Maharashtra Board Solutions Class 12

Maths

Chapter 1 Mathematical Logic

Exercise 1.1

1. State which of the following sentences are statements. Justify your answer. In case of the statement, write down the truth value:

i. $5+4=13$

ii. $x-3=14$

iii. Close the door

iv. Zero is a complex number

v. Please get me a breakfast

vi. Congruent triangles are also similar

vii. $x^2=x$

viii. A quadratic equation cannot have more than two roots.

ix. Do you like mathematics?

x. The sun sets in the west.

xi. All real numbers are whole numbers

xii. Can you speak in Marathi?

xiii. $x^2-6x-7=0$**, when** $x=7$

xiv. The sum of cube roots of unity is zero.

xv. It rains heavily.

Ans: (i) As, per the question we have given $5+4=13$, which is a statement that is false. Thus, its truth value is 'F'.

(ii) As, per the question we have $x-3=14$, which is an open sentence, thus it's not a sentence.

(iii) As per the question we have given, close the door, which is an imperative sentence, thus it's not a statement.

(iv) As per the question, we have given that zero is a complex number, which is a statement and also true, thus its truth value is 'T'.

(v) As per the question, we have given that please get me breakfast, which is an imperative sentence, thus it's not a statement.

(vi) As per the question, we have given that congruent triangles are also similar, which is a true statement, thus its truth value is 'T'.

(vii) As per the question, we have given that $x^2 = x$, which is an open sentence, thus it's not a statement.

(viii) As per the question, we have given that a quadratic cannot have more than more than two roots, so it is a statement which is true, hence, its truth value is 'T'.

(ix) As per the question, we have given that do you like mathematics, it is an interrogative sentence, thus, it's not a statement.

(x) As per the question, we have given that the sun sets in the west, it is a statement which is true, thus its truth value is 'T'.

(xi) As per the question, we have given that all numbers are whole numbers, it is a statement which is false, thus its truth value is 'F'.

(xii) As per the question, we have given that can you speak in Marathi, it is an interrogative sentence, thus it's not a statement.

(xiii) As per the question, we have given that $x^2 - 6x - 7 = 0$ when $x = 7$, it is a statement which is true, thus its truth value is 'T'.

(xiv) As per the question, we have given that the sum of cube roots of unity is zero, it is a statement which is true, thus its truth value is 'T'.

(xv) As per the question, we have given that it rains heavily, it's an open sentence, thus it's not a statement.

2. Write the following compound statements symbolically.

i.Nagpur is in Maharashtra and Chennai is in Tamil Nadu.

ii.Triangle is equilateral or isosceles.

iii.The angle is right angle if and only if it is of measure 90°.

iv.Angle is neither acute nor obtuse.

v.If ΔABC is right angled at B, then $m\angle A + m\angle C = 90^\circ$

vi.Hima Das wins the gold medal if and only if she runs fast.

vii.x Is not an irrational number but is a square of an integer.

Ans:

(i) Nagpur is in Maharashtra and Chennai is in Tamil Nadu

Assume p as Nagpur in Maharashtra.

q as Chennai in Tamil Nadu.

Thus, symbolic form of the given statement is $p \wedge q$.

(ii) Triangle is equilateral or isosceles

Consider p as the triangle is equilateral.

q The triangle is isosceles.

Thus, symbolic form of the given statement is $p \vee q$.

(iii) The angle is right angle if and only if it is of measure 90°

Consider p The angel is at the right angle.

q as it is measure of 90°.

Thus, symbolic form of the given statement is $p \leftrightarrow q$.

(iv) Angle is neither acute nor obtuse

Consider p as an acute angle.

q as the angle is obtuse.

Thus, symbolic form of the given statement is: $p \wedge : q$.

(v) If ΔABC is right angled at B, then $m\angle A + m\angle C = 90^\circ$

Consider p as ΔABC is right angle

q as $m\angle A + m\angle C = 90^\circ$

Thus, symbolic form of the given statement is $p \rightarrow q$.

(vi) Hima Das wins the gold medal if and only if she runs fast

Consider p as Hima Das wins the gold medal.

q as she runs fast.

Thus, symbolic form of the given statement is $p \leftrightarrow q$.

(vii) x Is not an irrational number but is a square of an integer.

Consider p as x is not an irrational number.

q as it is a square of an integer.

Thus, symbolic form of the given statement is $\sim p \wedge \sim q$.

3. Write the truth values of the following:

i. 4 is odd or 1 is prime.

ii. 64 is a perfect square and 46 is a prime number.

iii. 5 is a prime number and 7 divides 94.

iv. It is not true that $5-3i$ is a real number.

v. If $3 \times 5 = 8$, then $3+5=15$.

vi. Milk is white if and only if the sky is blue.

vii. 24 is a composite number or 17 is a prime number.

Ans:

(i) 4 is odd or 1 is prime.

Assume p as 4 is odd.

And q as 1 is prime.

Then the symbolic form of the given statement is $p \vee q$.

The truth values of both $p \& q$ are F.

The truth value of $p \vee q$ is F. ….. $[F \vee F = F]$

(ii) 64 is a perfect square and 46 is a prime number

Assume p as 64 is a perfect square.

And q as 46 is a prime number.

Then the symbolic form of the given statement is $p \wedge q$.

The truth values of both $p \& q$ are T.

The truth value of $p \wedge q$ is T. $\ldots..[T \wedge F \equiv F]$.

(iii) 5 is a prime number and 7 divides 94.

Assume p as 5 is a prime number.

And q as 7 divides 94.

Then the symbolic form of the given statement is $p \wedge q$.

The truth values of both $p \& q$ are T.

The truth value of $p \wedge q$ is T. $\ldots..[T \wedge F \equiv F]$.

(iv) It is not true that $5-3i$ is a real number

Assume p as $5-3i$ is a real number.

