \\ 3 & -1 \end{pmatrix}$$

9. $\begin{bmatrix} 2 & -2 \\ 4 & 3 \end{bmatrix}$

Ans: Suppose $A = \begin{bmatrix} 2 & -2 \\ 4 & 3 \end{bmatrix}$

$$|A| = \begin{vmatrix} 2 & -2 \\ 4 & 3 \end{vmatrix}$$

$$\Rightarrow 6 + 8$$

$$\Rightarrow 14 \neq 0$$

$\therefore A^{-1}$ exist

First we have to find the co-factor matrix

$= [A_{ij}]_{2\times 2}$ where $A_{ij} = (-1)^{i+j} M_{ij}$

Now, $A_{11} = (-1)^{1+1} M_{11}$

$$\Rightarrow 3$$

$$A_{12} = (-1)^{1+2} M$$

$$\Rightarrow -4$$

$$A_{21} = (-2)^{2+1} M_{21}$$

$$\rightarrow (-2)$$

$$\Rightarrow 2$$

$$A_{22} = (-1)^{2+2} M_{22}$$

$\Rightarrow 2$

Hence the co-factor matrix

$$\Rightarrow \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} = \begin{bmatrix} 3 & -4 \\ 2 & 2 \end{bmatrix}$$

$$\therefore adj\ A = \begin{bmatrix} 3 & 2 \\ -4 & 2 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|}(adj\ A)$$

$$\Rightarrow \frac{1}{14}\begin{pmatrix} 3 & 2 \\ -4 & 2 \end{pmatrix}$$

10. $\begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{3} & \mathbf{3} & \mathbf{0} \\ \mathbf{5} & \mathbf{2} & \mathbf{-1} \end{bmatrix}$

Ans: Suppose $A = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 3 & 0 \\ 5 & 2 & -1 \end{bmatrix}$

$$\therefore |A| = \begin{vmatrix} 1 & 0 & 0 \\ 3 & 3 & 0 \\ 5 & 2 & -1 \end{vmatrix}$$

$$\Rightarrow 1(-3-0)-0+0$$

$$\Rightarrow -3 \neq 0$$

$\therefore A^{-1}$ exists.

First we have to find the co-factor matrix

$= [A_{ij}]_{3\times 3}$, Where $A_{ij} = (-1)^{i+j} M_{ij}$

Now, $A_{11} = (-1)^{1+1} M_{11}$

$$\Rightarrow \begin{vmatrix} 3 & 0 \\ 2 & -1 \end{vmatrix}$$

$$\Rightarrow -3-0$$

$$\Rightarrow -3$$

$A_{12} = (-1)^{1+2} M_{12}$

$\Rightarrow -\begin{vmatrix} 3 & 0 \\ 5 & -1 \end{vmatrix}$

$\Rightarrow -(-3-0)$

$\Rightarrow 3$

$A_{13} = (-1)^{1+3} M_{13}$

$\Rightarrow \begin{vmatrix} 3 & 3 \\ 5 & 2 \end{vmatrix}$

$\Rightarrow 6-15$

$\Rightarrow -9$

$A_{21} = (-1)^{2+1} M_{21}$

$\Rightarrow -\begin{vmatrix} 0 & 0 \\ 2 & -1 \end{vmatrix}$

$\Rightarrow -(0-0)$

$\Rightarrow 0$

$A_{22} = (-1)^{2+2} M_{22}$

$\Rightarrow \begin{vmatrix} 1 & 0 \\ 5 & -1 \end{vmatrix}$

$\Rightarrow -1-0$

$\Rightarrow -1$

$A_{23} = (-1)^{2+3} M_{23}$

$\Rightarrow -\begin{vmatrix} 1 & 0 \\ 5 & 2 \end{vmatrix}$

$\Rightarrow -(2-0)$

$\Rightarrow -2$

$A_{31} = (-1)^{3-1} M_{31}$

$$\Rightarrow \begin{vmatrix} 0 & 0 \\ 3 & 0 \end{vmatrix}$$

$$\Rightarrow 0-0$$

$$\Rightarrow 0$$

$$A_{32} - (-1)^{3+2} M_{32}$$

$$\Rightarrow -\begin{vmatrix} 1 & 0 \\ 3 & 0 \end{vmatrix}$$

$$\Rightarrow -(0-0)$$

$$\Rightarrow 0$$

$$A_{33} - (-1)^{3+3} M_{33}$$

$$\Rightarrow \begin{vmatrix} 1 & 0 \\ 3 & 3 \end{vmatrix}$$

$$\Rightarrow 3-0$$

$$\Rightarrow 3$$

Therefore, the co – factor matrix

$$\Rightarrow \begin{pmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{pmatrix} = \begin{pmatrix} -3 & 3 & -9 \\ 0 & -1 & -2 \\ 0 & 0 & 3 \end{pmatrix}$$

$$\therefore adj\ A = \begin{pmatrix} -3 & 0 & 0 \\ 3 & -1 & 0 \\ -9 & -2 & 3 \end{pmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|}(adj\ A)$$

$$\rightarrow \frac{1}{-3}\begin{pmatrix} -3 & 0 & 0 \\ 3 & -1 & 0 \\ -9 & -2 & 3 \end{pmatrix}$$

$$\therefore A^{-1} = \frac{1}{3}\begin{bmatrix} 3 & 0 & 0 \\ -3 & 1 & 0 \\ 9 & 2 & -3 \end{bmatrix}$$

11. $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 5 \end{bmatrix}$

Ans: Suppose $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 5 \end{bmatrix}$

$\therefore |A| = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 5 \end{bmatrix}$

$\Rightarrow 1(10 \quad 0) \quad 0 + 0$

$\Rightarrow 1(10) \quad 0 + 0$

$\Rightarrow 10 \neq 0$

$\therefore A^{-1}$ exists.

First we have to find the co-factor matrix

$= [A_{ij}]_{3\times 3}$ Where $A_{ij} = (-1)^{i+j} M_{ij}$

Now, $A_{11} = (-1)^{1+1} M_{11}$

$\Rightarrow \begin{vmatrix} 2 & 4 \\ 0 & 5 \end{vmatrix}$

$\Rightarrow 10 - 0$

$\Rightarrow 10$

$A_{12} = (-1)^{1+2} M_{12}$

$\Rightarrow -\begin{vmatrix} 0 & 4 \\ 0 & 5 \end{vmatrix}$

$\Rightarrow 0 - 0$

$\Rightarrow 0$

$A_{13} = (-1)^{1+3} M_{13}$

$\Rightarrow \begin{vmatrix} 0 & 2 \\ 0 & 0 \end{vmatrix}$

$\Rightarrow 0-0$

$\Rightarrow 0$

$A_{21} = (-1)^{2+1} M_{21}$

$\Rightarrow -\begin{vmatrix} 2 & 3 \\ 0 & 5 \end{vmatrix}$

$\Rightarrow -10-0$

$\Rightarrow -10$

$A_{22} = (-1)^{2+2} M_{22}$

$\Rightarrow \begin{vmatrix} 1 & 3 \\ 0 & 5 \end{vmatrix}$

$\Rightarrow 5-0$

$\rightarrow 5$

$A_{23} = (-1)^{2+3} M_{23}$

$\Rightarrow -\begin{vmatrix} 1 & 2 \\ 0 & 0 \end{vmatrix}$

$\Rightarrow -0-0$

$\Rightarrow 0$

$A_{31} = (-1)^{3-1} M_{31}$

$\Rightarrow \begin{vmatrix} 2 & 3 \\ 0 & 4 \end{vmatrix}$

$\Rightarrow 8-6$

$\Rightarrow 2$

$A_{32} = (-1)^{3+2} M_{32}$

$\Rightarrow -\begin{vmatrix} 1 & 3 \\ 0 & 4 \end{vmatrix}$

$\Rightarrow -4-0$

$\Rightarrow -4$

$A_{33} - (-1)^{3+3} M_{33}$

$\Rightarrow \begin{vmatrix} 1 & 2 \\ 0 & 2 \end{vmatrix}$

$\Rightarrow 2-0$

$\Rightarrow 2$

Therefore, the co-factor matrix

$$\Rightarrow \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} = \begin{bmatrix} 10 & 0 & 0 \\ -10 & 5 & 0 \\ 2 & -4 & 2 \end{bmatrix}$$

$$\therefore adj\ A = \begin{bmatrix} 10 & -10 & 2 \\ 0 & 5 & -4 \\ 0 & 0 & 2 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|}(adj\ A)$$

$$\Rightarrow \frac{1}{10}\begin{pmatrix} 10 & -10 & 2 \\ 0 & 5 & -4 \\ 0 & 0 & 2 \end{pmatrix}$$

$$\Rightarrow \therefore A^{-1} = \frac{1}{10}\begin{pmatrix} 10 & -10 & 2 \\ 0 & 5 & -4 \\ 0 & 0 & 2 \end{pmatrix}$$

Find the inverse of the following matrices.

12. $\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$

Ans: Suppose $A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$

$\therefore |A| = \begin{vmatrix} 1 & 2 \\ 2 & -1 \end{vmatrix}$

$\Rightarrow -1-4$

$\Rightarrow -5 \neq 0$

$\therefore A^{-1}$ exists.

Consider $AA^{-1} = I$

$\therefore \begin{pmatrix} 1 & 2 \\ 2 & -1 \end{pmatrix} A^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

By $R_2 - 2R_1$, we get,

$\begin{pmatrix} 1 & 2 \\ 0 & -5 \end{pmatrix} A^{-1} = \begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix}$

By $\left(-\frac{1}{5}\right) R_2$, We get,

$\begin{pmatrix} 1 & 2 \\ 0 & -5 \end{pmatrix} A^{-1} = \begin{pmatrix} 1 & 0 \\ \frac{2}{5} & \frac{-1}{5} \end{pmatrix}$

By $R_1 - 2R_2$, we get,

$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} A^{-1} = \begin{pmatrix} \frac{1}{5} & \frac{2}{5} \\ \frac{2}{5} & -\frac{1}{5} \end{pmatrix}$

$\therefore A^{-1} = \frac{1}{5}\begin{pmatrix} 1 & 2 \\ 2 & -1 \end{pmatrix}$

The answer can be checked by finding the product AA^{-1}

$AA^{-1} = \begin{pmatrix} 1 & 2 \\ 2 & -1 \end{pmatrix} \begin{pmatrix} \frac{1}{5} & \frac{2}{5} \\ \frac{2}{5} & -\frac{1}{5} \end{pmatrix}$

$$\Rightarrow \begin{bmatrix} 1\left(\frac{1}{5}\right)+2\left(\frac{2}{5}\right) & 1\left(\frac{2}{5}\right)+2\left(-\frac{1}{5}\right) \\ 2\left(\frac{1}{5}\right)-1\left(\frac{2}{5}\right) & 2\left(\frac{2}{5}\right)-1\left(-\frac{1}{5}\right) \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \frac{1}{5}+\frac{4}{5} & \frac{2}{5}-\frac{2}{5} \\ \frac{2}{5}-\frac{2}{5} & \frac{4}{5}-\frac{1}{5} \end{bmatrix}$$

$$\Rightarrow \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$$

Hence, A^{-1} is the required answer.

13. $\begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$

Ans: Suppose $A = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$

$$\therefore |A| = \begin{vmatrix} 2 & -3 \\ -1 & 2 \end{vmatrix}$$

$$\Rightarrow 4-3$$

$$\Rightarrow 1 \neq 0$$

$\therefore A^{-1}$ exists.

Consider $AA^{-1} = I$

$$\therefore \begin{pmatrix} 2 & -3 \\ -1 & 2 \end{pmatrix} A^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

By $R_1 + R_2$, we get,

$$\therefore \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix} A^{-1} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

By $R_2 + R_1$, We get,

$$\begin{pmatrix}1 & -1\\ 0 & 1\end{pmatrix}A^{-1}=\begin{pmatrix}1 & 1\\ 1 & 2\end{pmatrix}$$

By $R_1 + R_2$, We get,

$$\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}A^{-1}=\begin{pmatrix}2 & 3\\ 1 & 2\end{pmatrix}$$

$$\therefore A^{-1}=\begin{pmatrix}2 & 3\\ 1 & 2\end{pmatrix}$$

14. $\begin{bmatrix}\mathbf{0} & \mathbf{1} & \mathbf{2}\\ \mathbf{1} & \mathbf{2} & \mathbf{3}\\ \mathbf{3} & \mathbf{1} & \mathbf{1}\end{bmatrix}$

Ans: Suppose $A=\begin{bmatrix}0 & 1 & 2\\ 1 & 2 & 3\\ 3 & 1 & 1\end{bmatrix}$

$$\therefore |A|=\begin{vmatrix}0 & 1 & 2\\ 1 & 2 & 3\\ 3 & 1 & 1\end{vmatrix}$$

$$\Rightarrow 0(2-3)-1(1-9)+2(1-6)$$

$$\Rightarrow 0+8-10$$

$$\Rightarrow -2\neq 0.$$

$\therefore A^{-1}$ exists.

Consider $AA^{-1}=I$

$$\begin{pmatrix}1 & 2 & 3\\ 0 & 1 & 2\\ 3 & 1 & 1\end{pmatrix}A^{-1}=\begin{pmatrix}0 & 1 & 0\\ 1 & 0 & 0\\ 0 & 0 & 1\end{pmatrix}$$

By $R_1 \leftrightarrow R_2$, we get,

$$\therefore\begin{pmatrix}0 & 1 & 2\\ 1 & 2 & 3\\ 3 & 1 & 1\end{pmatrix}A^{-1}=\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}$$

By $R_3 - 3R_1$, we get,

$$\begin{pmatrix}1 & 2 & 3\\0 & 1 & 2\\0 & -5 & -8\end{pmatrix}A^{-1}=\begin{pmatrix}0 & 1 & 0\\1 & 0 & 0\\0 & -3 & 1\end{pmatrix}$$

By $R_1 - R_2 and\ R_3 + 5R_2$, we get,

$$\begin{pmatrix}1 & 0 & -1\\0 & 1 & 2\\0 & 0 & 2\end{pmatrix}A^{-1}=\begin{pmatrix}-2 & 1 & 0\\1 & 0 & 0\\5 & -3 & 1\end{pmatrix}$$

By $\left(\frac{1}{2}\right)R_3$, we get,

$$\begin{pmatrix}1 & 0 & -1\\1 & 1 & 2\\3 & 0 & 1\end{pmatrix}A^{-1}=\begin{pmatrix}-2 & 1 & 0\\1 & 0 & 0\\\frac{5}{2} & -\frac{3}{2} & \frac{1}{2}\end{pmatrix}$$

By $R_1 + R_3 and\ R_2 - 2R_3$, we get,

$$\begin{pmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{pmatrix}A^{-1}=\begin{pmatrix}\frac{1}{2} & -\frac{1}{2} & \frac{1}{2}\\-4 & 3 & -1\\\frac{5}{2} & -\frac{3}{2} & \frac{1}{2}\end{pmatrix}$$

$$\therefore A^{-1}=\frac{1}{2}\begin{pmatrix}1 & -1 & 1\\-8 & 6 & -2\\5 & -3 & 1\end{pmatrix}$$

15. $\begin{bmatrix}\mathbf{2} & \mathbf{0} & \mathbf{-1}\\\mathbf{5} & \mathbf{1} & \mathbf{0}\\\mathbf{0} & \mathbf{1} & \mathbf{3}\end{bmatrix}$

Ans: Suppose $A=\begin{bmatrix}2 & 0 & -1\\5 & 1 & 0\\0 & 1 & 3\end{bmatrix}$

$$\therefore |A| = \begin{vmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{vmatrix}$$

$\Rightarrow 2(3-0)-0(15-0)-1(5-0)$

$\Rightarrow 6-0-5$

$\Rightarrow 1 \neq 0.$

$\therefore A^{-1}$ exists.

Consider $AA^{-1} = I$

$$\therefore \begin{pmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{pmatrix} A^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

By $3R_1$, we get,

$$\begin{pmatrix} 6 & 0 & -3 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{pmatrix} A^{-1} = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

By $R_1 - R_2$, we get,

$$\begin{pmatrix} 1 & -1 & -3 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{pmatrix} A^{-1} = \begin{pmatrix} 3 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

By $R_2 - 5R_1$, We get,

$$\begin{pmatrix} 1 & -1 & -3 \\ 0 & 6 & 15 \\ 0 & 1 & 3 \end{pmatrix} A^{-1} = \begin{pmatrix} 3 & -1 & 0 \\ -15 & 6 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

By $R_2 - 5R_3$, we get,

$$\begin{pmatrix} 1 & -1 & -3 \\ 0 & 1 & 0 \\ 0 & 1 & 3 \end{pmatrix} A^{-1} = \begin{pmatrix} 3 & -1 & 0 \\ -15 & 6 & -5 \\ 0 & 0 & 1 \end{pmatrix}$$

By $R_1 + R_2 and\ R_3 - R_2$, We get,

$$\begin{pmatrix}1 & 0 & -3\\ 0 & 1 & 0\\ 0 & 0 & 3\end{pmatrix} A^{-1} = \begin{pmatrix}-12 & 5 & -5\\ -15 & 6 & -5\\ 15 & -6 & 6\end{pmatrix}$$

By $\left(\frac{1}{3}\right) R_3$, we get,

$$\begin{pmatrix}1 & 0 & -3\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix} A^{-1} = \begin{pmatrix}-12 & 5 & -5\\ -15 & 6 & -5\\ 5 & -2 & 2\end{pmatrix}$$

By $R_1 + 3R_3$, we get,

$$\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix} A^{-1} = \begin{pmatrix}3 & -1 & -1\\ -15 & 6 & -5\\ 5 & -2 & 2\end{pmatrix}$$

$$\therefore A^{-1} = \begin{pmatrix}3 & -1 & -1\\ -15 & 6 & -5\\ 5 & -2 & 2\end{pmatrix}$$

Miscellaneous Exercise 2(A)

1. If $\mathbf{A} = \begin{pmatrix}\mathbf{1} & \mathbf{0} & \mathbf{0}\\ \mathbf{2} & \mathbf{1} & \mathbf{0}\\ \mathbf{3} & \mathbf{3} & \mathbf{1}\end{pmatrix}$**, then reduce it to I3 by using column transformations.**

Ans: The determinant of matrix A is,

$$|A| = \begin{vmatrix}1 & 0 & 0\\ 2 & 1 & 0\\ 3 & 3 & 1\end{vmatrix}$$

$\Rightarrow 1(1 \quad 0) \quad 0 + 0$

$\Rightarrow 1 \neq 0$

$\therefore A$ is a non-singular matrix.