Then the symbolic form of the given statement is $\sim p$.

The truth values of both $\sim p$ is F.

The truth value of $\sim p$ is T. $\ldots..[\sim F = T]$.

(v) If $3 \times 5 = 8$, then $3+5=15$.

Assume p as $3 \times 5 = 8$

And q as $3+5=15$.

Then the symbolic form of the given statement is $p \rightarrow q$.

The truth values of both $p \& q$ are F.

The truth value of $p \rightarrow q$ is T. $\ldots..[F \rightarrow F \equiv T]$.

(vi) Milk is white if and only if the sky is blue

Assume p as Milk is white.

And q as the sky is blue.

Then the symbolic form of the given statement is $p \leftrightarrow q$.

The truth values of both $p \& q$ are T.

The truth value of $p \leftrightarrow q$ is T. $\ldots\ldots [T \leftrightarrow T \equiv T]$.

(vii) 24 is a composite number or 17 is a prime number.

Assume p as 24 is a composite number.

And q as 17 is a prime number.

Then the symbolic form of the given statement is $p \vee q$.

The truth values of both $p \& q$ are T.

The truth value of $p \vee q$ is T. $\ldots\ldots [T \vee T \equiv T]$.

4. If the statement p, q are true statements and r, s are false statements, then determine the truth values of the following:

i. $p \vee (q \wedge r)$

ii. $(p \rightarrow q) \vee (r \rightarrow s)$

iii. $(q \wedge r) \vee (\sim p \wedge s)$

iv. $(p \rightarrow q) \wedge (\sim r)$

v. $(\sim r \leftrightarrow p) \rightarrow (\sim q)$

vi. $[\sim p \wedge (\sim q \wedge r) \vee (q \wedge r) \vee (p \wedge r)]$

vii. $[(\sim p \wedge q) \wedge (\sim r)] \vee [(q \rightarrow p) \rightarrow (\sim s \vee r)]$

viii. $\sim [(\sim p \wedge r) \vee (s \rightarrow \sim q)] \leftrightarrow (p \wedge r)$

Ans:

(i) $p \vee (q \wedge r)$

Truth values of the given p and q is T and truth values of r and s is F.

So,

$$p \vee (q \wedge r) \equiv T \vee (T \wedge F)$$
$$\equiv T \wedge F \equiv T$$

Thus, the truth values of the given statement is true.

(ii) Given $(p \to q) \vee (r \to s)$

$$(p \to q) \vee (r \to s) \equiv (T \to T) \wedge (F \to F)$$
$$\equiv T \vee T \equiv T$$

Therefore, the truth value of the given statement is true.

(iii) Given $(q \wedge r) \vee (: \ p \wedge s)$

$$(q \wedge r) \vee (\sim p \wedge s) \equiv (T \wedge F) \vee (\sim T \wedge F)$$
$$\equiv F \vee (F \wedge F)$$
$$\equiv F \vee F \equiv F$$

Therefore, the truth value of the given statement if false.

(iv) Given $(p \to q) \wedge (: \ r)$

$$\approx (p \to q) \wedge (\sim r) \equiv (T \to T) \wedge (\sim F)$$
$$\equiv T \wedge T \equiv T$$

Therefore, the truth value of the given statement is true.

(v) Given $(: \ r \leftrightarrow p) \to (: \ q)$

$$(: \ r \leftrightarrow p) \to (: \ q) \equiv (: \ F \leftrightarrow T) \to (: \ T)$$
$$\equiv (T \leftrightarrow T) \to F$$
$$\equiv T \to F$$
$$\equiv F$$

Therefore, the truth value of the given statement is false.

(vi) Given $[: \ p \wedge (: \ q \wedge r) \vee (q \wedge r) \vee (p \wedge r)]$

$$\approx[\sim p\wedge(\sim q\wedge r)\vee(q\wedge r)\vee(p\wedge r)]$$
$$\equiv[\sim T\wedge(\sim T\wedge F)]\vee((T\wedge F)\vee(T\wedge F))$$
$$\equiv[F\wedge(F\wedge F)]\vee[F\vee F]$$
$$\equiv(F\wedge F)\vee F$$
$$\equiv F\vee F$$
$$\equiv F$$

Therefore, the truth value of the given statement is false.

(vii)Given $[(:\ p\wedge q)\wedge(:\ r)]\vee[(q\to p)\to(:\ s\vee r)]$

$$\equiv[(:\ p\wedge q)\wedge(:\ r)]\vee[(q\to p)\to(:\ s\vee r)]$$
$$\equiv[(:\ T\wedge T)\wedge(:\ F)]\vee[(T\to T)\to(:\ F\vee F)]$$
$$\equiv[(F\wedge T)\wedge T]\vee[T\to(T\vee F)]$$
$$\equiv(F\wedge T)\vee(T\to T)$$
$$\equiv F\vee T$$
$$\equiv T$$

Thus, the truth values of the given statement is true.

(viii)Given $:\ [(:\ p\wedge r)\vee(s\to:\ q)]\leftrightarrow(p\wedge r)$

$$:\ [(:\ p\wedge r)\vee(s\to:\ q)]\leftrightarrow(p\wedge r)$$
$$:\ [(:\ T\wedge F)\vee(F\to:\ T)]\leftrightarrow(T\wedge F)$$
$$:\ [(F\wedge F)\vee(F\to F)]\leftrightarrow F$$
$$:\ (F\vee T)\leftrightarrow T$$
$$:\ T\leftrightarrow F$$
$$\equiv F\leftrightarrow F$$
$$\equiv T$$

Thus, the truth value is true.