Hence, the required transformation is possible.

Now, $A = \begin{bmatrix}1 & 0 & 0\\ 2 & 1 & 0\\ 3 & 3 & 1\end{bmatrix}$

$$A \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -3 & 3 & 1 \end{bmatrix}$$

By $C_1 - 2C_2$, we get,

By $C_1 + 3C_3$ and $C_2 - 3C_3$, we get,

$$A \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I_3.$$

2. If $\mathbf{A} = \begin{bmatrix} \mathbf{2} & \mathbf{1} & \mathbf{3} \\ \mathbf{1} & \mathbf{0} & \mathbf{1} \\ \mathbf{1} & \mathbf{1} & \mathbf{1} \end{bmatrix}$ **then reduce it to** $\mathbf{I_3}$ **by using row transformations.**

Ans: The determinant of matrix A is,

$$|A| = \begin{vmatrix} 2 & 1 & 3 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{vmatrix}$$

$$\Rightarrow 2(0 - 1) - 1(1 - 1) + 3(1 - 0)$$

$$\Rightarrow -2 - 0 + 3$$

$$\Rightarrow 1 \neq 0$$

$\therefore A$ is a non-singular matrix.

Hence, the required transformation is possible.

Now, $A = \begin{bmatrix} 2 & 1 & 3 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

By $R_1 - R_2$, we get,

$$A \sim \begin{pmatrix} 1 & 1 & 2 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

By $R_2 - R_1$ and $R_3 - R_1$, we get,

$$A \sim \begin{pmatrix} 1 & 1 & 2 \\ 0 & -1 & -1 \\ 0 & 0 & -1 \end{pmatrix}$$

By $(-1)\,R_2$ and $(-1)\,R_3$, we get

$$A \sim \begin{pmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

By $R_1 - R_2$, we get

$$A \sim \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

By $R_1 - R_3$ and $R_2 - R_3$, we get

$$A \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I_3.$$

Check whether the following matrices are invertible or not:

3. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Ans: Let $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Then, $|A| = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$

$\Rightarrow 1 - 0$

$\Rightarrow 1 \neq 0$

$\therefore A$ is a non-singular matrix.

Hence, A^{-1} exists.

4. $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

Ans: Let $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

Then, $|A| = \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix}$

$\Rightarrow 1 - 1 = 0.$

$\therefore A$ is a singular matrix.

Hence, A^{-1} does not exist.

5. $\begin{bmatrix} \mathbf{1} & \mathbf{2} \\ \mathbf{3} & \mathbf{3} \end{bmatrix}$

Ans: Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 3 \end{bmatrix}$

Then, $|A| = \begin{vmatrix} 1 & 2 \\ 3 & 3 \end{vmatrix}$

$\Rightarrow 3 - 6$

$\Rightarrow -3 \neq 0$

$\therefore A$ is a non-singular matrix.

Hence, A^{-1} exist.

6. $\begin{bmatrix} \mathbf{2} & \mathbf{3} \\ \mathbf{10} & \mathbf{15} \end{bmatrix}$

Ans: Let $A = \begin{bmatrix} 2 & 3 \\ 10 & 15 \end{bmatrix}$

Then, $|A| = \begin{vmatrix} 2 & 3 \\ 10 & 15 \end{vmatrix}$

$\Rightarrow 30 - 30 = 0$

$\therefore A$ is a singular matrix.

Hence, A^{-1} does not exist.

7. $\begin{bmatrix} \mathbf{\cos\theta} & \mathbf{\sin\theta} \\ \mathbf{\sin\theta} & \mathbf{\cos\theta} \end{bmatrix}$

Ans: Let $A = \begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

Then, $|A| = \begin{vmatrix} sec\theta & tan\theta \\ tan\theta & sec\theta \end{vmatrix}$

$\Rightarrow sec^2\theta - tan^2\theta$

$\Rightarrow 1 \neq 0$

$\therefore A$ is a non-singular matrix.

Hence, A^{-1} exist.

8. $\begin{bmatrix} \mathbf{sec}\theta & \mathbf{tan}\theta \\ \mathbf{tan}\theta & \mathbf{sec}\theta \end{bmatrix}$

Ans: Let $A = \begin{bmatrix} sec\theta & tan\theta \\ tan\theta & sec\theta \end{bmatrix}$

Then, $|A| = \begin{vmatrix} sec\theta & tan\theta \\ tan\theta & sec\theta \end{vmatrix}$

$\Rightarrow sec^2\theta - tan^2\theta$

$\Rightarrow 1 \neq 0$

$\therefore A$ is a non-singular matrix.

Hence, A^{-1} exist.

9. $\begin{bmatrix} \mathbf{3} & \mathbf{4} & \mathbf{3} \\ \mathbf{1} & \mathbf{1} & \mathbf{0} \\ \mathbf{1} & \mathbf{4} & \mathbf{5} \end{bmatrix}$

Ans: Let $A = \begin{bmatrix} 3 & 4 & 3 \\ 1 & 1 & 0 \\ 1 & 4 & 5 \end{bmatrix}$

Then, $|A| = \begin{vmatrix} 3 & 4 & 3 \\ 1 & 1 & 0 \\ 1 & 4 & 5 \end{vmatrix}$

$\Rightarrow 3(5 - 0) - 4(5 - 0) + 3(4 - 1)$

$\Rightarrow 15 - 20 + 9$

$\Rightarrow 4 \neq 0$

$\therefore A$ is a non-singular matrix.

Hence, A^{-1} exist.

10. $\begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 3 \\ 1 & 2 & 3 \end{bmatrix}$

Ans: Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 3 \\ 1 & 2 & 3 \end{bmatrix}$

Then, $|A| = \begin{vmatrix} 1 & 2 & 3 \\ 2 & -1 & 3 \\ 1 & 2 & 3 \end{vmatrix}$

$\Rightarrow 1(-3-6) - 2(6-3) + 3(4+1)$

$\Rightarrow -9 - 6 + 15$

$\Rightarrow -15+15=0$

$\therefore A$ is a singular matrix.

Hence, A^{-1} does not exist.

11. $\begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 4 & 6 & 8 \end{bmatrix}$

Ans: Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 4 & 6 & 8 \end{bmatrix}$

Then, $|A| = \begin{vmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 4 & 6 & 8 \end{vmatrix}$

$\Rightarrow 1(32-30) - 2(24-20) + 3(18-16)$

$\Rightarrow 2 - 8 + 6$

$\Rightarrow -8+8=0$

$\therefore A$ is a singular matrix.

Hence, A^{-1} does not exist.

12. Find AB, if $\mathbf{A}=\begin{bmatrix}1 & 2 & 3\\ 1 & -2 & -3\end{bmatrix}$ **and** $\mathbf{B}=\begin{bmatrix}1 & -1\\ 1 & 2\\ 1 & -2\end{bmatrix}$ **Examine whether AB has inverse or not.**

Ans: The multiplication of matrix AB is,

$$AB=\begin{pmatrix}1 & 2 & 3\\ 1 & -2 & -3\end{pmatrix}\times\begin{pmatrix}1 & -1\\ 1 & 2\\ 1 & -2\end{pmatrix}$$

$$\Rightarrow\begin{pmatrix}1(1)+2(1)+3(1) & 1(-1)+2(2)+3(-2)\\ 1(1)+(-2)(1)+(-3)(1) & 1(-1)+(-2)(2)+(-3)(-2)\end{pmatrix}$$

$$\Rightarrow\begin{pmatrix}1+2+3 & -1+4-6\\ 1-2-3 & -1-4+6\end{pmatrix}$$

$$\Rightarrow\begin{pmatrix}6 & -3\\ 4 & 1\end{pmatrix}$$

$$\therefore|AB|=\begin{vmatrix}6 & -3\\ 4 & 1\end{vmatrix}$$

$\Rightarrow 6-12$

$\Rightarrow -6\neq 0$

$\therefore A$ is a non-singular matrix.

Hence, $(AB)^{-1}$ exist.

13. If $\mathbf{A}=\begin{bmatrix}x & 0 & 0\\ 0 & y & 0\\ 0 & 0 & z\end{bmatrix}$ **is a nonsingular matrix then find** $\mathbf{A}^{-1}$ **by elementary row transformations.**

Hence, find the inverse of $\begin{bmatrix}2 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & -1\end{bmatrix}$

Ans: Since A is a non-singular matrix, then find A^{-1} by using elementary row

transformations.

We write $AA^{-1} = I$

$$\therefore \begin{pmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{pmatrix} A^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

By $\left(\frac{1}{x}\right)R_1, \left(\frac{1}{y}\right)R_2$ and $\left(\frac{1}{z}\right)R_3$, we get,

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} A^{-1} = \begin{pmatrix} \frac{1}{x} & 0 & 0 \\ 0 & \frac{1}{y} & 0 \\ 0 & 0 & \frac{1}{z} \end{pmatrix}$$

$$A^{-1} = \begin{pmatrix} \frac{1}{x} & 0 & 0 \\ 0 & \frac{1}{y} & 0 \\ 0 & 0 & \frac{1}{z} \end{pmatrix}$$

Comparing $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ with $\begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$

we get, $x = 2, y = 1, z = -1$

$\therefore 1x=12, 1y=11= 1, 1z=1-1= -1$

$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ is $\begin{pmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$

14. If $\mathbf{A} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $\mathbf{X}$ is a 2×2 matrix such that $\mathbf{AX} = \mathbf{I}$, then find $\mathbf{X}$.

Ans:

We will reduce the matrix A to the identity matrix by using row transformations. During this process, I will be converted to the matrix X.
We have $AX = I$.

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} X = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

By $R_2 \to 3R_1$, we get,

$$\begin{pmatrix} 1 & 2 \\ 0 & -2 \end{pmatrix} X = \begin{pmatrix} 1 & 0 \\ 3 & -1 \end{pmatrix}$$

By $\left(-\frac{1}{2}\right) R_2$, we get,

$$\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} X = \begin{pmatrix} 1 & 0 \\ \frac{3}{2} & -\frac{1}{2} \end{pmatrix}$$

By $R_1 \to 2R_2$, we get,

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} X = \begin{pmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{pmatrix}$$

$$\therefore X = \begin{pmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{pmatrix}$$

Find the inverse of each of the following matrices (if they exist).

15. $\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$

Ans: Let $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$

$$\therefore |A| = \begin{vmatrix} 1 & -1 \\ 2 & 3 \end{vmatrix}$$

$$\Rightarrow 3 + 2$$

$$\Rightarrow 5 \neq 0$$

$\therefore A^{-1}$ exists.

Consider $AA^{-1} = I$

$$\begin{pmatrix}1 & -1\\ 2 & 3\end{pmatrix} A^{-1} = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$$

By $R_2 - 2R_1$, we get,

$$\begin{pmatrix}1 & -1\\ 0 & 5\end{pmatrix} A^{-1} = \begin{pmatrix}1 & 0\\ -2 & 1\end{pmatrix}$$

By $\frac{1}{5} R_2$, we get,

$$\begin{pmatrix}1 & -1\\ 0 & 5\end{pmatrix} A^{-1} = \begin{pmatrix}1 & 0\\ -\frac{2}{5} & \frac{1}{5}\end{pmatrix}$$

By $R_1 + R_2$, we get,

$$\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} A^{-1} = \begin{pmatrix}\frac{3}{5} & \frac{1}{5}\\ -\frac{2}{5} & \frac{1}{5}\end{pmatrix}$$

$$A^{-1} = \frac{1}{5}\begin{pmatrix}3 & 1\\ -2 & 1\end{pmatrix}$$

16. $\begin{bmatrix}2 & 1\\ 1 & -1\end{bmatrix}$

Ans: Let $A = \begin{bmatrix}2 & 1\\ 1 & -1\end{bmatrix}$

$$\therefore |A| = \begin{vmatrix}2 & 1\\ 1 & -1\end{vmatrix}$$

$\Rightarrow -2 - 1$

$\Rightarrow -3 \neq 0$

$\therefore A^{-1}$ exists.

Consider $AA^{-1} = I$

$$\begin{bmatrix}2 & 1\\ 1 & -1\end{bmatrix} A^{-1} = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

By $R_1 \leftrightarrow R_2$, we get

$$\begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

By $R_2 - 2R_1$, we get

$$\begin{bmatrix} 1 & -1 \\ 0 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} 0 & 1 \\ 1 & -2 \end{bmatrix}$$

By $\left(\frac{1}{3}\right) R_2$, we get

$$\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 0 & 1 \\ \frac{1}{3} & -\frac{2}{3} \end{bmatrix}$$

By $R_1 + R_2$, we get,

$$\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & -\frac{2}{3} \end{bmatrix}$$

$$A^{-1} = \frac{1}{3}\begin{bmatrix} 1 & 1 \\ 1 & -2 \end{bmatrix}$$

17. $\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}$

Ans: Let $A = \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}$

$$\therefore |A| = \begin{vmatrix} 1 & 3 \\ 2 & 7 \end{vmatrix}$$

$\Rightarrow 7 - 6$

$\Rightarrow 1 \neq 0$

$\therefore A^{-1}$ exists.

Consider $AA^{-1} = I$

$$\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

By $R_2 - 2R_1$, we get,

$$\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$$

By $R_1 - 3R_2$, we get,

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 7 & -3 \\ -2 & 1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 7 & -3 \\ -2 & 1 \end{bmatrix}$$

18. $\begin{bmatrix} 2 & -3 \\ 7 & 5 \end{bmatrix}$

Ans: Let $A = \begin{bmatrix} 2 & -3 \\ 7 & 5 \end{bmatrix}$

$$\therefore |A| = \begin{vmatrix} 2 & -3 \\ 7 & 5 \end{vmatrix}$$

$\Rightarrow 14 + 15$

$\Rightarrow 29 \neq 0$

$\therefore A^{-1}$ exists.

Consider $AA^{-1} = I$

$$\therefore \begin{bmatrix} 2 & -3 \\ 5 & 7 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

By $3R_1$, we get,

$$\begin{bmatrix} 6 & -9 \\ 5 & 7 \end{bmatrix} A^{-1} = \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}$$

By $R_1 - R_2$, we get,

$$\begin{bmatrix} 1 & -16 \\ 5 & 7 \end{bmatrix} A^{-1} = \begin{bmatrix} 3 & -1 \\ 0 & 1 \end{bmatrix}$$

By $R_2 - 5R_1$, we get

$$\begin{bmatrix}1 & -16\\0 & 87\end{bmatrix}A^{-1}=\begin{bmatrix}3 & -1\\-15 & 6\end{bmatrix}$$

By $\left(\frac{1}{87}\right)R_2$, we get

$$\begin{bmatrix}1 & -16\\0 & 1\end{bmatrix}A^{-1}=\begin{bmatrix}3 & -1\\-\frac{5}{29} & \frac{2}{29}\end{bmatrix}$$

By R_1+16R_2, we get,

$$\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}A^{-1}=\begin{bmatrix}\frac{7}{29} & \frac{3}{29}\\-\frac{5}{29} & \frac{2}{29}\end{bmatrix}$$

$$\therefore A^{-1}=\frac{1}{29}\begin{bmatrix}7 & 3\\-5 & 2\end{bmatrix}$$

19. $\begin{bmatrix}\mathbf{2} & \mathbf{1}\\\mathbf{7} & \mathbf{4}\end{bmatrix}$

Ans: Let $A=\begin{bmatrix}2 & 1\\7 & 4\end{bmatrix}$

$$\therefore |A|=\begin{vmatrix}2 & 1\\7 & 4\end{vmatrix}$$

$$\Rightarrow 8-7$$

$$\Rightarrow 1\neq 0$$

$\therefore A^{-1}$ exists.

Consider $AA^{-1}=I$

$$\begin{bmatrix}2 & 1\\7 & 4\end{bmatrix}A^{-1}=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$$ By $R_1\rightarrow R_1-\frac{1}{7}R_2$, we get,

$$\begin{bmatrix}1 & \frac{3}{7}\\7 & 4\end{bmatrix}A^{-1}=\begin{bmatrix}1 & -\frac{1}{7}\\0 & 1\end{bmatrix}$$

By $R_2 \to R_2 - \frac{1}{7} R_1$, we get,

$$\begin{bmatrix} 1 & \frac{3}{7} \\ 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & -\frac{1}{7} \\ -7 & 2 \end{bmatrix}$$

By $R_1 \to R_1 - \frac{3}{7} R_2$, we get

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 4 & -1 \\ -7 & 2 \end{bmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix} 4 & -1 \\ -7 & 2 \end{bmatrix}$$

20. $\begin{bmatrix} \mathbf{3} & \mathbf{-10} \\ \mathbf{2} & \mathbf{-7} \end{bmatrix}$

Ans: Let $A = \begin{bmatrix} 3 & -10 \\ 2 & -7 \end{bmatrix}$

$$\therefore |A| = \begin{vmatrix} 3 & -10 \\ 2 & -7 \end{vmatrix}$$

$$\Rightarrow -21 + 20$$

$$\Rightarrow -1 \neq 0$$

$\therefore A^{-1}$ exists.

Consider $AA^{-1} = I$

$$\begin{bmatrix} 3 & -10 \\ 2 & -7 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

By $R_1 - R_2$, we get,

$$\begin{bmatrix} 1 & -3 \\ 2 & -7 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

By $R_2 - 2R_1$, we get,

$$\begin{bmatrix} 1 & -3 \\ 0 & -1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix}$$

By $(-1)R_2$, we get,

$$\begin{bmatrix}1 & -3\\ 0 & 1\end{bmatrix}A^{-1} = \begin{bmatrix}1 & -1\\ 2 & -3\end{bmatrix}$$

By $R_1 + 3R_2$, we get,

$$\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}A^{-1} = \begin{bmatrix}7 & -10\\ 2 & -3\end{bmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix}7 & -10\\ 2 & -3\end{bmatrix}$$

21. $\begin{bmatrix}\mathbf{2} & \mathbf{-3} & \mathbf{3}\\ \mathbf{2} & \mathbf{2} & \mathbf{3}\\ \mathbf{3} & \mathbf{-2} & \mathbf{2}\end{bmatrix}$

Ans: Let $A = \begin{bmatrix}2 & -3 & 3\\ 2 & 2 & 3\\ 3 & -2 & 2\end{bmatrix}$

$$\therefore |A| = \begin{vmatrix}2 & -3 & 3\\ 2 & 2 & 3\\ 3 & -2 & 2\end{vmatrix}$$

$$\Rightarrow 2(4+6) + 3(4-9) + 3(-4-6)$$

$$\Rightarrow 20 - 15 - 30$$

$$\Rightarrow -25 \neq 0$$

$\therefore A^{-1}$ exists.

Consider $AA^{-1} = I$

$$\begin{bmatrix}2 & -3 & 3\\ 2 & 2 & 3\\ 3 & -2 & 2\end{bmatrix}A^{-1} = \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

By $R_1 \leftrightarrow R_3$, we get,

$$\begin{bmatrix}3 & -2 & 2\\ 2 & 2 & 3\\ 2 & -3 & 3\end{bmatrix}A^{-1} = \begin{bmatrix}0 & 0 & 1\\ 0 & 1 & 0\\ 1 & 0 & 0\end{bmatrix}$$

By $R_1 - R_2$, we get,

$$\begin{bmatrix}1 & -4 & -1\\ 2 & 2 & 3\\ 2 & -3 & 3\end{bmatrix} A^{-1} = \begin{bmatrix}0 & -1 & 1\\ 0 & 1 & 0\\ 1 & 0 & 0\end{bmatrix}$$

By $R_2 - 2R_1$ and $R_3 - 2R_1$, we get,

$$\begin{bmatrix}1 & -4 & -1\\ 0 & 10 & 5\\ 0 & 5 & 5\end{bmatrix} A^{-1} = \begin{bmatrix}0 & -1 & 1\\ 0 & 3 & -2\\ 1 & 2 & -2\end{bmatrix}$$

By $\left(\frac{1}{10}\right) R_2$, we get

$$\begin{bmatrix}1 & -4 & -1\\ 0 & 1 & \frac{1}{2}\\ 0 & 5 & 5\end{bmatrix} A^{-1} = \begin{bmatrix}0 & -1 & 1\\ 0 & \frac{3}{10} & -\frac{1}{5}\\ 1 & 2 & -2\end{bmatrix}$$

By $R_1 + 4R_2$ and $R_3 - 5R_2$, we get,

$$\begin{bmatrix}1 & 0 & 1\\ 0 & 1 & \frac{1}{2}\\ 0 & 0 & \frac{5}{2}\end{bmatrix} A^{-1} = \begin{bmatrix}0 & \frac{1}{5} & \frac{1}{5}\\ 0 & \frac{3}{10} & -\frac{1}{5}\\ 1 & \frac{1}{2} & -1\end{bmatrix}$$

By $\left(\frac{2}{5}\right) R_1$, we get,

$$\begin{bmatrix}1 & 0 & 1\\ 0 & 1 & \frac{1}{2}\\ 0 & 0 & 1\end{bmatrix} A^{-1} = \begin{bmatrix}0 & \frac{1}{5} & \frac{1}{5}\\ 0 & \frac{3}{10} & -\frac{1}{5}\\ \frac{2}{5} & \frac{1}{5} & -\frac{2}{5}\end{bmatrix}$$

By $R_1 - R_3$ and $R_2 - \left(\frac{1}{2}\right)R_3$, we get,

$$\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}A^{-1} = \begin{bmatrix}-\frac{2}{5} & 0 & \frac{3}{5}\\ -\frac{1}{5} & \frac{1}{5} & 0\\ \frac{2}{5} & \frac{1}{5} & -\frac{2}{5}\end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{5}\begin{bmatrix}-2 & 0 & 3\\ -1 & 1 & 0\\ 2 & 1 & -2\end{bmatrix}$$

22. $\begin{bmatrix}\mathbf{1} & \mathbf{3} & \mathbf{-2}\\ \mathbf{-3} & \mathbf{0} & \mathbf{-5}\\ \mathbf{2} & \mathbf{5} & \mathbf{0}\end{bmatrix}$

Ans: Let $A = \begin{bmatrix}1 & 3 & -2\\ -3 & 0 & -5\\ 2 & 5 & 0\end{bmatrix}$

$$\therefore |A| = \begin{vmatrix}1 & 3 & -2\\ -3 & 0 & -5\\ 2 & 5 & 0\end{vmatrix}$$

$\Rightarrow 1(0 + 25) + 3(0 + 10) + 2(-15 - 0)$

$\Rightarrow 25 + 30 - 30$

$\Rightarrow 25 \neq 0$

$\therefore A^{-1}$ exists.

Consider $AA^{-1} = I$

$$\therefore \begin{bmatrix}1 & 3 & -2\\ -3 & 0 & -5\\ 2 & 5 & 0\end{bmatrix}A^{-1} = \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

By $R_2 \to R_2 + 3R_1$, we get

$$\begin{bmatrix}1 & 3 & -2\\ 0 & 9 & -11\\ 2 & 5 & 0\end{bmatrix} A^{-1} = \begin{bmatrix}1 & 0 & 0\\ 3 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

By $R_3 \to R_3 - 2R_1$, we get

$$\begin{bmatrix}1 & 3 & -2\\ 0 & 9 & -11\\ 0 & -1 & 4\end{bmatrix} A^{-1} = \begin{bmatrix}1 & 0 & 0\\ 3 & 1 & 0\\ -2 & 0 & 1\end{bmatrix}$$

By $R \to 1/9\, R_2$, we get

$$\begin{bmatrix}1 & 3 & -2\\ 0 & 1 & -\frac{11}{9}\\ 0 & -1 & 4\end{bmatrix} A^{-1} = \begin{bmatrix}1 & 0 & 0\\ \frac{1}{3} & \frac{1}{9} & 0\\ -2 & 0 & 1\end{bmatrix}$$

By $R_3 \to R_3 + R_2$, we get

$$\begin{bmatrix}1 & 3 & -2\\ 0 & 1 & -\frac{11}{9}\\ 0 & 0 & \frac{25}{9}\end{bmatrix} A^{-1} = \begin{bmatrix}1 & 0 & 0\\ \frac{1}{3} & \frac{1}{9} & 0\\ -\frac{5}{3} & \frac{1}{9} & 1\end{bmatrix}$$

By $R_1 \to R_1 + 3R_2$, we get

$$\begin{bmatrix}1 & 0 & \frac{5}{3}\\ 0 & 1 & -\frac{11}{9}\\ 0 & 0 & \frac{25}{9}\end{bmatrix} A^{-1} = \begin{bmatrix}0 & -\frac{1}{3} & 0\\ \frac{1}{3} & \frac{1}{9} & 0\\ -\frac{5}{3} & \frac{1}{9} & 1\end{bmatrix}$$

By $R_3 \to \left(\frac{9}{25}\right) R_3$, we get

$$\begin{bmatrix} 1 & 0 & \frac{5}{3} \\ 0 & 1 & -\frac{11}{9} \\ 0 & 0 & \frac{25}{9} \end{bmatrix} A^{-1} = \begin{bmatrix} 0 & -\frac{1}{3} & 0 \\ \frac{1}{3} & \frac{1}{9} & 0 \\ -\frac{3}{5} & \frac{1}{25} & \frac{9}{25} \end{bmatrix}$$

By $R_2 \to R_2 + \left(\frac{11}{9}\right) R_3$, we get

$$\begin{bmatrix} 1 & 0 & \frac{5}{3} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 0 & -\frac{1}{3} & 0 \\ -\frac{2}{5} & \frac{4}{25} & \frac{11}{25} \\ -\frac{3}{5} & \frac{1}{25} & \frac{9}{25} \end{bmatrix}$$

By $R_1 \to R_1 - \left(\frac{5}{3}\right) R_3$, we get

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} \frac{5}{5} & -\frac{6}{15} & -\frac{3}{5} \\ -\frac{2}{5} & \frac{4}{5} & \frac{11}{25} \\ -\frac{3}{5} & \frac{1}{25} & \frac{9}{25} \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 1 & -\frac{2}{5} & -\frac{3}{5} \\ -\frac{2}{5} & \frac{4}{25} & \frac{11}{25} \\ -\frac{3}{5} & \frac{1}{25} & \frac{9}{25} \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{25} \begin{bmatrix} 25 & -10 & -15 \\ -10 & 4 & 11 \\ -15 & 1 & 9 \end{bmatrix}$$

23. $\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$

Ans: Let $A = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$

$\therefore |A| = \begin{vmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{vmatrix}$

$\Rightarrow 2(3 - 0) - 0 - 1(5 - 0)$

$\Rightarrow 6 - 0 - 5$

$\Rightarrow 1 \neq 0$

$\therefore A^{-1}$ exists.

Consider $AA^{-1} = I$

$$\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By $R_1 \leftrightarrow R_2$, we get,

$$\begin{bmatrix} 5 & 1 & 0 \\ 2 & 0 & -1 \\ 0 & 1 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By $R_1 - 2R_2$, we get,

$$\begin{bmatrix} 1 & 1 & 2 \\ 2 & 0 & -1 \\ 0 & 1 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By $R_2 - 2R_1$, we get,

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & -2 & -5 \\ 0 & 1 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} -2 & 1 & 0 \\ 5 & -2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By $R_2 + 3R_3$, we get,

$$\begin{bmatrix}1 & 1 & 2\\ 0 & 1 & 4\\ 0 & 1 & 3\end{bmatrix} A^{-1} = \begin{bmatrix}-2 & 1 & 0\\ 5 & -2 & 3\\ 0 & 0 & 1\end{bmatrix}$$

By $R_1 - R_2$ and $R_3 - R_2$, we get,

$$\begin{bmatrix}1 & 0 & -2\\ 0 & 1 & 4\\ 0 & 0 & -1\end{bmatrix} A^{-1} = \begin{bmatrix}-7 & 3 & -3\\ 5 & -2 & 3\\ -5 & 2 & -2\end{bmatrix}$$

By $(-1)\, R_3$, we get,

$$\begin{bmatrix}1 & 0 & -2\\ 0 & 1 & 4\\ 0 & 0 & 1\end{bmatrix} A^{-1} = \begin{bmatrix}-7 & 3 & -3\\ 5 & -2 & 3\\ 5 & -2 & 2\end{bmatrix}$$

By $R_1 + 2R_3$ and $R_2 - 4R_3$, we get,

$$\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix} A^{-1} = \begin{bmatrix}3 & -1 & 1\\ -15 & 6 & -5\\ 5 & -2 & 2\end{bmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix}3 & -1 & 1\\ -15 & 6 & -5\\ 5 & -2 & 2\end{bmatrix}$$

24. $\begin{bmatrix}\mathbf{1} & \mathbf{2} & \mathbf{-2}\\ \mathbf{0} & \mathbf{-2} & \mathbf{1}\\ \mathbf{-1} & \mathbf{3} & \mathbf{0}\end{bmatrix}$

Ans: Let $A = \begin{bmatrix}1 & 2 & -2\\ 0 & -2 & 1\\ -1 & 3 & 0\end{bmatrix}$

$$\therefore A^{-1} = \begin{bmatrix}1 & 2 & -2\\ 0 & -2 & 1\\ -1 & 3 & 0\end{bmatrix}$$

$$\Rightarrow 1\begin{vmatrix}-2 & 1\\ 3 & 0\end{vmatrix} - 2\begin{vmatrix}0 & 1\\ -1 & 1\end{vmatrix} - 2\begin{vmatrix}0 & -2\\ -1 & 3\end{vmatrix}$$

$|A| = 1(0 - 3) - 2(0 + 1) - 2(0 - 2)$

$\Rightarrow -3 - 2 + 4$

$\Rightarrow -1 \neq 0$

$\therefore A^{-1}$ exists.

We have $AA^{-1} = I$

$$\begin{bmatrix} 1 & 2 & -2 \\ 0 & -2 & 1 \\ -1 & 3 & 0 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By $R_3 \to R_3 + R_1$, we get

$$\begin{bmatrix} 1 & 2 & -2 \\ 0 & -2 & 1 \\ 0 & 5 & -2 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

By $R_3 \to R_3 + 2R_2$, we get

$$\begin{bmatrix} 1 & 2 & -2 \\ 0 & -2 & 1 \\ 0 & 1 & -0 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

By $R_2 \leftrightarrow R_3$, we get

$$\begin{bmatrix} 1 & 2 & -2 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

By $R_1 \to R_1 - 2R_2$ and $R_3 \to R_3 + 2R_2$, we get

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} -1 & -4 & -2 \\ 1 & 2 & 1 \\ 2 & 5 & 2 \end{bmatrix}$$

By $R_1 \to R_1 + 2R_3$, we get

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 3 & 6 & 2 \\ 1 & 2 & 1 \\ 2 & 5 & 2 \end{bmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix} 3 & 6 & 2 \\ 1 & 2 & 1 \\ 2 & 5 & 2 \end{bmatrix}$$

25. Find the inverse of $\mathbf{A} = \begin{bmatrix} \mathbf{Cos}\theta & -\mathbf{Sin}\theta & \mathbf{0} \\ \mathbf{Sin}\theta & \mathbf{Cos}\theta & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} \end{bmatrix}$ **by**

(i) elementary row transformations

Ans: The determinant of matrix A is,

$$|A| = \begin{vmatrix} Cos\ \theta & -Sin\ \theta & 0 \\ Sin\ \theta & Cos\ \theta & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$\Rightarrow cos\theta\ (cos\theta - 0) + sin\theta\ (sin\theta - 0) + 0$$

$$\Rightarrow cos^2\theta + sin^2\theta$$

$$\Rightarrow 1 \neq 0$$

$\therefore A^{-1}$ exists.

Consider $AA^{-1} = I$

$$\begin{bmatrix} Cos\ \theta & -Sin\ \theta & 0 \\ Sin\ \theta & Cos\ \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By $Cos\theta \times R_1$, we get,

$$\begin{bmatrix} Cos^2\theta & -Sin\ \theta Cos\ \theta & 0 \\ Sin\ \theta & Cos\ \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} Cos\ \theta & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By $R_1 + Sin\ \theta \times R_2$, we get,

$$\begin{bmatrix} 1 & 0 & 0 \\ Sin\ \theta & Cos\ \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} Cos\ \theta & Sin\ \theta & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By $R_2 - Sin\ \theta \times R_1$, we get,

$$\begin{bmatrix}1 & 0 & 0\\ 0 & Cos\ \theta & 0\\ 0 & 0 & 1\end{bmatrix} A^{-1} = \begin{bmatrix}Cos\ \theta & Sin\ \theta & 0\\ -Sin\theta\ Cos\theta & Cos^2\theta & 0\\ 0 & 0 & 1\end{bmatrix}$$

By $\left(\frac{1}{Cos\theta}\right) \times R_1$, we get

$$\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix} A^{-1} = \begin{bmatrix}Cos\theta & Sin\theta & 0\\ -Sin\theta & Cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix}Cos\theta & Sin\theta & 0\\ -Sin\theta & Cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}$$

(ii) elementary column transformations

Ans: The determinant of matrix A is,

$$|A| = \begin{vmatrix}Cos\ \theta & -Sin\ \theta & 0\\ Sin\ \theta & Cos\ \theta & 0\\ 0 & 0 & 1\end{vmatrix}$$

$$\Rightarrow cos\theta\ (cos\theta - 0) + sin\theta\ (sin\theta - 0) + 0$$

$$\Rightarrow cos^2\theta + sin^2\theta$$

$$\Rightarrow 1 \neq 0$$

$\therefore A^{-1}$ exists.