5. Write the negations of the following:

i.Tirupati is in Andhra Pradesh.

ii. 3 **is not a root of the equation** $x^2+3x-18=0$**.**

iii. $\sqrt{2}$ **is a rational number.**

iv.Polygon ABCDE is a pentagon.

v. $7+3>5$

Ans:

(i) Negation Tirupati is in Andhra Pradesh is, Tirupati is not in Andhra Pradesh.

(ii) Negation of 3 is not a root of the equation $x^2+3x-18=0$ is , 3 is a root of the equation $x^2+3x-18=0$.

(iii) Negation of $\sqrt{2}$ is a rational number $\sqrt{2}$ is not a rational number.

(iv) Negation of polygon ABCDE is a pentagon if polygon ABCDE is not a pentagon.

(v) Negation of $7+3>5$ is $7+3\not>5$.

Exercise 1.2

1. Construct the truth table for each of the following statements patterns:

i. $[(p\rightarrow q)\wedge q]\rightarrow p$

ii. $(p\wedge\sim q)\leftrightarrow(p\rightarrow q)$

iii. $(p\wedge q)\leftrightarrow(q\vee r)$

iv. $p\rightarrow[\sim(q\wedge r)]$

v. $\sim p\wedge[(p\vee\sim q)\wedge q]$

vi. $(\sim p\rightarrow\sim q)\wedge(\sim q\rightarrow\sim p)$

vii. $(q\rightarrow p)\vee(\sim p\leftrightarrow q)$

viii. $[p\rightarrow(q\rightarrow r)]\leftrightarrow[(p\wedge q)\rightarrow r]$

ix. $p\rightarrow[\sim(q\wedge r)]$

x. $(p\vee\sim q)\rightarrow(r\wedge p)$

Ans:

(i) Drawing the table for $[(p \to q) \wedge q] \to p$

p	q	$p \to q$	$(p \to q) \wedge q$	$[(p \to q) \wedge q] \to p$
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	T

(ii) Drawing the table for $(p \wedge : q) \leftrightarrow (p \to q)$

p	q	$: q$	$p \wedge : q$	$p \to q$	$(p \wedge : q) \leftrightarrow (p \to q)$
T	T	F	F	T	F
T	F	T	T	F	F
F	T	F	F	T	F
F	F	T	F	T	F

(iii) Drawing the table for $(p \wedge q) \leftrightarrow (q \vee r)$

p	q	r	$p \wedge q$	$q \vee r$	$(p \wedge q) \leftrightarrow (q \vee r)$
T	T	T	T	T	T
T	T	F	T	T	T
T	F	T	F	T	F
T	F	F	F	F	T
F	T	T	F	T	F
F	T	F	F	T	F
F	F	T	F	T	F
F	F	F	F	F	T

(iv) Drawing the table for $P \to [: (q \wedge r)]$

p	q	r	$q \wedge r$	$: (q \wedge r)$	$p \rightarrow [: (q \wedge r)]$
T	T	T	T	F	F
T	T	F	F	T	T
T	F	T	F	T	T
T	F	F	F	T	T
F	T	T	T	F	T
F	T	F	F	T	T
F	F	T	F	T	T
F	F	F	F	T	T

(v) Drawing the table for $: p \wedge [(p \vee : q) \wedge q]$

p	q	$: p$	$: q$	$p \vee : q$	$(p \vee : q) \wedge q$	$: p \wedge [(p \vee : q) \wedge q]$
T	T	F	F	T	T	F
T	F	T	T	T	F	F
F	T	F	F	F	F	F
F	F	T	T	T	F	F

(vi) Drawing the table for $(: p \rightarrow : q) \wedge (: q \rightarrow : p)$

p	q	$: p$	$: q$	$: p \rightarrow : q$	$: q \rightarrow : p$	$(: p \rightarrow : q) \wedge (: q \rightarrow : p)$
T	T	F	T	T	T	T
T	F	F	T	T	F	F
F	T	T	F	F	T	F
F	F	T	T	T	T	T

(vii) Drawing the table for $(q \rightarrow p) \vee (: p \leftrightarrow q)$

p	q	$: p$	$q \rightarrow p$	$: p \leftrightarrow q$	$(q \rightarrow p) \vee (: p \leftrightarrow q)$
T	T	F	T	F	T
T	F	F	T	T	T
F	T	T	F	T	T
F	F	T	T	F	T

(viii) Drawing the table for $[p \rightarrow (q \rightarrow r)] \leftrightarrow [(p \wedge q) \rightarrow r]$

p	q	r	$q \to r$	$p \to (q \to r)$	$p \wedge q$	$(p \wedge q) \to r$	$[p \to (q \to r)] \leftrightarrow [(p \wedge q) \to r]$
T	T	T	T	T	T	T	T
T	T	F	F	F	T	F	T
T	F	T	T	T	F	T	T
T	F	F	T	T	F	T	T
F	T	T	T	T	F	T	T
F	T	F	F	T	F	T	T
F	F	T	T	T	F	T	T
F	F	F	T	T	F	T	T

(ix) Drawing the table for $p \to [: (q \wedge r)]$

p	q	r	$q \wedge r$	$: (q \wedge r)$	$p \to [: (q \wedge r)]$
T	T	T	T	F	F
T	T	F	F	T	T
T	F	T	F	T	T
T	F	F	F	T	T
F	T	T	T	F	T
F	T	F	F	T	T
F	F	T	F	T	T
F	F	F	F	T	T

(x) Drawing the table for $(p \vee : q) \to (r \wedge p)$

p	q	r	$: q$	$p \vee : q$	$r \wedge p$	$(p \vee : q) \to (r \wedge p)$
T	T	T	F	T	T	T
T	T	F	F	T	F	F
T	F	T	T	T	T	T
T	F	F	T	T	F	F
F	T	T	F	F	F	T
F	T	F	F	F	F	T
F	F	T	T	T	F	F
F	F	F	T	T	F	F