Consider $A^{-1}A = I$

$$A^{-1}\begin{bmatrix}Cos\ \theta & -Sin\ \theta & 0\\ Sin\ \theta & Cos\ \theta & 0\\ 0 & 0 & 1\end{bmatrix} = \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

By $(Cos\ \theta) \times C_1$, we get,

$$A^{-1}\begin{bmatrix}Cos^2\theta & -Sin\theta & 0\\ Sin\theta Cos\theta & Cos\theta & 0\\ 0 & 0 & 1\end{bmatrix} = \begin{bmatrix}Cos\ \theta & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

By $C_1 - Sin\theta \times C_2$, we get

$$A^{-1}\begin{bmatrix}1 & -Sin\theta & 0\\ 0 & Cos\theta & 0\\ 0 & 0 & 1\end{bmatrix} = \begin{bmatrix}Cos\theta & 0 & 0\\ -Sin\theta & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

By $C_2 + Sin\,\theta \times C_1$, we get

$$A^{-1}\begin{bmatrix}1 & 0 & 0\\ 0 & Cos\theta & 0\\ 0 & 0 & 1\end{bmatrix} = \begin{bmatrix}Cos\,\theta & Sin\theta\,Cos\theta & 0\\ -Sin\theta & Cos^2\theta & 0\\ 0 & 0 & 1\end{bmatrix}$$

By $\left(\frac{1}{Cos\theta}\right) C_2$, we get

$$A^{-1}\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix} = \begin{bmatrix}Cos\theta & Sin\theta & 0\\ -Sin\theta & Cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix}Cos\theta & Sin\theta & 0\\ -Sin\theta & Cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}$$

26. If $\mathbf{A} = \begin{bmatrix}2 & 3\\ 1 & 2\end{bmatrix}$, $\mathbf{B} = \begin{bmatrix}1 & 0\\ 3 & 1\end{bmatrix}$ **find** $\mathbf{AB}$ **and** $(\mathbf{AB})^{-1}$**. Verify that** $(\mathbf{AB})^{-1} = \mathbf{B^{-1}A^{-1}}$

Ans: The multiplication of matrix AB is,

$$AB = \begin{bmatrix}2 & 3\\ 1 & 2\end{bmatrix}\begin{bmatrix}1 & 0\\ 3 & 1\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}2+9 & 0+3\\ 1+6 & 0+2\end{bmatrix} = \begin{bmatrix}11 & 3\\ 7 & 2\end{bmatrix}$$

$$|AB| = \begin{vmatrix}11 & 3\\ 7 & 2\end{vmatrix}$$

$$\Rightarrow 22 - 21$$

$$\Rightarrow 1 \neq 0$$

$\therefore (AB)^{-1}$ exists.

Now, $(AB)(AB)^{-1} = I$

$$\begin{bmatrix}11 & 3\\ 7 & 2\end{bmatrix}(AB)^{-1} = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

By $2R_1$, we get,

$$\begin{bmatrix}22 & 6\\ 7 & 2\end{bmatrix}(AB)^{-1} = \begin{bmatrix}2 & 0\\ 0 & 1\end{bmatrix}$$

By $R_1 - 3R_2$, we get,

$$\begin{bmatrix}1 & 0\\ 7 & 2\end{bmatrix}(AB)^{-1} = \begin{bmatrix}2 & -3\\ 0 & 1\end{bmatrix}$$

By $R_2 - 7R_1$, we get,

$$\begin{bmatrix}1 & 0\\ 0 & 2\end{bmatrix}(AB)^{-1} = \begin{bmatrix}2 & -3\\ -14 & 22\end{bmatrix}$$

By $\left(\frac{1}{2}\right)R_2$, we get,

$$\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}(AB)^{-1} = \begin{bmatrix}2 & -3\\ -7 & 11\end{bmatrix}$$

$$(AB)^{-1} = \begin{bmatrix}2 & -3\\ -7 & 11\end{bmatrix} \quad \text{.....(1)}$$

$$\therefore |A| = \begin{vmatrix}2 & 3\\ 1 & 2\end{vmatrix}$$

$$\Rightarrow 4 - 3$$

$$\Rightarrow 1 \neq 0$$

$\therefore A^{-1}$ exists

Consider $A^{-1}A = I$

$$\begin{bmatrix}2 & 3\\ 1 & 2\end{bmatrix}A^{-1} = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

By $R_1 \leftrightarrow R_2$, we get,

$$\begin{bmatrix}1 & 2\\ 2 & 3\end{bmatrix}A^{-1} = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

By $R_2 - 2R_1$, we get,

$$\begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 1 \\ 1 & -2 \end{bmatrix}$$

By $(-1)R_2$, we get,

$$\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$$

By $R_1 - 2R_2$, we get,

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$$

$$|B| = \begin{vmatrix} 1 & 0 \\ 3 & 1 \end{vmatrix}$$

$$\Rightarrow 1 - 0$$

$$\Rightarrow 1 \neq 0$$

B^{-1} exists.

Consider $BB^{-1} = I$

$$\begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix} B^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

By $R_2 - 3R_1$, we get,

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} B^{-1} = \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix}$$

$$B^{-1} = \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix}$$

$$\therefore B^{-1} \cdot A^{-1} = \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix} \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2-0 & -3+0 \\ -6-1 & 9+2 \end{bmatrix}$$

$\Rightarrow \begin{bmatrix} 2 & -3 \\ -7 & 11 \end{bmatrix}$ .....(2)

From (1) and (2), $(AB)^{-1} = B^{-1} \cdot A^{-1}$.

27. If $\mathbf{A} = \begin{bmatrix} 4 & 5 \\ 2 & 1 \end{bmatrix}$**, then show that** $\mathbf{A}^{-1} = \frac{1}{6}(\mathbf{A} - 5\mathbf{I})$

Ans: The determinant of matrix A is,

$|A| = \begin{vmatrix} 4 & 5 \\ 2 & 1 \end{vmatrix}$

$\Rightarrow 4 - 10$

$\Rightarrow -6 \neq 0$

$\therefore A^{-1}$ exists.

Consider $AA^{-1} = I$

$\therefore \begin{bmatrix} 4 & 5 \\ 2 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

By $\left(\frac{1}{4}\right) R_1$, we get,

$\begin{bmatrix} 1 & \frac{5}{4} \\ 2 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} \frac{1}{4} & 0 \\ 0 & 1 \end{bmatrix}$

By $R_2 - 2R_1$, We get,

$\begin{bmatrix} 1 & \frac{5}{4} \\ 0 & -\frac{3}{2} \end{bmatrix} A^{-1} = \begin{bmatrix} \frac{1}{4} & 0 \\ -\frac{1}{2} & 1 \end{bmatrix}$

By $-\frac{2}{3} R_2$, We get,

$\begin{bmatrix} 1 & \frac{5}{4} \\ 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} \frac{1}{4} & 0 \\ \frac{1}{3} & -\frac{2}{3} \end{bmatrix}$

By $R_1 - \frac{5}{4}R_2$, We get,

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} -\frac{1}{6} & \frac{5}{6} \\ \frac{1}{3} & -\frac{2}{3} \end{bmatrix}$$

$$A^{-1} = \frac{1}{6}\begin{bmatrix} -1 & 5 \\ 2 & -4 \end{bmatrix}$$

$$\frac{1}{6}(A - 5I) = \frac{1}{6}\left\{\begin{bmatrix} 4 & 5 \\ 2 & 1 \end{bmatrix} - 5\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right\}$$

$$\Rightarrow \frac{1}{6}\left\{\begin{bmatrix} 4 & 5 \\ 2 & 1 \end{bmatrix} - \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}\right\}$$

$$\Rightarrow \frac{1}{6}\begin{bmatrix} -1 & 5 \\ 2 & 4 \end{bmatrix}$$

From (1) and (2) $A^{-1} = 1/6\,(A - 5I)$

28. Find matrix $\mathbf{X}$ such that $\mathbf{AX = B}$, where $\mathbf{A} = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 0 & 1 \\ 2 & 4 \end{bmatrix}$

Ans: The matrix equation can be written as,

$$AX = B$$

$$\begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix} X = \begin{bmatrix} 0 & 1 \\ 2 & 4 \end{bmatrix}$$

By $R_2 + R_1$, we get

$$\begin{bmatrix} 1 & 2 \\ 0 & 5 \end{bmatrix} X = \begin{bmatrix} 0 & 1 \\ 2 & 5 \end{bmatrix}$$

By $\left(\frac{1}{5}\right)R_2$, we get

$$\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} X = \begin{bmatrix} 0 & 1 \\ \frac{2}{5} & 1 \end{bmatrix}$$

By $R_1 - 2R_2$, we get

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} X = \begin{bmatrix} -\frac{4}{5} & -1 \\ \frac{2}{5} & 1 \end{bmatrix}$$

$$\therefore X = \begin{bmatrix} -\frac{4}{5} & -1 \\ \frac{2}{5} & 1 \end{bmatrix}$$

29. Find X, if $AX = B$ where $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix}$ **and** $B = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$.

Ans: The matrix equation can be written as,

$$AX = B$$

$$\begin{bmatrix} 1 & 2 & 3 \\ -1 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix} X = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

By $R_2 + R_1$ and $R_3 - R_1$, we get

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 3 & 5 \\ 0 & 0 & 1 \end{bmatrix} X = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$$

By $\left(\frac{1}{3}\right) R_2$, we get,

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & \frac{5}{3} \\ 0 & 0 & 1 \end{bmatrix} X = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

By $R_1 - 2R_2$, we get,

$$\begin{bmatrix} 1 & 0 & -\frac{1}{3} \\ 0 & 1 & \frac{5}{3} \\ 0 & 0 & 1 \end{bmatrix} X = \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}$$

By $R_1 + \frac{1}{3}R_3$ and $R_2 - \frac{5}{3}R_1$, we get

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} X = \begin{bmatrix} -\frac{1}{3} \\ \frac{7}{3} \\ 2 \end{bmatrix}$$

$$\therefore X = \begin{bmatrix} -\frac{1}{3} \\ \frac{7}{3} \\ 2 \end{bmatrix}$$

30. If $A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 1 \\ 3 & 1 \end{bmatrix}$ and $C = \begin{bmatrix} 24 & 7 \\ 31 & 9 \end{bmatrix}$ then find matrix X such that $AXB - C$.

Ans: The matrix equation can be written as,

$$AXB - C$$

$$\therefore \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}(XB) = \begin{bmatrix} 24 & 7 \\ 31 & 9 \end{bmatrix}$$

First we perform the row transformations.

By $R_2 - R_1$, we get

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} (XB) = \begin{bmatrix} 24 & 7 \\ 7 & 2 \end{bmatrix}$$

By $R_1 - R_2$, we get

$$\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}(XB) = \begin{bmatrix}17 & 5\\7 & 2\end{bmatrix}$$

$$\therefore XB = \begin{bmatrix}17 & 5\\7 & 2\end{bmatrix}$$

$$\therefore X\begin{bmatrix}4 & 1\\3 & 1\end{bmatrix} = \begin{bmatrix}17 & 5\\7 & 2\end{bmatrix}$$

Now, we perform the column transformations.

By $C_1 \leftrightarrow C_2$, we get,

$$X\begin{bmatrix}1 & 4\\1 & 3\end{bmatrix} = \begin{bmatrix}5 & 17\\2 & 7\end{bmatrix}$$

By $C_2 - 4C_1$, we get,

$$X\begin{bmatrix}1 & 0\\1 & -1\end{bmatrix} = \begin{bmatrix}5 & -3\\2 & -1\end{bmatrix}$$

By $(-1)C_2$, we get,

$$X\begin{bmatrix}1 & 0\\1 & 1\end{bmatrix} = \begin{bmatrix}5 & 3\\2 & 1\end{bmatrix}$$

By $C_1 - C_2$, we get,

$$X\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} = \begin{bmatrix}2 & 3\\1 & 1\end{bmatrix}$$

$$\therefore X = \begin{bmatrix}2 & 3\\1 & 1\end{bmatrix}$$

31. Find the inverse of $\begin{bmatrix}1 & 2 & 3\\1 & 1 & 5\\2 & 4 & 7\end{bmatrix}$ **by adjoint method.**

Ans: Let the matrix be A,

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 5 \\ 2 & 4 & 7 \end{bmatrix}$$

$$\therefore |A| = \begin{vmatrix} 1 & 2 & 3 \\ 1 & 1 & 5 \\ 2 & 4 & 7 \end{vmatrix}$$

$$\Rightarrow 1(7 - 20) - 2(7 - 10) + 3(4 - 2)$$

$$\Rightarrow -13 + 6 + 6 = -1 \neq 0$$

$\therefore A^{-1}$ exists.

First we have to find the cofactor matrix

$= \left[A_{ij}\right]_{3\times 3}$ where $A_{ij} - (-1)^{i+j} M_{ij}$

$$A_{11} = (-1)^{1+1} M_{11} = \begin{vmatrix} 1 & 5 \\ 4 & 7 \end{vmatrix} = 7 - 20 = -13$$

Now,

$$A_{12} = (-1)^{1+2} M_{12} = \begin{vmatrix} 1 & 5 \\ 2 & 7 \end{vmatrix} = -(7 - 10) = 3$$

$$A_{13} = (-1)^{1+3} M_{13} = \begin{vmatrix} 1 & 1 \\ 2 & 4 \end{vmatrix} = 4 - 2 = 2$$

$$A_{21} = (-1)^{2+1} M_{21} = -\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = -(14 - 12) = -2$$

$$A_{22} = (-1)^{2+2} M_{22} = \begin{vmatrix} 1 & 3 \\ 2 & 7 \end{vmatrix} = 7 - 6 = 1$$

$$A_{23} = (-1)^{2+3} M_{23} = -\begin{vmatrix} 1 & 2 \\ 2 & 4 \end{vmatrix} = -(4 - 4) = 0$$

$$A_{31} = (-1)^{3+1} M_{31} = \begin{vmatrix} 2 & 3 \\ 1 & 5 \end{vmatrix} = 10 - 3 = 7$$

$$A_{32} = (-1)^{3+2} M_{32} = -\begin{vmatrix} 1 & 3 \\ 1 & 5 \end{vmatrix} = -(5 - 3) = -2$$

$$A_{33} = (-1)^{3+3} M_{33} = \begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix} = 1 - 2 = -1$$

Co – matrix will be

$$\begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} = \begin{bmatrix} -13 & 3 & 2 \\ -2 & 1 & 0 \\ 7 & -2 & -1 \end{bmatrix}$$

$$adj\ A = \begin{bmatrix} -13 & -2 & 7 \\ 3 & 1 & -2 \\ 2 & 0 & -1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}(adj\ A)$$

$$\Rightarrow \frac{1}{-1}\begin{bmatrix} -13 & -2 & 7 \\ 3 & 1 & -2 \\ 2 & 0 & -1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 13 & 2 & -7 \\ -3 & -1 & 2 \\ -2 & 0 & 1 \end{bmatrix}$$

32. Find the inverse of $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$ **by adjoint method.**

Ans: Let the matrix A be,

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$$

$$|A| = 1(2 - 6) - 0(0 - 3) + 1(0 - 2)$$

$$\Rightarrow -4 - 2$$

$$\Rightarrow -6 \neq 0$$

$\therefore A^{-1}$ exists.

First we have to find the cofactor matrix

$= [A_{ij}]_{3\times 3}$, where $A_{ij} - (-1)^{i+j} M_{ij}$

Now $A_{11} = (-1)^{1+1} M_{11} = \begin{vmatrix} 2 & 3 \\ 2 & 1 \end{vmatrix} = 2 - 6 = -4$

$$A_{12} = (-1)^{1+2} M_{12} = -\begin{vmatrix} 0 & 3 \\ 1 & 1 \end{vmatrix} = 0 - 3 = -3$$

$$A_{13} = (-1)^{1+3} M_{13} = \begin{vmatrix} 0 & 2 \\ 1 & 2 \end{vmatrix} = 0 - 2 = -2$$

$$A_{21} = (-1)^{2+1} M_{21} = -\begin{vmatrix} 0 & 1 \\ 2 & 1 \end{vmatrix} = -0-2 = 2$$

$$A_{22} = (-1)^{2+2} M_{22} = -\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 1 - 1 = 0$$

$$A_{23} = (-1)^{2+3} M_{23} = -\begin{vmatrix} 1 & 0 \\ 1 & 2 \end{vmatrix} = -2 - 0 = -2$$

$$A_{31} = (-1)^{3+1} M_{31} = \begin{vmatrix} 1 & 0 \\ 2 & 3 \end{vmatrix} = 0 - 2 = -2$$

$$A_{32} = (-1)^{3+2} M_{32} = -\begin{vmatrix} 1 & 1 \\ 0 & 3 \end{vmatrix} = -3 - 0 = -3$$

$$A_{33} = (-1)^{3+3} M_{33} = \begin{vmatrix} 1 & 0 \\ 0 & 2 \end{vmatrix} = 2 - 0 = 2$$

$$\therefore adj\ (A) = \begin{bmatrix} -4 & -3 & -2 \\ -2 & 0 & -2 \\ -2 & -3 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \cdot adj(A)$$

$$A^{-1} = \frac{1}{6}\begin{bmatrix} -4 & -3 & -2 \\ -2 & 0 & -2 \\ -2 & -3 & 2 \end{bmatrix}$$

33. Find A^{-1} by adjoint method and by elementary transformations if

$$\mathbf{A} = \begin{bmatrix} \mathbf{1} & \mathbf{2} & \mathbf{3} \\ \mathbf{1} & \mathbf{1} & \mathbf{2} \\ \mathbf{1} & \mathbf{2} & \mathbf{4} \end{bmatrix}$$

Ans: The determinant of matrix A is,

$$|A| = \begin{vmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 1 & 2 & 4 \end{vmatrix}$$

$\Rightarrow 1(4 - 4) - 2(-4 - 2) + 3(-2 - 1)$

$\Rightarrow 0 + 12 - 9$

$\Rightarrow 3 \neq 0$

$\therefore A^{-1}$ exists.

A^{-1} by adjoint method :

We have to find the cofactor matrix

$= \left[A_{ij}\right]_{3\times3}$, where $A_{ij} = (-1)^{i+j} M_{ij}$

Now, $A_{11} = (-1)^{1+1} M_{11}$

$\Rightarrow \begin{vmatrix} 1 & 2 \\ 2 & 4 \end{vmatrix}$

$\Rightarrow 4 - 4 = 0$

$A_{12} = (-1)^{1+2} M_{12}$

$\Rightarrow \begin{vmatrix} -1 & 2 \\ 1 & 4 \end{vmatrix}$

$\Rightarrow (4 \quad 2) = 6$

$A_{13} = (-1)^{1+3} M_{13}$

$\Rightarrow \begin{vmatrix} -1 & 1 \\ 1 & 2 \end{vmatrix}$

$\Rightarrow -2 - 1 = -3$

$A_{21} = (-1)^{2+1} M_{21}$

$\Rightarrow \begin{vmatrix} 2 & 3 \\ 2 & 4 \end{vmatrix}$

$\Rightarrow (8 \quad 6) = 2$

$A_{22} = (-1)^{2+2} M_{22}$

$\Rightarrow \begin{vmatrix} 1 & 3 \\ 1 & 4 \end{vmatrix}$

$\Rightarrow 4 - 3 = 1$

$A_{23} = (-1)^{2+3} M_{23}$

$\Rightarrow \begin{vmatrix} 1 & 2 \\ 1 & 2 \end{vmatrix}$

$\Rightarrow (2 \quad 2) = 0$

$A_{31} = (-1)^{3+1} M_{31}$

$\Rightarrow \begin{vmatrix} 2 & 3 \\ 1 & 2 \end{vmatrix}$

$\Rightarrow 4 - 3 = 1 \; A_{32} = (-1)^{3+2} M_{32}$

$\Rightarrow \begin{vmatrix} 1 & 3 \\ 1 & 2 \end{vmatrix}$

$\Rightarrow (2 + 3) = 5$

$A_{33} = (-1)^{3-3} M_{33}$

$\Rightarrow \begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix}$

$\Rightarrow 1 + 2 = 3$

The cofactor matrix

$$\begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} = \begin{bmatrix} 0 & 6 & -3 \\ -2 & 1 & 0 \\ 1 & -5 & 3 \end{bmatrix}$$

$$adj\ A = \begin{bmatrix} 0 & -2 & 1 \\ 6 & 1 & 5 \\ -3 & 0 & 3 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}(adj\ A)$$

$$\Rightarrow \frac{1}{3}\begin{bmatrix} 0 & -2 & 1 \\ 6 & 1 & 5 \\ -3 & 0 & 3 \end{bmatrix}$$

A^{-1} by elementary transformations:

Consider $AA^{-1} = I$

By $R_2 + R_1$ and $R_3 - R_1$, we get,

$$\therefore \begin{bmatrix} 1 & 2 & 3 \\ 0 & 3 & 5 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

By $\left(\frac{1}{3}\right)R_2$, we get,

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & \frac{5}{3} \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{3} & \frac{1}{3} & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

By $R_1 - 2R_2$, we get,

$$\begin{bmatrix} 1 & 0 & -\frac{1}{3} \\ 0 & 1 & \frac{5}{3} \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} \frac{1}{3} & -\frac{2}{3} & 0 \\ \frac{1}{3} & \frac{1}{3} & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

By $R_1 + \frac{1}{3}R_3$ and $R_2 - \frac{5}{3}R_3$, we get,

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 0 & -\frac{2}{3} & \frac{1}{3} \\ 2 & \frac{1}{3} & -\frac{5}{3} \\ -1 & 0 & 1 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{3}\begin{bmatrix} 0 & -2 & 1 \\ 6 & 1 & -5 \\ -3 & 0 & 3 \end{bmatrix}$$

34. Find the inverse of $\mathbf{A} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$ **by elementary column transformations.**

Ans: The determinant of matrix A is,

$$|A| = \begin{vmatrix} 1 & 0 & 1 \\ 0 & 2 & 3 \\ 1 & 2 & 1 \end{vmatrix}$$

$$\Rightarrow 1(2-6) - 0 + 1(0-2)$$

$$\Rightarrow -4 - 2$$

$$\Rightarrow -6 \neq 0$$

$\therefore A^{-1}$ exists.

Consider $A^{-1}A = I$

$$\therefore A^{-1}\begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By $C_3 - C_1$, we get,

$$A^{-1}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 3 \\ 1 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By $C_2 \leftrightarrow C_3$, we get,

$$A^{-1}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 2 \\ 1 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

By $C_2 - C_3$, we get,

$$A^{-1}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 1 & -2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & -1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

By $C_3 - 2C_2$, we get,

$$A^{-1}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & -2 & 6 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 2 \\ 0 & -1 & 3 \\ 0 & 1 & -2 \end{bmatrix}$$

By $\left(\frac{1}{6}\right)C_3$, we get,

$$A^{-1}\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 1 & -2 & 1\end{bmatrix} = \begin{bmatrix}1 & -1 & \frac{1}{3}\\ 0 & -1 & \frac{1}{2}\\ 0 & 1 & -\frac{1}{3}\end{bmatrix}$$

By $C_1 - C_3$, and $C_2 + 2C_3$ we get,

$$A^{-1}\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix} = \begin{bmatrix}\frac{2}{3} & -\frac{1}{3} & \frac{1}{3}\\ -\frac{1}{2} & 0 & \frac{1}{2}\\ \frac{1}{3} & \frac{1}{3} & -\frac{1}{3}\end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{6}\begin{bmatrix}4 & -2 & 2\\ 3 & 0 & 3\\ 2 & 2 & -2\end{bmatrix}$$

35. Find the inverse of by $\begin{bmatrix}\mathbf{1} & \mathbf{2} & \mathbf{3}\\ \mathbf{1} & \mathbf{1} & \mathbf{5}\\ \mathbf{2} & \mathbf{4} & \mathbf{7}\end{bmatrix}$ **elementary row transformations.**

Ans: Let the matrix A be,

$$A = \begin{bmatrix}1 & 2 & 3\\ 1 & 1 & 5\\ 2 & 4 & 7\end{bmatrix}$$

$$\therefore |A| = \begin{vmatrix}1 & 2 & 3\\ 1 & 1 & 5\\ 2 & 4 & 7\end{vmatrix}$$

$$\Rightarrow 1(7 - 20) - 2(7 - 10) + 3(4 - 2)$$

$$\Rightarrow -13 + 6 + 6$$

$$\Rightarrow -1 \neq 0$$

$\therefore A^{-1}$ exists.

Consider $AA^{-1} = I$

$$\begin{bmatrix}1 & 2 & 3\\1 & 1 & 5\\2 & 4 & 7\end{bmatrix} A^{-1} = \begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$

By $R_2 - R_1$ and $R_3 - 2R_1$, we get,

$$\begin{bmatrix}1 & 2 & 3\\0 & -1 & 2\\0 & 0 & 1\end{bmatrix} A^{-1} = \begin{bmatrix}1 & 0 & 0\\-1 & 1 & 0\\-2 & 0 & 1\end{bmatrix}$$

By $(-1)R_2$, we get,

$$\begin{bmatrix}1 & 2 & 3\\0 & 1 & -2\\0 & 0 & 1\end{bmatrix} A^{-1} = \begin{bmatrix}1 & 0 & 0\\1 & -1 & 0\\-2 & 0 & 1\end{bmatrix}$$

By $R_1 - 2R_2$, we get,

$$\begin{bmatrix}1 & 0 & 7\\0 & 1 & -2\\0 & 0 & 1\end{bmatrix} A^{-1} = \begin{bmatrix}-1 & 2 & 0\\1 & -1 & 0\\-2 & 0 & 1\end{bmatrix}$$

By $R_1 - 7R_3$, and $R_2 + 2R_3$, we get,

$$\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix} A^{-1} = \begin{bmatrix}13 & 2 & -7\\-3 & -1 & 2\\-2 & 0 & 1\end{bmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix}13 & 2 & -7\\3 & 1 & 2\\-2 & 0 & 1\end{bmatrix}$$

36. Show with usual notations that for any matrix $\mathbf{A} = [a_{ij}]_{3\times3}$

(i) $\mathbf{a_{11}A_{21} + a_{12}A_{22} + a_{13}A_{23} = 0}$

Ans: Say the matrix A is,

$$A = [a_{ij}]_{3\times3} = \begin{bmatrix}a_{11} & a_{12} & a_{13}\\a_{21} & a_{22} & a_{23}\\a_{31} & a_{32} & a_{33}\end{bmatrix}$$

$$A_{21} = (-1)^{2+1} M_{21} = -\begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix}$$

$$\Rightarrow -(a_{12}a_{33} \quad a_{13}a_{32})$$

$$\Rightarrow -a_{12}a_{33} + a_{13}a_{32}$$

$$A_{22} = (-1)^{2+2} M_{22} = \begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix}$$

$$\Rightarrow a_{11}a_{33} - a_{13}a_{31}$$

$$A_{23} = (-1)^{2+3} M_{23} = -\begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix}$$

$$\Rightarrow -(a_{11}a_{32} \quad a_{12}a_{31})$$

$$\Rightarrow -a_{11}a_{32} + a_{12}a_{31}$$

$$\therefore a_{11}A_{21} + a_{12}A_{22} + a_{13}A_{23}$$

$$\Rightarrow a_{11}(-a_{12}a_{33} + a_{13}a_{32}) + a_{12}(a_{11}a_{33} - a_{13}a_{31}) + a_{13}(-a_{11}a_{32} + a_{12}a_{31})$$

$$\Rightarrow -a_{11}a_{12}a_{33} + a_{11}a_{13}a_{32} + a_{11}a_{12}a_{33} - a_{12}a_{13}a_{31} - a_{11}a_{13}a_{32} + a_{12}a_{13}a_{31}$$

$$\Rightarrow 0$$

(ii) $\mathbf{a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13} = |A|}$

Ans: The value of A11 is,

$$A_{11} = (-1)^{1+1} M_{11} = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}$$

$$A_{12} = (-1)^{1+2} M_{12} = \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}$$

$$A_{13} = (-1)^{1+3} M_{13} = -\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

$$\therefore a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13}$$

$$\Rightarrow a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{32} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

$$\Rightarrow \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = |A|$$

37. If $\mathbf{A}=\begin{bmatrix}1 & 0 & 1\\ 0 & 2 & 3\\ 1 & 2 & 1\end{bmatrix}$ **and** $\mathbf{B}=\begin{bmatrix}1 & 2 & 3\\ 1 & 1 & 5\\ 2 & 4 & 7\end{bmatrix}$**, then find a matrix** $\mathbf{X}$ **such that** $\mathbf{XA}-\mathbf{B}$.

Ans: Consider $XA = B$

$$X\begin{bmatrix}1 & 0 & 1\\ 0 & 2 & 3\\ 1 & 2 & 1\end{bmatrix}=\begin{bmatrix}1 & 2 & 3\\ 1 & 1 & 5\\ 2 & 4 & 7\end{bmatrix}$$

By $C_3 - C_1$, we get,

$$X\begin{bmatrix}1 & 0 & 0\\ 0 & 2 & 3\\ 1 & 2 & 0\end{bmatrix}=\begin{bmatrix}1 & 2 & 2\\ 1 & 1 & 4\\ 2 & 4 & 5\end{bmatrix}$$

By $\left(\frac{1}{2}\right)C_2$, we get,

$$X\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 3\\ 1 & 1 & 0\end{bmatrix}=\begin{bmatrix}1 & 1 & 2\\ 1 & \frac{1}{2} & 4\\ 2 & 2 & 5\end{bmatrix}$$

By $C_3 - 3C_2$, we get,

$$X\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 1 & 1 & -3\end{bmatrix}=\begin{bmatrix}1 & 1 & -1\\ 1 & \frac{1}{2} & \frac{5}{2}\\ 2 & 2 & -1\end{bmatrix}$$

By $\left(\frac{-1}{3}\right)C_3$, we get,

$$X\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 1 & 1 & 1\end{bmatrix}=\begin{bmatrix}1 & 1 & \frac{1}{3}\\ 1 & \frac{1}{2} & -\frac{5}{6}\\ 2 & 2 & \frac{1}{3}\end{bmatrix}$$

By $C_1 - C_3$ and $C_2 - C_3$, we get,

$$X\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 1 & 1 & 1\end{bmatrix} = \begin{bmatrix}\frac{2}{3} & \frac{2}{3} & \frac{1}{3}\\ \frac{11}{6} & \frac{4}{3} & -\frac{5}{6}\\ \frac{5}{3} & \frac{5}{3} & \frac{1}{3}\end{bmatrix}$$

$$\therefore X = \frac{1}{6}\begin{bmatrix}4 & 4 & 2\\ 11 & 8 & 5\\ 10 & 10 & 2\end{bmatrix}$$

Practice set 2.3

Solve the following equations by the inversion method.

1. $\mathbf{x + 2y = 2,\ 2x + 3y = 3}$

Ans: According to the given question, the equations can be written in the matrix form as :

$$\begin{bmatrix}1 & 2\\ 2 & 3\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix} = \begin{bmatrix}2\\ 3\end{bmatrix}$$

This is of the form $AX = B$, where

$$A = \begin{bmatrix}1 & 2\\ 2 & 3\end{bmatrix},\ X = \begin{bmatrix}x\\ y\end{bmatrix} \text{ and } B = \begin{bmatrix}2\\ 3\end{bmatrix}$$

Now, we have to find A^{-1}

$$|A| = \begin{vmatrix}1 & 2\\ 2 & 1\end{vmatrix}$$

$$\Rightarrow 3 - 4$$

$$\Rightarrow -1 \neq 0$$

A^{-1} exists.

Consider $AA^{-1} = I$

$$\begin{bmatrix}1 & 2\\ 2 & 3\end{bmatrix}A^{-1} = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

By $R_2 - 2R_1$, we get

$$\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

By $(-1)R_2$, we get

$$\begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 \\ -2 & -1 \end{bmatrix}$$

By $R_1 - 2R_2$, we get

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} -3 & 2 \\ 2 & 1 \end{bmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix} -3 & 2 \\ 2 & 1 \end{bmatrix}$$

Now, multiplying $AX = B$ by A^{-1}, we get,

$$A^{-1}(AX) = A^{-1}B$$

$$\therefore (A^{-1}A)X = A^{-1}B$$

$$\therefore IX = A^{-1}B$$

$$\therefore X = \begin{bmatrix} -3 & 2 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -6 + 6 \\ 4 \quad 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

So, So, So, by equality of matrices

$x = 0, y = 1$ is the required value.

2. $\mathbf{x + y = 4,\ 2x \quad y = 5}$

Ans: The given equations are:

$x + y = 4,\ 2x \quad y = 5$

The given equations can be written in the matrix form as:

$$\begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

Now, this is of the form $AX = B$

$$\Rightarrow X = A^{-1}B$$

$$A = \begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix}$$

$|A| = -1 - 2 = -3 \neq 0$

$Adj\ A = \begin{bmatrix} -1 & -1 \\ 2 & 1 \end{bmatrix}$

$\Rightarrow \frac{1}{-3}\begin{bmatrix} -1 & -1 \\ 2 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}$

$X = A^{-1}B = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} 4 \\ 5 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 4 & 5 \\ 8 & 5 \end{bmatrix} = \begin{bmatrix} 9 \\ 3 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 3 \\ 1 \end{bmatrix}$

So, by equality of matrices.

$x = 3, y = 1$

3. $\mathbf{2x + 6y = 8, x + 3y = 5}$

Ans: According to the given question, the equations can be written in the matrix form as :

$\begin{bmatrix} 2 & 6 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 5 \end{bmatrix}$

This is of the form $AX = B$, where

$A = \begin{bmatrix} 2 & 6 \\ 1 & 3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}$ and $B = \begin{bmatrix} 8 \\ 5 \end{bmatrix}$

Suppose us find A^{1}.

$|A| = \begin{vmatrix} 2 & 6 \\ 1 & 3 \end{vmatrix}$

$\Rightarrow 6 - 6 = 0$

$\therefore A^{1}$ does not exist.

Therefore, x and y do not exist.

Solve the following equations by reduction method.

4. $2x + y = 5,\ 3x + 5y = -3$

Ans: According to the given question, the equations can be written in the matrix form as :

$$\begin{bmatrix} 2 & 1 \\ 3 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$$

By $2R_2$, we get,

$$\begin{bmatrix} 2 & 1 \\ 6 & 10 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \end{bmatrix}$$

By $R_2 - 3R_1$, we get,

$$\begin{bmatrix} 2 & 1 \\ 0 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 21 \end{bmatrix}$$

$$\begin{bmatrix} 2x+y \\ 0+7y \end{bmatrix} = \begin{bmatrix} 5 \\ 21 \end{bmatrix}$$

So, by equality of matrices.