2. Using truth tables prove the following logical equivalences.

I. $\sim p \wedge q \equiv (p \vee q) \wedge \sim p$

II. $\sim (p \vee q) \vee (\sim p \wedge q) \equiv \sim p$

III. $p \leftrightarrow q \equiv \sim [(p \vee q) \wedge \sim (p \wedge q)]$

IV. $p \rightarrow (q \rightarrow p) \equiv \sim p \rightarrow (p \rightarrow q)$

V. $(p \vee q) \rightarrow r \equiv (p \rightarrow r) \wedge (q \rightarrow r)$

VI. $p \rightarrow (q \wedge r) \equiv (p \rightarrow q) \wedge (q \rightarrow r)$

VII. $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

VIII. $[\sim (p \vee q) \vee (p \vee q)] \wedge r \equiv r$

IX. $\sim (p \leftrightarrow q) \equiv (p \wedge \sim q) \vee (q \wedge \sim p)$

Ans:

(i) $\sim p \wedge q \equiv (p \vee q) \wedge \sim p$

1	2	3	4	5	6
p	q	$\sim p$	$\sim p \wedge q$	$p \vee q$	$(p \vee q) \wedge \sim p$
T	T	F	F	T	F
T	F	F	F	T	F
F	T	T	T	T	T
F	F	T	F	F	F

So, columns 4 and 6 are identical.

$\sim p \wedge q \equiv (p \vee q) \wedge \sim p$

(ii) $\sim (p \vee q) \vee (\sim p \wedge q) \equiv \sim p$

1	2	3	4	5	6	7
p	q	$\sim p$	$p \vee q$	$\sim (p \vee q)$	$\sim p \wedge q$	$\sim (p \vee q) \vee (\sim p \wedge q)$
T	T	F	T	F	F	F
T	F	F	T	F	F	F
F	T	T	T	F	T	T
F	F	T	F	T	F	T

So, column 3 and 7 are identical.

$: (p \vee q) \vee (: p \wedge q) \equiv: p$

(iii) $p \leftrightarrow q \equiv: [(p \vee q) \wedge : (p \wedge q)]$

1	2	3	4	5	6	7	8
p	q	$p \leftrightarrow q$	$p \vee q$	$p \wedge q$	$: (p \vee q)$	$(p \wedge q) \wedge : (p \wedge q)$	$: [(p \vee q) \wedge : (p \wedge q)]$
T	T	T	T	T	F	F	T
T	F	F	T	F	T	T	F
F	T	F	T	F	T	T	F
F	F	T	F	F	T	F	T

So, column 3 and 8 are identical.

$p \leftrightarrow q \equiv: [(p \vee q) \wedge : (p \wedge q)]$

(iv) $p \rightarrow (q \rightarrow p) \equiv: p \rightarrow (p \rightarrow q)$

1	2	3	4	5	6	7
p	q	$q \rightarrow p$	$p \rightarrow (q \rightarrow p)$	$: p$	$p \rightarrow q$	$: p \rightarrow (p \rightarrow q)$
T	T	T	T	F	T	T
T	F	T	T	F	F	T
F	T	F	T	T	T	T
F	F	T	T	T	T	T

So, column 4 and 7 are identical.

$p \rightarrow (q \rightarrow p) \equiv: p \rightarrow (p \rightarrow q)$

(v) $(p \vee q) \rightarrow r \equiv (p \rightarrow r) \wedge (q \rightarrow r)$

1	2	3	4	5	6	7	8
p	q	r	$p \vee q$	$(p \vee q) \rightarrow r$	$p \rightarrow r$	$q \rightarrow r$	$(p \rightarrow r) \wedge (q \rightarrow r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F
T	F	T	T	T	T	T	T
T	F	F	T	F	F	T	F
F	T	T	T	T	T	T	T
F	T	F	T	F	T	F	F
F	F	T	F	T	T	T	T
F	F	F	T	T	T	T	T

So, column 5 and 8 identical.

$(p \vee q) \rightarrow r \equiv (p \rightarrow r) \wedge (q \rightarrow r)$

(vi) $p \rightarrow (q \wedge r) \equiv (p \rightarrow q) \wedge (q \rightarrow r)$

1	2	3	4	5	6	7	8
p	q	r	$q \wedge r$	$p \rightarrow (q \wedge r)$	$p \rightarrow q$	$p \rightarrow r$	$(p \rightarrow q) \wedge (p \rightarrow r)$
T	T	T	T	T	T	T	T
T	T	F	F	F	T	F	F
T	F	T	F	F	F	T	F
T	F	F	F	F	F	F	F
F	T	T	T	T	T	T	T
F	T	F	F	T	T	T	T
F	F	T	F	T	T	T	T
F	F	F	F	T	T	T	T

So, column 5 and 8 are identical.

$p \rightarrow (q \wedge r) \equiv (p \rightarrow q) \wedge (q \rightarrow r)$

(vii) $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

1	2	3	4	5	6	7	8
p	q	r	$q \vee r$	$p \wedge (q \vee r)$	$p \wedge q$	$p \wedge r$	$(p \wedge q) \vee (p \wedge r)$
T	T	T	T	T	T	T	T
T	T	F	T	T	T	F	T
T	F	T	T	T	F	T	T
T	F	F	F	F	F	F	F
F	T	T	T	F	F	F	F
F	T	F	T	F	F	F	F
F	F	T	T	F	F	F	F
F	F	F	F	T	F	F	F

So, column 5 and 8 are identical.

$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$.

(viii) $[: (p \vee q) \vee (p \vee q)] \wedge r \equiv r$

1	2	3	4	5	6	7
p	q	r	$p \vee q$	$: (p \vee q)$	$: (p \vee q) \vee (p \vee q)$	$[: (p \vee q) \vee (p \vee q)] \wedge r$
T	T	T	T	F	T	T
T	T	F	T	F	T	F
T	F	T	T	F	T	T
T	F	F	T	F	T	F
F	T	T	T	F	T	T
F	T	F	T	F	T	F
F	F	T	F	T	T	T
F	F	F	F	T	T	F

So, column 3 and 7 are identical.

$[: (p \vee q) \vee (p \vee q)] \wedge r \equiv r$

(ix) $: (p \leftrightarrow q) \equiv (p \wedge : q) \vee (q \wedge : p)$

1	2	3	4	5	6	7	8	9
p	q	$: p$	$: q$	$p \leftrightarrow q$	$: (p \leftrightarrow q)$	$p \wedge : q$	$q \wedge : p$	$(p \wedge : q) \vee (q \wedge : p)$
T	T	F	F	T	F	F	F	F
T	F	F	T	F	T	T	F	T
F	T	T	F	F	T	F	T	T
F	F	T	T	T	F	F	F	F

So, column 6 and 9 are identical.

$: (p \leftrightarrow q) \equiv (p \wedge : q) \vee (q \wedge : p)$.

3. Examine whether each of the following statement patterns is a tautology or a contradiction or a contingency.

i. $(p \wedge q) \rightarrow (q \vee p)$

ii. $(p \rightarrow q) \leftrightarrow (: p \wedge q)$

iii. $[: (: p \wedge : q)] \vee q$

iv. $[(p \rightarrow q) \wedge q] \rightarrow p$

v. $[(p \rightarrow q) \wedge : q] \rightarrow : p$

vi. $(p \leftrightarrow q) \wedge (p \rightarrow : q)$

vii. $: (: q \wedge p) \wedge q$

viii. $(p \wedge : q) \leftrightarrow (p \rightarrow q)$

ix. $(: p \rightarrow q) \wedge (p \wedge r)$

x. $[p \rightarrow (: q \vee r)] \leftrightarrow : [p \rightarrow (q \rightarrow r)]$

Ans:

(i) $(p \wedge q) \rightarrow (q \vee p)$

p	q	$p \wedge q$	$q \vee p$	$(p \wedge q) \rightarrow (q \wedge p)$
T	T	T	T	T
T	F	F	T	T
F	T	F	T	T
F	F	F	F	T

In the last column of the truth table mentioned above all the entries are T.

So, $(p \wedge q) \rightarrow (q \vee p)$ is a tautology.

(ii) $(p \rightarrow q) \leftrightarrow (: \ p \wedge q)$

p	q	$: p$	$p \rightarrow q$	$: p \vee q$	$(p \rightarrow q) \leftrightarrow (: \ p \vee q)$
T	T	F	T	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	T	T

In the last column of the truth table mentioned above all the entries are T.

So, $(p \rightarrow q) \leftrightarrow (: \ p \wedge q)$ is a tautology.

(iii) $[: \ (: \ p \wedge : \ q)] \vee q$

p	q	$: p$	$: q$	$: p \wedge : q$	$: (: p \wedge : q)$	$[: (: p \wedge : q)] \vee q$
T	T	F	F	F	T	T
T	F	F	T	F	T	T
F	T	T	F	F	T	T
F	F	T	T	T	F	F

In the last column of the truth table mentioned above all the entries are not T nor F.

So, $[: \ (: \ p \wedge : \ q)] \vee q$ is a contingency.

(iv) $[(p \rightarrow q) \wedge q] \rightarrow p$

p	q	$p \to q$	$(p \to q) \wedge q$	$[(p \to q) \wedge q] \to p$
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	T

In the last column of the truth table mentioned above all the entries are not T nor F.

So, $[(p \to q) \wedge q] \to p$ is a contingency.

(v) $[(p \to q) \wedge : q] \to : p$

p	q	$: p$	$: q$	$p \to q$	$(p \to q) \wedge : q$	$[(p \to q) \wedge : q] \to : p$
T	T	F	F	T	F	T
T	F	F	T	F	F	T
F	T	T	F	T	F	T
F	F	T	T	T	T	T

In the last column of the truth table mentioned above all the entries are T.

So, $[(p \to q) \wedge : q] \to : p$ is a tautology.

(vi) $(p \leftrightarrow q) \wedge (p \to : q)$

p	q	$: q$	$p \leftrightarrow q$	$p \to : q$	$(p \leftrightarrow q) \wedge (p \to : q)$
T	T	F	T	F	F
T	F	T	F	T	F
F	T	F	F	T	F
F	F	T	T	T	T

In the last column of the truth table mentioned above all the entries are not T and nor F.

So, $(p \leftrightarrow q) \wedge (p \to : q)$ is a contingency.

(vii) $: (: q \wedge p) \wedge q$

p	q	$\sim q$	$q \wedge p$	$\sim(q \wedge p)$	$(\sim(q \wedge p)) \wedge q$
T	T	F	F	T	T
T	F	T	T	F	F
F	T	F	F	T	T
F	F	T	F	T	F

In the last column of the truth table mentioned above all the entries are not T and nor F.

So, $(\sim(q \wedge p)) \wedge q$ is a contingency.

(viii) $(p \wedge \sim q) \leftrightarrow (p \rightarrow q)$

p	q	$\sim q$	$p \wedge \sim q$	$p \rightarrow q$	$(p \wedge \sim q) \leftrightarrow (p \rightarrow q)$
T	T	F	F	T	F
T	F	T	T	F	F
F	T	F	F	T	F
F	F	T	F	T	F

In the last column of the truth table mentioned above all the entries are F.

So, $(p \wedge \sim q) \leftrightarrow (p \rightarrow q)$ is a contradiction.