$2x + y = 5 \ldots(1)$

$7y = -21 \ldots(2)$

From (2), $y = 3$

On substituting $y = 3$ in (1), we get,

$2x - 3 = 5$

$\therefore 2x = 8$

$\therefore x = 4$

Therefore, $x = 4, y = -3$ is the required value.

5. $\mathbf{x + 3y = 2,\ 3x + 5y = 4.}$

Ans: According to the given question, the equations can be written in the matrix form as :

$$\begin{bmatrix} 1 & 3 \\ 3 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

By $R_2 - 3R_1$, we get

$$\begin{bmatrix} 1 & 3 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$$

$$\therefore \begin{bmatrix} x+3 \\ 0 \quad 4y \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

So, by equality of matrices

$x + 3y = 2 \ldots(1)$

$-4y = -2$

From (2), $y = \frac{1}{2}$

On substituting $y = \frac{1}{2}$ in (1), we get,

$x - \frac{3}{2} = 2$

$\therefore x = 2 - \frac{3}{2} = \frac{1}{2}$

Therefore, $x = \frac{1}{2}, y = \frac{1}{2}$ is the required value.

6. $3x \quad y = 1, 4x + y = 6$

Ans: According to the given question, the equations can be written in the matrix form as :

$$\begin{bmatrix} 3 & -1 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 6 \end{bmatrix}$$

By $4R_1$ and $3R_2$, we get,

$$\begin{bmatrix} 12 & -4 \\ 12 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 18 \end{bmatrix}$$

By $R_2 - R_1$, we get,

$$\begin{bmatrix} 12 & -4 \\ 0 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 14 \end{bmatrix}$$

$$\begin{bmatrix} 12x - 4y \\ 0 + 7y \end{bmatrix} = \begin{bmatrix} 4 \\ 14 \end{bmatrix}$$

So, by equality of matrices

$12x \quad 4y = 4 \ldots (1)$

$7y = 14 \ldots (2)$

From (2), $y = 2$

On, substituting $y = 2$ in (1), we get,

$12x - 8 = 4$

$\therefore 12x = 12$

$\therefore x = 1$

Therefore, $x = 1, y = 2$ is the required value.

7. $\mathbf{5x + 2y = 4,\ 7x + 3y = 5}$

Ans: The given equations are,

$5x + 2y = 4$ .....(1)

$7x + 3y = 5$ .....(2)

Multiplying Eq. (1) with 7 and Eq. (2) with 5

$35x + 14y = 28$

$35x + 15y = 25$

$- \quad -$

$-1y = 3$

$y = -3$

By putting $y = -3$ into Eq. (1)

$5x + 2y = 4$

$\Rightarrow 5x + 2(-3) = 4$

$\Rightarrow 5x - 6 = 4$

$\Rightarrow 5x = 4 + 6$

$\Rightarrow 5x = 10$

$\Rightarrow x = 105$

$\Rightarrow x = 2$

Therefore, $x = 2, y = -3$ is the required value.

8. The cost of 4 pencils, 3 pens and 2 erasers is $Rs\ 60$. The cost of 2 pencils, 4 pens and 6 erasers is $Rs\ 90$, whereas the cost of 6 pencils, 2 pens and 3 erasers is $Rs\ 70$. Find the cost of each item by using matrices.

Ans: Suppose the cost of 1 pencil, 1 pen and 1 eraser be $Rs\ x$, $Rs\ y$ and $Rs\ z$ respectively.

Then, according to the given question,

$4x + 3y + 2z = 60$
$2x + 4y + 6z = 90,$

$i.e., x + 2y + 3z = 45$
$6x + 2y + 3z = 70$

These equations can be written in the matrix form as :

$$\begin{bmatrix} 4 & 3 & 2 \\ 1 & 2 & 3 \\ 6 & 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 60 \\ 45 \\ 70 \end{bmatrix}$$

By $R_1 \leftrightarrow R_2$, we get,

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 3 & 2 \\ 6 & 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 45 \\ 60 \\ 70 \end{bmatrix}$$

By $R_2 - 4R_1$ and $R_3 - 6R_1$, we get,

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -5 & -10 \\ 0 & -10 & -15 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 45 \\ -120 \\ -200 \end{bmatrix}$$

By $R_3 - 2R_2$, we get,

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -5 & -10 \\ 0 & 0 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 45 \\ -120 \\ 40 \end{bmatrix}$$

$$\begin{bmatrix} x + 2y + 3z \\ 0 \quad 5y \quad 10z \\ 0 + 0 + 5z \end{bmatrix} = \begin{bmatrix} 45 \\ 120 \\ 40 \end{bmatrix}$$

So, by equality of matrices

$x + 2y + 3z = 45 \ldots\ldots(1)$

$-5y - 10z = -120 \ldots\ldots(2)$

$5z - 40$

From (3), $z - 8$

On substituting $z - 8$ in (2), we get,

$5y \quad 80 = -120$

$\therefore \ 5y = -40$

$\therefore y = 8$

On substituting $y = 8, z = 8$ in (1), we get,

$x + 16 + 24 = 45$

$\therefore x + 40 = 45$

$\therefore x = 5$

$\therefore x = 5, y = 8, z = 8$

Therefore, the cost is $Rs\ 5$ for a pencil, $Rs\ 8$ for a pen and $Rs\ 8$ for an eraser.

9. If three numbers are added, their sum is 2. If 2 times the second number is subtracted from the sum of first and third numbers, we get 8 and if three times the first number is added to the sum of second and third numbers, we get 4. Find the numbers using matrices.

Ans: Suppose, the three numbers be x, y and z. According to the given conditions,

$x + y + z = 2$

$x + z \quad 2y = 8,$

i.e., $x \quad 2y + 2 = 8$

And $y + z + 3x = 4,$

i.e., $3x + y + z = 4$

Therefore, the system of linear equations is

$x + y + z = 2$

$x \quad 2y + z = 8$

$3x + y + z = 4$

These equations can be written in the matrix form as :

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 3 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 8 \\ 4 \end{bmatrix}$$

By $R_2 - R_1$ and $R_3 - 3R_1$, we get,

$$\begin{bmatrix}1 & 1 & 1\\ 0 & 3 & 0\\ 0 & -2 & -2\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix} = \begin{bmatrix}2\\ 6\\ -2\end{bmatrix}$$

$$\begin{bmatrix}x + y + z\\ 0 \quad 3y + 0\\ 0 - 2y - 2z\end{bmatrix} = \begin{bmatrix}2\\ 6\\ -2\end{bmatrix}$$

So, by equality of matrices

$x + y + z = 2$ .....(1)

$-3y = 6$ .....(2)

$-2y - 2z = -2$ .....(3)

From (2), $y = -2$

On substituting $y = -2$ in (3), we get,

$-2(-2) \quad 2z = -2$

$\therefore -2z - -6$

$\therefore z - 3$

On substituting $y = -2, z = 3$ in (1), we get,

$x - 2 + 3 = 2$

$\therefore x - 1$

Therefore, the required numbers are $1, -2$ and 3.

10. The total cost of 3 T.V. sets and 2 V.C.R.s is $\text{Rs } 35000$. The shop-keeper wants profit of $\text{Rs } 1000$ per television and $\text{Rs } 500$ per V.C.R. He can 2 T. V. sets and 1 V.C.R. and get the total revenue as $\text{Rs } 21,500$. Find the cost price and the selling price of a T.V. sets and a V.C.R.

Ans: Suppose, the cost of each T.V. set be $Rs\ x$ and each V.C.R. be $Rs\ y$. Then the total cost of 3 T.V. sets and 2 V.C.R.'s is $Rs\ (3x + 2y)$ which is given to be $Rs\ 35,000$.

$\therefore 3x + 2y = 35000$

The shopkeeper wants profit of $Rs\ 1000$ per T.V. set and of $Rs\ 500$ per V.C.R.

So, the selling price of each T.V. set is $Rs\ (x + 1000)$ and of each V.C.R. is $Rs\ (y + 500)$.

So, selling price of 2 T.V. set and 1 V.C.R. is $Rs[2(x + 1000) + (y + 500)]$ which is given to be $Rs\ 21,500.$

$\therefore 2(x + 1000) + (y + 500) = 21500$

$\therefore 2x + 2000 + y + 500 = 21500$

$\therefore 2x + y = 19000$

Therefore, the system of linear equations is

$3x + 2y = 35000$

$2x + y = 19000$

These equations can be written in the matrix form as :

$$\begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 35000 \\ 19000 \end{bmatrix}$$

By $R_1 - R_2$, we get,

$$\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 19000 \\ 35000 \end{bmatrix}$$

By $R_2 - 2R_1$, we get,

$$\begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 19000 \\ 3000 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 2x+y \\ x+0 \end{bmatrix} = \begin{bmatrix} 19000 \\ 3000 \end{bmatrix}$$

So, by equality of matrices

$2x + y = 19000 \ldots\ldots\ldots(1)$

$-x = -3000 \ldots\ldots\ldots(2)$

From (2), $x - 3000$

On substituting $x - 3000$ in (1), we get,

$2(3000) + y = 19000$

$\therefore y = 13000$

So, the cost price of one T.V. set is $Rs\ 3000$ and of one V.C.R. is $Rs\ 13000$. So, the selling price of one T.V. set is $Rs\ 4000$ and of one V.C.R. is $Rs\ 13500$.

Miscellaneous Exercise 2(B)

Choose the correct answer from the given alternatives in each of the following questions

1. If $\mathbf{A} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $\mathbf{adj\ A} = \begin{bmatrix} 4 & a \\ -3 & b \end{bmatrix}$ **then the values of** a **and** b **are,**

a. $a = -2, b = 1$

b. $a = 2, b = 4$

c. $a = 2, b = -1$

d. $a = 1, b = -2$

Ans: Given A is 2×2 matrix

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$adj\ A = \begin{bmatrix} 4 & a \\ 3 & b \end{bmatrix}$$

let us suppose a matrix X such that

$$X = \begin{bmatrix} u & v \\ w & x \end{bmatrix}$$

So now adjoint of this 2×2 matrix X will be said as $adj\ X$ which is

$$adj\ X = \begin{bmatrix} x & -v \\ w & u \end{bmatrix}$$

Now we compare the two matrix X and $adj\ X$.

We see that the right diagonal elements are interchanged and in the left diagonal only the element are multiplied by -1

So comparing the Matrix A and $adj\ A$, in this matrix the property will satisfy.

according to that,

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$adj\ A = \begin{bmatrix} 4 & a \\ 3 & b \end{bmatrix}$$

So the left diagonals element interchange then the value of $b-1$ and since the row 1 column 2 element remains but multiplied with -1 therefore $a--2$.

Therefore the value of $a--2$ and $b-1$.

The correct answer is (a).

2. The inverse of $\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$ is

a. $\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}$

b. $\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$

c. $\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$

d. None of these

Ans: The matrix B is,

$$B=\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$$

$$|B| = 0-1$$

$$\Rightarrow -1 \neq 0$$

B is a singular matrix; therefore, it is invertible.

Let C_{ij} be a cofactor of b_{ij} in B.

Now,

$$C_{11}=0$$

$$C_{12}=-1$$

$$C_{21}=-1$$

$C_{22} = 0$

$$adj(B) = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}^T = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$$

$$B^{-1} = \frac{1}{|B|} adjB$$

$$\Rightarrow \frac{1}{-1}\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

The correct option is **(b).**

3. If $\mathbf{A} = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ **and** $\mathbf{A(adj\ A) = k\ I}$**, then the value of** $\mathbf{k}$ **is**

a. $\mathbf{1}$

b. $\mathbf{-1}$

c. $\mathbf{0}$

d. $\mathbf{-3}$

Ans: The given matrix A is,

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$$

$$adjA = \begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}$$

$$A(adjA) = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ -2 & 1 \end{bmatrix}$$

$$A(adjA) = \begin{bmatrix} -3 & 0 \\ 0 & 3 \end{bmatrix}$$

$$A(adjA) = 3\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$A\ (\ adj\ A) = -3\ I$$

$$A(adj\ A) = kI$$

$$\Rightarrow k = -3$$

The correct option is **(d)**.

4. If $\mathbf{A} = \begin{bmatrix}2 & -4\\ 3 & 1\end{bmatrix}$, then the adjoint of matrix $\mathbf{A}$ is

a. $\begin{bmatrix}-1 & 3\\ 4 & 1\end{bmatrix}$

b. $\begin{bmatrix}1 & 4\\ 3 & 2\end{bmatrix}$

c. $\begin{bmatrix}1 & 3\\ 4 & 2\end{bmatrix}$

d. $\begin{bmatrix}-1 & -3\\ 4 & 2\end{bmatrix}$

Ans: The matrix A be,

$$A = \begin{bmatrix}2 & -4\\ 3 & 1\end{bmatrix}$$

Let C be the cofactor matrix of matrix A.

$$C_{11} = (-1)^{1+1} M_{11} = 1$$

$$C_{12} = (-1)^{1+2} M_{12} = 4$$

$$C_{21} = (-1)^{2+1} M_{21} = -3$$

$$C_{22} = (-1)^{2+2} M_{22} = 2$$

$$C_{ij} = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$$

$$Adj(C_{ij}) = \begin{bmatrix} 1 & 4 \\ -3 & 2 \end{bmatrix}^T$$

$$\Rightarrow \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$$

The correct option is **(b)**.

5. If $\mathbf{A} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ **and** $\mathbf{A(adj\ A) = k\ I}$**, then the value of** $\mathbf{k}$ **is**

a. 2

b. −2

c. 10

d. −10

Ans: The give matrix is,

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

Adjoint of $A = \begin{bmatrix} 4 & -2 \\ 3 & 1 \end{bmatrix}$

$$A(adjA) = KI$$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix} = K \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} K & 0 \\ 0 & K \end{bmatrix}$$

$$\therefore K = -2$$

The correct otion is **(b)**.

6. If $\mathbf{A} = \begin{bmatrix} \lambda & 1 \\ -1 & -\lambda \end{bmatrix}$**, then** $\mathbf{A^{-1}}$ **does not exist if** $\lambda =$ **……..**

a. 0

b. ± 1

c. 2

d. 3

Ans: Given,

$$A = \begin{bmatrix} \lambda & 1 \\ -1 & \lambda \end{bmatrix}$$

Now $A^{-1} = \frac{1}{|A|} adj(A)$

If $|A| = 0$ then A^{-1} does not exist.

$$|A| = \lambda \times -\lambda - (1 \times -1) = 0$$

$$|A| = -\lambda^2 + 1 = 0$$

$$\lambda^2 = 1$$

$\lambda - \pm\lambda$ (A^{-1} does not exist)

Therefore, option **(b)** is correct.

7. If $\mathbf{A} = \begin{bmatrix} \cos\alpha & \sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$ **then** $\mathbf{A^{-1}} =$ ….

a. $\begin{bmatrix} \frac{1}{\cos\alpha} & -\frac{1}{\sin\alpha} \\ \frac{1}{\sin\alpha} & \cos\alpha \end{bmatrix}$

b. $\begin{bmatrix} \cos\alpha & \sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

c. $\begin{bmatrix} -\cos\alpha & \sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

d. $\begin{bmatrix} -\cos\alpha & \sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

Ans: Correct option is $(b)\begin{bmatrix} \frac{\cos\alpha}{\cos(2\alpha)} & \frac{-\sin\alpha}{\cos(2\alpha)} \\ \frac{-\sin\alpha}{\cos(2\alpha)} & \frac{\cos\alpha}{\cos(2\alpha)} \end{bmatrix}$

$$A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

We know that $A^{-1} = \frac{1}{|A|} adj(A)$

$$|A| = \cos^2\alpha - \sin^2\alpha$$

$$\Rightarrow \cos(2\alpha)$$

$$adj(A) = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$$

$$A^{-1} = \frac{1}{\cos(2\alpha)} \begin{bmatrix} \cos\alpha & -\sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \frac{\cos\alpha}{\cos(2\alpha)} & \frac{-\sin\alpha}{\cos(2\alpha)} \\ \frac{-\sin\alpha}{\cos(2\alpha)} & \frac{\cos\alpha}{\cos(2\alpha)} \end{bmatrix}$$

8. If $F(\alpha) = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$ **where** $\alpha \in R$ **then** $[F(\alpha)]^{-1}$ **is =**

a. $F(-\alpha)$

b. $F(\alpha^{-1})$

c. $F(2\alpha)$

d. None of these

Ans: The multiplication of given matrix is,

$$F(\alpha)F(-\alpha)=\begin{bmatrix}\cos\alpha & -\sin\alpha & 0\\ \sin\alpha & \cos\alpha & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\cos\alpha & \sin\alpha & 0\\ -\sin\alpha & \cos\alpha & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$\therefore F(\alpha)F(-\alpha)=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}=I$$

$$\therefore F(-\alpha)-\left[F(\alpha)\right]^{-1}$$

Correct option is **(a)**.

9. The inverse of $\mathbf{A}=\begin{bmatrix}\mathbf{0} & \mathbf{1} & \mathbf{0}\\ \mathbf{1} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{1}\end{bmatrix}$ **is**

a. **I**

b. **A**

c. **A'**

d. **-I**

Ans: Let the matrix A be,

Let $A=\begin{bmatrix}0 & 1 & 0\\ 1 & 0 & 0\\ 0 & 0 & 1\end{bmatrix}$

$$|\Lambda|=-1\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=-1$$

$$Adj.A=\begin{bmatrix}0 & -1 & 0\\ -1 & 0 & 0\\ 0 & 0 & -1\end{bmatrix}$$

We have,

$$A^{-1} = \frac{1}{|A|} \cdot AdjA$$

$$\Rightarrow A^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = A$$

Correct option is **(b)**.