(ix) $(\sim p \rightarrow q) \wedge (p \wedge r)$

p	q	r	$\sim p$	$\sim p \rightarrow q$	$p \wedge r$	$(\sim p \rightarrow q) \wedge (p \wedge r)$
T	T	T	F	T	T	T
T	T	F	F	T	F	F
T	F	T	F	T	T	T
T	F	F	F	T	F	F
F	T	T	T	T	F	F
F	T	F	T	T	F	F
F	F	T	T	F	F	F
F	F	F	T	F	F	F

In the last column of the truth table mentioned above all the entries are not T nor F.

So, $(\sim p \rightarrow q) \wedge (p \wedge r)$ is a contingency.

(x) $[p \to (: q \vee r)] \leftrightarrow : [p \to (q \to r)]$

p	q	r	$: q$	$: q \vee r$	$p \to (: q \vee r)$	$q \to r$	$p \to (q \to r)$	$: [p \to (q \to r)]$	$[p \to (: q \vee r)] \leftrightarrow : [p \to (q \to r)]$
T	T	T	F	T	T	T	T	F	F
T	T	F	F	F	F	F	F	T	F
T	F	T	T	T	T	T	T	F	F
T	F	F	T	T	T	T	T	F	F
F	T	T	F	T	T	T	T	F	F
F	T	F	F	F	T	F	T	F	F
F	F	T	T	T	T	T	T	F	F
F	F	T	T	T	T	T	T	F	F

In the last column of the truth table mentioned above all the entries are F.

So, $[p \to (: q \vee r)] \leftrightarrow : [p \to (q \to r)]$ is a contradiction.

Exercise 1.3

1. If $A = \{3,5,7,9,11,12\}$ determine the truth value of each of the following.

i. $\exists x \in A$ **such that** $x - 8 = 1$**.**

ii. $\forall x \in A, x^2 + x$ **is an even number.**

iii. $\exists x \in A$ **such that** $x^2 < 0$**.**

iv. $\forall x \in A, x$ **is an even number.**

v. $\exists x \in A$ **such that** $3x + 8 > 40$**.**

vi. $\forall x \in A, 2x + 9 > 14$

Ans:

(i) If $A = \{3,5,7,9,11,12\}$, so it is seen clearly that $x - 9 \in A$ satisfies $x - 8 = 1$ Thus, the given statement is true and the truth value is T.

(ii) If $A = \{3,5,7,9,11,12\}$, so $x \in A, x^2 + x$ is an even number for each, thus it's true and the truth value is T.

(iii) If $A=\{3,5,7,9,11,12\}$, so there is no $x \in A$ so, $x^2<0$, hence the statement is false and the truth value is F.

(iv) If $A=\{3,5,7,9,11,12\}$, so $x=3 \in A, x=5 \in A, x=7 \in A, x=9 \in A, x=11 \in A$ not satisfy that x is an even number, hence the statement is false and the truth value is F.

(v) If $A=\{3,5,7,9,11,12\}$, so $x=11 \in A$ and also $x=12 \in A$ which shows $3x+8>40$ Hence, the given statement is true, and the truth value is T.

(vi) If $A=\{3,5,7,9,11,12\}$ so, $x \in A, 2x+9>14$ Thus, the given statement is true and the truth value is T.

2. Write duals of each of the following.

i. $p \vee (q \wedge r)$

ii. $p \wedge (q \wedge r)$

iii. $(p \vee q) \wedge (r \vee s)$

iv. $p \wedge : q$

v. $(: p \vee q) \wedge (: r \wedge s)$

vi. $: p \wedge (: q \wedge (p \vee q) \wedge : r)$

vii. $[: (p \vee q)] \wedge [p \vee : (q \wedge : s)]$

viii. $c \vee \{p \wedge (q \wedge r)\}$

ix. $: p \vee (q \wedge r) \wedge t$

x. $(p \vee q) \vee c$

Ans:

(i) The dual pattern of $p \vee (q \wedge r)$ is $p \wedge (q \vee r)$.

(ii) The dual pattern of $p \wedge (q \wedge r)$ is $p \vee (q \vee r)$.

(iii) The dual pattern of $(p \vee q) \wedge (r \vee s)$ is $(p \wedge q) \vee (r \wedge s)$.

(iv) The dual pattern of $p \wedge : q$ is $p \vee : q$.

(v) The dual pattern of $(: p \vee q) \wedge (: r \wedge s)$ is $(: p \wedge q) \vee (: r \vee s)$

(vi) The dual pattern of $: p \wedge (: q \wedge (p \vee q) \wedge : r)$ is $: p \vee (: q \vee (p \wedge q) \vee : r)$

(vii) The dual pattern of $[: (p \vee q)] \wedge [p \vee : (q \wedge : s)]$ is $[: (p \wedge q)] \vee [p \wedge : (q \vee : s)]$

(viii) The dual pattern of $c \vee \{p \wedge (q \wedge r)\}$ is $t \wedge \{p \wedge (q \mathrm{A} r)\}$

(ix) The dual pattern of $: p \vee (q \wedge r) \wedge t$ is $: p \wedge (q \vee r) \vee c$

(x) The dual pattern of $(p \vee q) \vee c$ is $(p \wedge q) \wedge t$.

3. Write the negations of the following.

i. $x+8>11$ **or** $y-3=6$

ii. $11<15$ **and** $25>20$

iii. Quadrilateral is a square if and only if it is a rhombus.

iv. It is cold and raining.

v. If it is raining then we will go and play football.

vi. $\sqrt{2}$ **is a rational number.**

vii. All natural numbers are whole numbers.

viii. $\forall n \in N, n^2+n+2$ **is divisible by** 4**.**

ix. $\exists x \in N$ **such that** $x-17<20$**.**

Ans:

(i) $x+8>11$ or $y-3=6$

Assume $x+8>11$ as p and $y-3=6$ as q,

So, the symbolic form will be $p \vee q$.

As, $\sim (p \vee q) \equiv \sim p \wedge \sim q$, then the negations of the statement is,

$x+8>11$ is $x+8 \ngtr 11$ and $y-3=6$ is $y-3 \neq 6$.

(ii) $11<15$ and $25>20$

Assume p as $11<15$ and q as $25>20$.

So, the symbolic form will be $p \wedge q$.

As, $\sim (p \wedge q) \equiv \sim p \vee \sim q$, so the negation of the statements are

$11<15$ is $11 \ngtr 15$ and $25>20$ is $25 \not< 20$.

(iii) Quadrilateral is a square if and only if it is a rhombus

Consider p as quadrilateral is a square and q is a rhombus.

Thus the symbolic form is $p \leftrightarrow q$ so, $\sim (p \leftrightarrow q) \equiv (p \wedge \sim q) \vee (q \wedge \sim p)$.

The negation of Quadrilateral is a square if and only if it is a rhombus is 'quadrilateral is a square but it is not a rhombus or quadrilateral is a rhombus but it is not a square.

(iv) It is cold and raining.

Assume p as it is cold and q as it is raining.

So, the symbolic form is $p \wedge q$.

Thus, $\sim (p \wedge q) \equiv \sim p \vee \sim q$ so, the negation will be 'it is not cold or not raining.

(v) If it is raining then we will go and play football

Assume p as it is raining and q as we will go and r as we play football.

Thus, symbolic form is $p \rightarrow (q \wedge r)$.

As, $\sim [p \rightarrow (q \wedge r)] \equiv p \wedge \sim (q \wedge r) \equiv (p \wedge (q \wedge r))$, hence the negation will be 'it is raining and we will not go or not play football.

(vi) $\sqrt{2}$ is a rational number.

The negation of the statement will be '$\sqrt{2}$ is not a rational number'.

(vii) All natural numbers are whole numbers.

The negation of the statement will be 'some natural numbers are not whole numbers'.

(viii) $\forall n \in N, n^2+n+2$ is divisible by 4.

The negation of the statement will be '$\exists n \in N$, such that n^2+n+2 is not divisible by 4.

(ix) $\exists x \in N$ such that $x-17<20$.

The negation of the statement will be $\forall x \in N, x-17 \not> 20$.

4. Write converse, inverse and contrapositive of the following statements.

i.If $x<y$ then $x^2<y^2 (x, y \in R)$.

ii.A family becomes literate if the woman is literate.

iii.If the surface decreases then pressure increases.

iv.If voltage increases then current decreases.

Ans:

(i) Assume p will be $x<y, x^2<y^2$

Thus, symbolic form is $\approx p \rightarrow q$.

Converse will be $\approx q \rightarrow p$, so, $x^2<y^2$ will be $x<y$.

Inverse will be $\approx: \ p \rightarrow q$ of $p \rightarrow q$ if $x \not> y$ then $x^2 \not> y^2$.

Contrapositive $\approx: \ q \rightarrow p$ of $p \rightarrow q$, if $x^2 \not> y^2$ then $x \not> y$.

(ii) Suppose p will be the family is literate.

q Will be a family become literate.

Thus, the symbolic form is $p \rightarrow q$.

Converse will be $q \to p$ so, if a family becomes literate then the women in it is literate.

Inverse will be $\sim p \to \sim q$, if the woman in the family is not literate, then the family does not become literate.

Contrapositive: $\sim q \to \sim p$, so if a family does not become literate, then the women in it are not literate.

(iii) Assume p as the surface decreases.

q as the pressure increases.

So, the symbolic form is $p \to q$.

Converse will be $q \to p$, so if the pressure increases, then the surface area decreases.

Inverse will be $\sim p \to \sim q$, so if the surface area does not decrease, then the pressure does not increase.

Contrapositive will be $\sim q \to \sim p$, so if the pressure does not increase, then the surface area does not decrease.

(iv) If voltage decreases then current decreases.

Assume p voltage increases.

q is currently decreasing.

Thus, symbolic form will be $p \to q$.

Converse will be $q \to p$, so if current decreases, then voltage increases.

Inverse will be $\sim p \to \sim q$, so if voltage does not increase, the current does not decrease.

Contrapositive will be $\sim q \to \sim p$, so if current does not decrease, so voltage does not increase.

Exercise 1.4

1. Using rules of negation write the negations of the following with justifications.

I. $: q \to p$

II. $p \wedge : q$

III. $p \vee : q$

IV. $(p \vee : q) \wedge r$

V. $p \to (p \vee : q)$

VI. $: (p \wedge q) \vee (p \vee : q)$

VII. $(p \vee : q) \to (p \wedge : q)$

VIII. $(: p \vee : q) \vee (p \wedge : q)$

Ans:

(i) As per the question it is given $: q \to p$, so negation of $: q \to p$ is $: (: q \to p) \equiv : q \wedge : p$ (negation of implication).

(ii)As per the question given equation is $p \wedge : q$ so, negation of $p \wedge : q$ will be $: (p \wedge : q) \equiv : p \vee : (: q)$ (negation of conjunction) and $\equiv : p \vee q$ (negation of negation).

(iii)As per the question the given equation is $p \vee : q$ so, negation of $p \vee : q$ is $: (p \vee : (q) \equiv : p \wedge : (: q))$ (negation of disjunction) and $: p \wedge q$ (negation of negation).

(iv) As per the question it is given that $(p \vee : q) \wedge r$ so, negation of $(p \vee : q) \wedge r$ is $: [(p \vee : q) \wedge r] \equiv : (p \vee : q) \vee : r$ (negation of conjunction) and $\equiv [: p \wedge : (: q)] \vee : r$ (negation of disjunction) and $\equiv (: p \wedge q) \wedge : r$ (negation of negation).