10. The inverse of a symmetric matrix is

a. Symmetric

b. Non-symmetric

c. Null matrix

d. Diagonal matrix

Ans: Let A be a symmetric matrix.

$A^T = A$

$\Rightarrow (A^T)^{-1} - A^{-1}$

$\Rightarrow (A^{-1})^T - A^{-1}$

$\because (A^T)^{-1} = (A^{-1})^T$

Therefore, A^{-1} is symmetric.

Correct option is **(a)**.

11. For a 2×2 matrix A, if $A(adjA) = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$ then determinant A equals

a. 20

b. 10

c. 30

d. **40**

Ans: The given part is,

$$A[AdjA] = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$$

$$A(AdjA) = 10I \ldots\ldots(1)$$

$$A^{-1} = \frac{AdjA}{|A|}$$

$$\frac{I}{A} = \frac{AdjA}{|A|}$$

$$|A|I = A.AdjA \ldots\ldots(2)$$

Comparing (1) and (2)

$$\Rightarrow |A = 10$$

Correct option is **(b)**.

Solve the following equations by the methods of inversion.

12. $2x - y = -2,\ 3x + 4y = 5$

Ans: The given equations can be written in the matrix form as :

$$\begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$$

This is of the form $AX = B$, where

$$A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix} \text{ and } B = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$$

Let us find A^{-1}.

$$|A| = \begin{vmatrix} 2 & -1 \\ 3 & 4 \end{vmatrix}$$

$$\Rightarrow 8 + 3$$

$$\Rightarrow 11 \neq 0$$

$\therefore A^{-1}$ exists.

Consider $AA^{-1} = I$

$$\begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

By $R_1 \leftrightarrow R_2$, we get,

$$\begin{bmatrix} 3 & 4 \\ 2 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

By $R_1 - R_2$, we get,

$$\begin{bmatrix} 1 & 5 \\ 2 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} -1 & 1 \\ 1 & 0 \end{bmatrix}$$

By $R_2 - 2R_1$, we get,

$$\begin{bmatrix} 1 & 5 \\ 0 & -11 \end{bmatrix} A^{-1} = \begin{bmatrix} -1 & 1 \\ 3 & -2 \end{bmatrix}$$

By $\left(\frac{1}{11}\right) R_2$, we get,

$$\begin{bmatrix} 1 & 5 \\ 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} -1 & 1 \\ -\frac{3}{11} & \frac{2}{11} \end{bmatrix}$$

By $R_1 - 5R_2$, we get,

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A^{-1} - \begin{bmatrix} \frac{4}{11} & \frac{1}{11} \\ -\frac{3}{11} & \frac{2}{11} \end{bmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix} \frac{4}{11} & \frac{1}{11} \\ -\frac{3}{11} & \frac{2}{11} \end{bmatrix}$$

Now, multiplying $AX = B$ by A^{-1}, we get,

$$A^{-1}(AX) = A^{-1}B$$

$$(A^{-1}A)X = A^{-1}B$$

$$IX = A^{-1}B$$

$$\therefore X = \begin{bmatrix} \frac{4}{11} & \frac{1}{11} \\ -\frac{3}{11} & \frac{2}{11} \end{bmatrix} \begin{bmatrix} -2 \\ 3 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -\frac{8}{11} + \frac{3}{11} \\ \frac{6}{11} + \frac{6}{11} \end{bmatrix} = \begin{bmatrix} -\frac{5}{11} \\ \frac{12}{11} \end{bmatrix}$$

By equality of matrices,

$x = -\frac{5}{11}, y = \frac{12}{11}$ is the required value.

13. $\mathbf{x + y + z = 1,\ 2x + 3y + 2z = 2}$ **and** $\mathbf{ax + ay + 2az = 4,\ a \neq 0.}$

Ans: The given equations can be written in the matrix form as :

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ a & a & 2a \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$$

This is of the form $AX = B$, where

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ a & a & 2a \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$$

Let us find A^{-1}.

$$|A| = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ a & a & 2a \end{vmatrix}$$

$$\Rightarrow 1(6a - 2a) - 1(4a - 2a) + 1(2a - 3a)$$

$$\Rightarrow 4a - 2a - a$$

$$\Rightarrow a \neq 0$$

$\therefore A^{-1}$ exists.

Consider $AA^{-1} = I$

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ a & a & 2a \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By $R_2 - 2R_1$ and $R_3 - aR_1$, we get,

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & a \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -a & 0 & 1 \end{bmatrix}$$

By $R_1 - R_2$, we get,

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & a \end{bmatrix} A^{-1} = \begin{bmatrix} 3 & -1 & 0 \\ -2 & 1 & 0 \\ -a & 0 & 1 \end{bmatrix}$$

By $\left(\frac{1}{3}\right) R_3$, we get

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 3 & -1 & 0 \\ -2 & 1 & 0 \\ -1 & 0 & \frac{1}{a} \end{bmatrix}$$

By $R_1 - R_3$, we get,

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 4 & -1 & -\frac{1}{a} \\ -2 & 1 & 0 \\ -1 & 0 & \frac{1}{a} \end{bmatrix}$$

$$A^{-1}=\begin{bmatrix}4 & -1 & -\frac{1}{a}\\ -2 & 1 & 0\\ -1 & 0 & \frac{1}{a}\end{bmatrix}$$

Now, multiplying $AX = B$ by A^{-1}, we get,

$$A^{-1}(AX) = A^{-1}B$$

$$(A^{-1}A)X = A^{-1}B$$

$$IX = A^{-1}B$$

$$X = \begin{bmatrix}4 & -1 & -\frac{1}{a}\\ -2 & 1 & 0\\ 1 & 0 & \frac{1}{a}\end{bmatrix}\begin{bmatrix}1\\2\\4\end{bmatrix}$$

$$\therefore \begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}4-2-\frac{4}{a}\\ -2+2+0\\ -1+0+\frac{4}{a}\end{bmatrix} = \begin{bmatrix}2-\frac{4}{a}\\ 0\\ \frac{4}{a}-1\end{bmatrix}$$

By equality of matrices,

$\therefore x = 2-\frac{4}{a}, y = 0, z = \frac{4}{a} - 1$ is the required value.

14. $\mathbf{5x - y + 4z = 5,\ 2x + 3y + 5z = 2}$ **and** $\mathbf{5x - 2y + 6z = -1}$

Ans: The given equations can be written in the matrix form as :

$$\begin{bmatrix}5 & -1 & 4\\ 2 & 3 & 5\\ 5 & -2 & 6\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}5\\2\\-1\end{bmatrix}$$

This is of the form $AX = B$, where

$$A = \begin{bmatrix} 5 & -1 & 4 \\ 2 & 3 & 5 \\ 5 & -2 & 6 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 5 \\ 2 \\ -1 \end{bmatrix}$$

Let us find A^{-1}.

$$|A| = \begin{vmatrix} 5 & -1 & 4 \\ 2 & 3 & 5 \\ 5 & -2 & 6 \end{vmatrix}$$

$$\Rightarrow 5(18 + 10) + 1(12 - 25) + 4(-4 - 15)$$

$$\Rightarrow 140 - 13 - 76$$

$$\Rightarrow 51 \neq 0$$

$\therefore A^{-1}$ exists.

Now, we have to find the cofactor matrix

$= A_{ij3\times3}$, where $A_{ij} = (-1)^{i+j} M_{ij}$

$$A_{11} = (-1)^{1+1} M_{11} = \begin{vmatrix} 3 & 5 \\ -2 & 6 \end{vmatrix} = 18+10 = 28$$

$$A_{12} = (-1)^{1+2} M_{12} = -\begin{vmatrix} 2 & 5 \\ 5 & 6 \end{vmatrix} = -(12 - 25) = 13$$

$$A_{13} = (-1)^{1+3} M_{13} = \begin{vmatrix} 2 & 3 \\ 5 & -2 \end{vmatrix} = -4 - 15 = -19$$

$$A_{21} = (-1)^{2+1} M_{21} = -\begin{vmatrix} -1 & 4 \\ -2 & 6 \end{vmatrix} = -(-6 + 8) = -2$$

$$A_{22} = (-1)^{2+2} M_{22} = \begin{vmatrix} 5 & 4 \\ 5 & 6 \end{vmatrix} = 30 - 20 = 10$$

$$A_{23} = (-1)^{2+3} M_{23} = -\begin{vmatrix} 5 & -1 \\ 5 & -2 \end{vmatrix} = -(-10 + 5) = 5$$

$$A_{31} = (-1)^{3+1} M_{31} = \begin{vmatrix} -1 & 4 \\ 3 & 5 \end{vmatrix} = -5 - 12 = -17$$

$$A_{32} = (-1)^{3+2} M_{32} = -\begin{vmatrix} 5 & 4 \\ 2 & 5 \end{vmatrix} = -(25 - 8) = -17$$

$$A_{33} = (-1)^{3+3} M_{33} = \begin{vmatrix} 5 & -1 \\ 2 & 3 \end{vmatrix} = 15 + 2 = 17$$

$\therefore$ Cofactor matrix

$$\begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} = \begin{bmatrix} 28 & 13 & -19 \\ -2 & 10 & 5 \\ -17 & -17 & 17 \end{bmatrix}$$

$$Adj\ A = \begin{bmatrix} 28 & -2 & -17 \\ 13 & 10 & -17 \\ -19 & 5 & 17 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} (adj\ A)$$

$$\Rightarrow \frac{1}{51} \begin{bmatrix} 28 & -2 & -17 \\ 13 & 10 & -17 \\ -19 & 5 & 17 \end{bmatrix}$$

Now, pre multiply $AX = B$ by A^{-1}, we get,

$$A^{-1}(AX) = A^{-1}B$$

$$\therefore (A^{-1}A)X = A^{-1}B$$

$$\therefore IX = A^{-1}B$$

$$\therefore X = \frac{1}{51} \begin{bmatrix} 28 & -2 & -17 \\ 13 & 10 & -17 \\ -19 & 5 & 17 \end{bmatrix} \begin{bmatrix} 5 \\ 2 \\ -1 \end{bmatrix}$$

$$\Rightarrow \frac{1}{51} \begin{bmatrix} 140 - 4 + 17 \\ 65 + 20 + 17 \\ -95 + 10 - 17 \end{bmatrix}$$

$$\Rightarrow \frac{1}{51} \begin{bmatrix} 153 \\ 102 \\ -102 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ -2 \end{bmatrix}$$

By equality of matrices,

$x = 3, y = 2, z = -2$ is the required values.

15. $\mathbf{2x + 3y = -5, 3x + y = 3}$

Ans: We know if A and X and B are 3 matrix such that $AX = B$ then $X = A^{-1}B$

$2x - 3y = -5$

$3x + y = 3$

$$\therefore A = \begin{bmatrix} 2 & 3 \\ 3 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} -5 \\ 3 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x \\ y \end{bmatrix} = A^{-1} \begin{bmatrix} -5 \\ 3 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 3 \\ 3 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} Adjoint\ (A)$$

For 2×2 matrix, we exchange the principal diagonal elements and multiply the other two by -1

$$\therefore A^{-1} = \frac{1}{2 \times 1 - 3 \times 3} \begin{bmatrix} 1 & -3 \\ -3 & 2 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{-7} \begin{bmatrix} 1 & -3 \\ -3 & 2 \end{bmatrix}$$

$$x = A^{-1}B = \frac{1}{-7} \begin{bmatrix} 1 & -3 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} -5 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-7} \begin{bmatrix} -5 - 9 \\ 15 + 6 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \frac{-14}{-7} \\ \frac{21}{-7} \end{bmatrix} - \begin{bmatrix} 2 \\ -3 \end{bmatrix}$$

$\therefore x = 2,\ y = -3$

16. $\mathbf{x + y + z = -1,\ y + z = 2}$ **and** $\mathbf{x + y - z = 3}$

Ans: The given equations can be written in the matrix form as :

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$$

This is of the form $AX = B$, where

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & -1 \end{bmatrix},\ X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$$

Let us find A^{-1}.

$$|A| = \begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & -1 \end{vmatrix}$$

$$\Rightarrow 1(-1 - 1) - 1(0 - 1) + 1(0 - 1)$$

$$\Rightarrow -2 + 1 - 1$$

$$\Rightarrow -2 \neq 0$$

$\therefore A^{-1}$ exists.

Consider $AA^{-1} = I$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & -1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By $R_3 - R_1$, we get,

$$\begin{bmatrix}1 & 1 & 1\\0 & 1 & 1\\0 & 0 & -2\end{bmatrix} A^{-1} = \begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\-1 & 0 & 1\end{bmatrix}$$

By $R_1 - R_2$, we get,

$$\begin{bmatrix}1 & 0 & 0\\0 & 1 & 1\\0 & 0 & -2\end{bmatrix} A^{-1} = \begin{bmatrix}1 & -1 & 0\\0 & 1 & 0\\-1 & 0 & 1\end{bmatrix}$$

By $\left(\frac{1}{2}\right) R_3$, we get,

$$\begin{bmatrix}1 & 0 & 0\\0 & 1 & 1\\0 & 0 & 1\end{bmatrix} A^{-1} = \begin{bmatrix}1 & -1 & 0\\0 & 1 & 0\\\frac{1}{2} & 0 & -\frac{1}{2}\end{bmatrix}$$

By $R_2 - R_3$, we get,

$$\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix} A^{-1} = \begin{bmatrix}1 & -1 & 0\\-\frac{1}{2} & 1 & \frac{1}{2}\\\frac{1}{2} & 0 & -\frac{1}{2}\end{bmatrix}$$

$$A^{-1} = \begin{bmatrix}1 & -1 & 0\\-\frac{1}{2} & 1 & \frac{1}{2}\\\frac{1}{2} & 0 & -\frac{1}{2}\end{bmatrix}$$

Now, multiply $AX = B$ by A^{-1}, we get,

$$A^{-1}(AX) = A^{-1}B$$

$$\therefore (A^{-1}A)X = A^{-1}B$$

$$\therefore IX = A^{-1}B$$

$$\therefore X = \begin{bmatrix} 1 & -1 & 0 \\ -\frac{1}{2} & 1 & \frac{1}{2} \\ \frac{1}{2} & 0 & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -1-2+0 \\ \frac{1}{2}+2+\frac{3}{2} \\ -\frac{1}{2}+0-\frac{3}{2} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3 \\ 4 \\ -2 \end{bmatrix}$$

$\therefore$ by equality of the matrices, $x = -3, y = 4, z = -2$ is the required value.

Express the following equation in matrix from and solve them by the method of reduction.

17. x – y + z = 1, 2x – y = 1, 3x + 3y – 4z = 2

Ans: The given equations can be written in the matrix form as :

$$\begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 3 & 3 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

By $R_2 - 2R_1$ and $R_3 - 3R_1$, we get,

$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & -2 \\ 0 & 6 & -7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$$

By $R_3 - 6R_2$, we get,

$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x - y + z \\ 0 + y - 2z \\ 0 + 0 + 5z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 5 \end{bmatrix}$$

By equality of matrices,

$x - y + z = 1$ .....(1)

$y - 2z = -1$ .....(2)

$5z = 5$ ....(3)

From (3), $z = 1$

Substituting $z = 1$ in (2), we get,

$y - 2 = -1$

$y = -1 + 2$

$\therefore y = 1$

Substituting $y = 1, z = 1$ in (1), we get,

$x - 1 + 1 = 1$

$\therefore x = 1$

Therefore, $x = 1, y = 1, z = 1$ is the required Values.

18. $\mathbf{x + y = 1, y + z = 53, z + x = 43.}$

Ans: The given equations can be written in the matrix form as :

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ \frac{5}{3} \\ \frac{4}{3} \end{bmatrix}$$

By $R_3 - R_1$, we get,

$$\begin{bmatrix}1 & 1 & 0\\ 0 & 1 & 1\\ 0 & -1 & 1\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix} = \begin{bmatrix}1\\ \frac{5}{3}\\ \frac{1}{3}\end{bmatrix}$$

By $R_3 + R_2$, we get,

$$\begin{bmatrix}1 & 1 & 0\\ 0 & 1 & 1\\ 0 & 0 & 2\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix} = \begin{bmatrix}1\\ \frac{5}{3}\\ 2\end{bmatrix}$$

$$\begin{bmatrix}x + y + 0\\ 0 + y + z\\ 0 + 0 + 2z\end{bmatrix} = \begin{bmatrix}1\\ \frac{5}{3}\\ 2\end{bmatrix}$$

By equality of matrices,

$x + y = 1$ .....(1)

$y + z \ -\frac{5}{3}$...(2)

$2z = 2$ .....(3)

From (3), $z = 1$

Substituting $z = 1$ in (2), we get,

$y + 1 \ -\frac{5}{3}$

$y \ -\frac{5}{3} - 1$

$\therefore y \ -\frac{2}{3}$

Substituting $y \ -\frac{2}{3}$ in (1), we get,

$x + \frac{2}{3} - 1$

$x - 1 - \frac{2}{3}$

$\therefore x = \frac{1}{3}$

Therefore, $x = -\frac{1}{3}, y = -\frac{2}{3}, z = 1$ is the required Values.

19. $2x - y + z = 1, x + 2y + 3z = 8$ **and** $3x + y - 4z = 1$

Ans: The given equations can be written in the matrix form as :

$$\begin{bmatrix} 2 & -1 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 8 \\ 1 \end{bmatrix}$$

By $R_1 \leftrightarrow R_2$, we get,

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 1 \\ 3 & 1 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 1 \\ 1 \end{bmatrix}$$

By $R_2 - 2R_1$ and $R_3 - 3R_1$, we get,

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -5 & -5 \\ 0 & -5 & -13 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ -15 \\ -23 \end{bmatrix}$$

By $R_3 - R_2$, we get,

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -5 & -5 \\ 0 & 0 & -8 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ -15 \\ -8 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x+2y+3z \\ 0-5y-5z \\ 0+0-8z \end{bmatrix} = \begin{bmatrix} 8 \\ -15 \\ -8 \end{bmatrix}$$

By equality of matrices,

$x + 2y + 3z = 8 \ldots..(1)$

$-5y - 5z = -15 \ldots.(2)$

$-8z = -8 \ldots..(3)$

From (3), $z = 1$

Substituting $z = 1$ in (2), we get,

$-5y - 5 = -15$

$-5y = -10$

$y = \frac{-10}{-5}$

$\therefore y = 2$

Substituting $y = 2, z = 1$ in (1), we get,

$x + 4 + 3 = 8$

$x + 7 = 8$

$x = 8 - 7$

$\therefore x = 1$

Therefore, $x = 1, y = 2, z = 1$ is the required value.

20. $\mathbf{x + 2y + z = 8, 2x + 3y - z = 11}$ **and** $\mathbf{3x - y - 2z = 5}$

Ans:

The given equations can be written in the matrix form as :

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & -1 \\ 3 & -1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 11 \\ 5 \end{bmatrix}$$

By $R_2 - 2R_1$ and $R_3 - 3R_1$, we get,

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & -1 & -3 \\ 0 & -7 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ -5 \\ -19 \end{bmatrix}$$

By $R_3 - 7R_2$, we get,

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & -1 & -3 \\ 0 & 0 & 16 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ -5 \\ -16 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x + 2y + z \\ 0 - y - 3z \\ 0 + 0 + 16z \end{bmatrix} = \begin{bmatrix} 8 \\ -5 \\ 16 \end{bmatrix}$$

By equality of matrices,

$x + 2y + z = 8 \ldots (1)$

$-y - 3z = -5 \ldots (2)$

$16z = 16 \ldots (3)$

From (3), $z = 1$

Substituting $z = 1$ in (2), we get,

$-y - 3 = -5,$

$-y = -5 + 3$

$-y = -2$

$\therefore y = 2$

Substituting $y = 2, z = 1$ in (1), we get,

$x + 4 + 1 = 8$

$x + 5 = 8$

$x = 8 - 5$

$\therefore x = 3$

Therefore, $x = 3, y = 2, z = 1$ is the required values.

21. $\mathbf{x + 3y + 2z = 6, 3x - 2y + 5z = 5}$ **and** $\mathbf{2x - 3y + 6z = 7}$.

Ans: The given equations can be written in the matrix form as :

$$\begin{bmatrix} 1 & 3 & 2 \\ 3 & -2 & 5 \\ 2 & -3 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 5 \\ 7 \end{bmatrix}$$

By $R_3 - 2R_1, R_2 - 3R_1$ we get

$$\begin{bmatrix} 1 & 3 & 2 \\ 0 & 11 & 1 \\ 0 & -9 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 13 \\ -5 \end{bmatrix}$$

By $R_2 + R_3$, we get

$$\begin{bmatrix} 1 & 3 & 2 \\ 0 & 2 & 3 \\ 0 & -9 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 8 \\ -5 \end{bmatrix}$$

By $\frac{R_2}{2}$ we get

$$\begin{bmatrix} 1 & 3 & 2 \\ 0 & 1 & \frac{3}{2} \\ 0 & -9 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 4 \\ -5 \end{bmatrix}$$

By $R_3 + 9R_2$, we get

$$\begin{bmatrix} 1 & 3 & 2 \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & \frac{31}{2} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 4 \\ 31 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x + 3y + 2z \\ 0 + y + \frac{3}{2}z \\ 0 + 0 + \frac{31}{2}z \end{bmatrix} = \begin{bmatrix} 6 \\ 4 \\ 31 \end{bmatrix}$$

By equality of matrices,

$x + 3y + 2z = 6 \ldots(1)$

$y + \frac{3}{2}z - 4 \ldots(2)$

$\frac{31}{2}z - 31 \ldots..(3)$

From (3), $z = 2$

Substituting $z = 2$ in (2), we get,

$y + \frac{3}{2}z - 4$

$y + \frac{3}{2}(2) - 4$

$y - 3 = 4$

$y = 1$

Substituting $y = 1, z = 2$ in (2), we get,

$x + 3y + 2z = 6$

$x + 3(1) + 2(2) = 6$

$x + 3 + 4 - 6$

$x + 7 - 6$

$x - 6 - 7$

$x - -1$

Therefore, $x = -1, y = 1, z = 2$ is the required values.

22. The sum of three numbers is 6. If we multiply third number by 3 and add it to the second number we get 11. By adding first and the third numbers we get a number which is double the second number. Use this information and find a system of linear equations. Find the three numbers using matrices.

Ans: Let the three numbers be x, y and z. According to the given conditions,

$x + y + z = 6$

$3z + y = 11,$

$i.e., y + 3z = 11$

and $x + z = 2y,$

$i.e., x - 2y + z = 0$

Therefore, the system of the linear equations is

$x + y + z = 6$

$y + 3z = 11$

$x - 2y + z = 0$

These equations can be written in the matrix form as :

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 11 \\ 0 \end{bmatrix}$$

By $R_3 - R_1$, we get,

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 0 & -3 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 11 \\ -6 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x+y+z \\ 0+y+3z \\ 0-3z+0 \end{bmatrix} = \begin{bmatrix} 6 \\ 11 \\ -6 \end{bmatrix}$$

By equality of matrices,

$x + y + z = 6 \ldots(1)$

$y + 3z = 11 \ldots(2)$

$-3y = -6 \ldots(3)$

From (3), $y = 2$

Substituting $y = 2$ in (2), we get,

$2 + 3z = 11$

$\therefore 3z = 9$

$\therefore z = 3$

Put $y = 2, z = 3$ in (1), we get,

$x + 2 + 3 = 6$

$\therefore x = 1$

$\therefore x = 1, y = 2, z = 3$

Therefore, the required numbers are $1, 2$ *and* 3.

23. The cost of 4 pencils, 3 pens and 2 books is Rs. 150. The cost of 1 pencil, 2 pens and 3 books is Rs. 125. The cost of 6 pencils, 2 pens and 3 books is Rs. 175. Find the cost of each item by using Matrices.

Ans: Let the cost of 1 pencil, 1 pen and 1 book be $Rs.\, x, Rs.\, y, Rs.\, z$ respectively.

According to the given conditions,

$4x + 3y + 2z = 150$

$x + 2y + 3z = 125$

$6x + 2y + 3z = 175$

The equations can be written in matrix form as :

$$\begin{bmatrix} 4 & 3 & 2 \\ 1 & 2 & 3 \\ 6 & 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 150 \\ 125 \\ 175 \end{bmatrix}$$

By $R_1 \leftrightarrow R_2$, we get

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 3 & 2 \\ 6 & 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 125 \\ 150 \\ 175 \end{bmatrix}$$

By $R_2 - 4R_1$ and $R_3 - 6R_1$, we get

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -5 & -10 \\ 0 & -10 & -15 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 125 \\ -350 \\ -575 \end{bmatrix}$$

By $R_3 - 2R_2$, we get,

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -5 & -10 \\ 0 & 0 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 125 \\ -350 \\ 125 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x + 2y + 3z \\ 0 - 5y - 10z \\ 0 + 0 + 5z \end{bmatrix} = \begin{bmatrix} 125 \\ -350 \\ 125 \end{bmatrix}$$

By equality of matrices,

$x + 2y + 3z = 125 \ldots(1)$

$-5y - 10z = -350 \ldots(2)$

$5z = 125 \ldots(3)$

From (3), $z - 25$

Substituting $z - 25$ in (2), we get

$-5y - 10(25) = -350$

$\therefore -5y = -350 + 250 = -100$

$\therefore y = 20$

Substituting $y = 20, z = 25$ in (1), we get

$x + 2(20) + 3(25) = 125$

$\therefore x = 125 - 40 - 75 = 10$

$\therefore x = 10, y = 20, z = 25$

Therefore, the cost of 1 pencil is $Rs.\ 10$, 1 pen is $Rs.\ 20$ and 1 book is $Rs.\ 25$.

24. The sum of three numbers is 6. Thrice the third number when added to the first number, gives 7. On adding three times first number to the sum of second and third number, we get 12. Find the three numbers by using Matrices.

Ans: Let the numbers be x, y and z.

According to the given conditions,

$x + y + z = 6$

$3z + x = 7,$

$i.e.,\ x + 3z = 7$

and $3x + y + z = 12$

Therefore, the system of linear equations is

$x + y + z = 6$

$x + 3z - 7$

$3x + y + z = 12$

These equations can be written in matrix form as :

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 3 \\ 3 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 7 \\ 12 \end{bmatrix}$$

By $R_2 - R_1$ and $R_3 - 3R_1$, we get,

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -1 & 2 \\ 0 & -2 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 1 \\ -6 \end{bmatrix}$$

By $R_3 + R_2$, we get,

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -1 & 2 \\ 0 & -3 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 1 \\ -5 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x+y+z \\ 0-y+2z \\ 0-3y+0 \end{bmatrix} = \begin{bmatrix} 6 \\ 1 \\ -5 \end{bmatrix}$$

By equality of matrices,

$x + y + z = 6 \ldots(1)$

$-y + 2z = 1 \ldots(2)$

$-3y = -5 \ldots(3)$

From (3), $y = -\frac{5}{3}$

Substituting $y = -\frac{5}{3}$ in (2), we get,

$-\frac{5}{3} + 2z = 1$

$\therefore 2z = 1 + \frac{5}{3} = \frac{8}{3}$

$\therefore z = \frac{4}{3}$

Substituting $y = -\frac{5}{3}, z = \frac{4}{3}$ in (1), we get,

$x + \frac{5}{3} + \frac{4}{3} = 6$

$\therefore x = 3$

$\therefore x = 3, y = -\frac{5}{3}, z = \frac{4}{3}$

Therefore, the required numbers are $3, -\frac{5}{3}$ and $\frac{4}{3}$.

25. The sum of three numbers is 2. If twice the second number is added to the sum of first and third number, we get 1 adding five times the first number to the sum of second and third we get 6. Find the three numbers by using matrices.

Ans: Let the three numbers be x, y and z.

According to the question,

$x + y + z = 2$

$x + 2y + z = 1$

$5x + y + z = 6$

The given system of equations can be written in matrix form as follows:

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 5 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 6 \end{bmatrix}$$

$AX = B$

Here,

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 5 & 1 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad B = \begin{bmatrix} 2 \\ 1 \\ 6 \end{bmatrix}$$ and

$|A| = 1(2 - 1) - 1(1 - 5) + 1(1-10)$

$\Rightarrow 1 + 4 - 9$

$\Rightarrow -4$

Let C_{ij} be the cofactors of the elements a_{ij} in $A = [a_{ij}]$. Then,

$$C_{11} = (-1)^{1+1}\begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} = 1$$

$$C_{12} = (-1)^{1+2}\begin{vmatrix} 1 & 1 \\ 5 & 1 \end{vmatrix} = 4$$

$$C_{13} = (-1)^{1+3}\begin{vmatrix} 1 & 2 \\ 5 & 1 \end{vmatrix} = -9$$

$$C_{21} = (-1)^{2+1}\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 0$$

$$C_{22} = (-1)^{2+2}\begin{vmatrix} 1 & 1 \\ 5 & 1 \end{vmatrix} = -4$$

$$C_{23} = (-1)^{2+3}\begin{vmatrix} 1 & 1 \\ 5 & 1 \end{vmatrix} = 4$$

$$C_{31} = (-1)^{3+1}\begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} = -1$$

$$C_{32} = (-1)^{1-1}\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 0$$

$$C_{33} = (-1)^{1+1}\begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} = 1$$

$$Adj\ A = \begin{bmatrix} 1 & 4 & -9 \\ 0 & -4 & 4 \\ -1 & 0 & 1 \end{bmatrix}^T$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & -1 \\ 4 & -4 & 0 \\ -9 & 4 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}\ adj\ A$$

$$\Rightarrow \frac{1}{-4}\begin{bmatrix} 1 & 0 & -1 \\ 4 & -4 & 0 \\ -9 & 4 & 1 \end{bmatrix}$$

$$X = A^{-1}B$$

$$\Rightarrow X = \frac{1}{-4}\begin{bmatrix} 1 & 0 & -1 \\ 4 & -4 & 0 \\ -9 & 4 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \\ 6 \end{bmatrix}$$

$$\Rightarrow X = \frac{1}{-4}\begin{bmatrix} 2+0-6 \\ 8-4+0 \\ -18+4+6 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{-4}\begin{bmatrix} -4 \\ 4 \\ -8 \end{bmatrix}$$

$$\therefore x = 1,\ y = -1\ and\ z = 2$$

26. An amount of Rs. 5000 is invested in three types of investments, at interest rates 6%, 7%, 8% per annum respectively. The total annual income from these investimest is Rs. 350. If the total annual income from first two investment is Rs. 70 more than the income from the third, find the amount of each investment using matrix method.

Ans: Let the amounts in three investments by $Rs.\ x,\ Rs.\ y\ and\ Rs.\ z$ respectively.

Then $x + y + z = 5000$

Since the rate of interest in these investments are $6\%,\ 7\%\ and\ 8\%$ respectively, the annual income of the three investments are $\frac{6x}{100}, \frac{7y}{100}\ and\ \frac{8z}{100}$ respectively.

According to the given conditions,

$$\frac{6x}{100} + \frac{7y}{100} + \frac{8z}{100} = 350$$

$$i.e.\ 6x + 7y + 8z = 35000$$

Also, $\frac{6x}{100} + \frac{7y}{100} - \frac{8z}{100} = 70$

i.e. $6x + 7y - 8z = 7000$

Therefore, the system of linear equation is

$$x + y + z = 5000$$

$$6x + 7y + 8z = 35000$$

$$6x + 7y - 8z = 7000$$

These equations can be written in matrix form as :

$$\begin{bmatrix} 1 & 1 & 1 \\ 6 & 7 & 8 \\ 6 & 7 & -8 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5000 \\ 35000 \\ 7000 \end{bmatrix}$$

By $R_3 - R_2$, we get

$$\begin{bmatrix} 1 & 1 & 1 \\ 6 & 7 & 8 \\ 6 & 7 & -8 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5000 \\ 35000 \\ -28000 \end{bmatrix}$$

By $R_2 - 6R_1$, we get

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & -16 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5000 \\ 5000 \\ -28000 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x+y+z \\ 0+y+2z \\ 0+0-16z \end{bmatrix} = \begin{bmatrix} 5000 \\ 5000 \\ -28000 \end{bmatrix}$$

By equality of matrices,

$x + y + z = 5000 \ldots(1)$

$y + 2z = 5000 \ldots(2)$

$-16z = -28000 \ldots.(3)$

From (3), $z - 1750$

Substituting $z - 1750$ in (2), we get,

$y + 2(1750) = 5000$

$\therefore y = 5000 - 3500 = 1500$

Substituting $y = 1500, z = 1750$ in (1), we get,

$x + 1500 + 1750 = 5000$

$\therefore x = 5000 - 3250 = 1750$

$\therefore x = 1750, y = 1500, z = 1750$

Therefore, the amounts of the three investments are $Rs.1750, Rs.\ 1500\ and\ Rs.1750$ respectively.

27. The sum of the costs of one book each of Mathematics, Physics and Chemistry is $\text{Rs. } 210$. Total cost of a mathematics book, 2 physics books, and a chemistry book is $\text{Rs. } 240$ Also the total cost of a Mathematics book, 3 physics book and chemistry books is $\text{Rs. } 300/-$. Find the cost of each book, using Matrices.

Ans:

Let cost of 1 math book, 1 physics book and 1 chemistry book are $x, y\ and\ z$ respectively.

As per the given question

$x + y + z = 210 \ldots..(1)$

$x + 2y + z = 240 \ldots..(2)$

$x + 3y + 2z = 300 \ldots..(3)$

On solving we get

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 3 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 210 \\ 240 \\ 300 \end{bmatrix}$$

By $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$, we get

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 210 \\ 30 \\ 90 \end{bmatrix}$$

By $R_3 \to R_3 - 2R_2$, we get

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 210 \\ 30 \\ 30 \end{bmatrix}$$

$$\begin{bmatrix} x+y+z \\ y \\ z \end{bmatrix} = \begin{bmatrix} 210 \\ 30 \\ 30 \end{bmatrix}$$

$y = 30 \text{ and } z = 30$

on substituting this value in equation (1)

$x + y + z = 210$

$\Rightarrow x + 30 + 30 = 210$

$\Rightarrow x + 60 = 210$

$\Rightarrow x = 210 - 60$

$\therefore x - 150$

The cost of each math, physics and chemistry book are $Rs.150, Rs.30 \text{ and } Rs.30$ respectively.