(v)As per the question it is given that $p \to (p \vee : q)$ so, negation of $p \to (p \vee : q)$ is $: [p \to (p \vee : q)] \equiv p \wedge : (p \wedge : p)$ (negation of implication) and $\equiv p \wedge [: p \wedge : (: q)]$ (negation of disjunction) and $\equiv p \wedge (: p \wedge q)$ (negation of negation).

(vi)As per the question the given equation is $: (p \wedge q) \vee (p \vee : q)$ so, negation of $: (p \wedge q) \vee (p \vee : q)$ will be $: [: (p \wedge q) \vee (p \vee : q)] \equiv : [: (p \wedge q)] \wedge : (p \vee : q)$ (negation of disjunction) and $\equiv (p \wedge q) \wedge (: p \wedge q)$ (negation of negation).

(vii)As per the question it is given that $(p \vee : q) \rightarrow (p \wedge : q)$ so, negation of $(p \vee : q) \rightarrow (p \wedge : q)$ will be $: [(p \vee : q) \rightarrow (p \vee : q)]$ and $\equiv (p \vee : q) \wedge : (p \wedge : q)$ (negation of implication) $\equiv (p \vee : q) \wedge : (p \wedge : q)$ (negation of conjunction) and $\equiv (p \vee : q) \wedge (: p \vee q)$ (negation of negation).

(viii)As per the question it is given that $(: p \vee : q) \vee (p \wedge : q)$ so, negation of $(: p \vee : q) \vee (p \wedge : q)$ will be $: [(: p \vee : q) \vee (p \wedge : q)]$ and
$\equiv : (: p \vee : q) \wedge : (p \wedge : q)$ (negation of disjunction) and
$\equiv [: (: p) \wedge : (: q)] \wedge [: p \vee : q]$ (negation of conjunction) and
$\equiv (p \wedge q) \wedge (: p \vee q)$ (negation of negation).

2. Rewrite the following statements without using if.. Then.

I.If a man is a judge then he is honest.

II.If 2 is a rational number then $\sqrt{2}$ is irrational number.

III.If $f(2) = 0$ then $f(x)$ is divisible by $(x-2)$.

Ans:

(i) Writing the given statement without using if and then the given statement is if a man is a judge then he is honest so, $p \rightarrow \equiv : p \vee q$ then, the statement will be A man is not a judge or he is honest.

(ii) Writing the given statement without using if and then the given statement is if 2 is a rational number then $\sqrt{2}$ is irrational number then the statement will be 2 is not a rational number or $\sqrt{2}$ is an irrational number.

(iii) Writing the given statement without using if and then the given statement is if $f(2)=0$ then $f(x)$ is divisible by $(x-2)$ so, the statement will be $f(2)\neq 0$ or $f(x)$ is divisible by $(x-2)$.

3. Without using truth table prove that:

I. $p\leftrightarrow q\equiv(p\wedge q)\vee(:\ p\wedge:\ q)$

II. $(p\vee q)\wedge(p\vee:\ q)\equiv p$

III. $(p\wedge q)\vee(:\ p\wedge q)\vee(p\wedge:\ q)\equiv p\vee q$

IV. $:\ [(p\vee:\ q)\rightarrow(p\wedge:\ q)]\equiv(p\vee:\ q)\wedge(:\ p\vee q)$

Ans:

(i) Without using truth table proving $p\leftrightarrow q\equiv(p\wedge q)\vee(:\ p\wedge:\ q)$

So, the left hand side will be $\Rightarrow p\leftrightarrow q$

$\equiv(p\leftrightarrow q)\wedge(q\leftrightarrow p)$ (Biconditional law)

$\equiv(:\ p\vee q)\wedge(:\ q\vee p)$ (Conditional law)

$\equiv[:\ p\wedge(:\ q\vee p)]\vee[q\wedge(:\ q\vee p)]$ (Distributive law)

$\equiv[(:\ p\wedge:\ q)\vee(:\ p\wedge p)]\vee[(q\wedge:\ q)\vee(q\wedge p)]$ (Distributive law)

$\equiv[(:\ p\wedge:\ q)\vee F]\vee[F\vee(q\wedge p)]$ (Complement law)

$\equiv(:\ p\wedge:\ q)\vee(q\wedge p)$ (Identity law)

$\equiv(:\ p\wedge:\ q)\vee(p\wedge q)$ (Commutative law)

$\equiv(p\wedge q)\vee(:\ p\wedge:\ q)$ (Commutative law)

Hence, $LHS=RHS$.

(ii) Without using truth table proving $(p\vee q)\wedge(p\vee:\ q)\equiv p$

So, left hand side will be $\Rightarrow(p\vee q)\wedge(p\vee:\ q)$

$\equiv p \vee (q \wedge : q)$ (Distributive law)

$\equiv p \vee F$ (complement law)

$\equiv p$ (Identity law)

Hence, $LHS = RHS$

(iii) without using the truth table proving $(p \wedge q) \vee (: p \wedge q) \vee (p \wedge : q) \equiv p \vee q$

So, left hand side will be $\Rightarrow (p \wedge q) \vee (: p \wedge q) \vee (p \wedge : q)$

$\equiv [(p \vee : p) \wedge q] \vee (p \wedge : q)$ (Distributive law)

$\equiv (T \wedge q) \vee (p \wedge : q)$ (Complement law)

$\equiv q \vee (p \wedge : q)$ (identity law)

$\equiv (q \vee p) \wedge (q \vee : q)$ (Distributive law)

$\equiv (q \vee p) \wedge T$ (Complement law)

$\equiv q \vee p$ (Identity law)

$\equiv p \vee q$ (Commutative law)

Hence, $LHS = RHS$

(iv) Without using truth table proving

$: [(p \vee : q) \rightarrow (p \wedge : q)] \equiv (p \vee : q) \wedge (: p \vee q)$

So, left hand side will be $\Rightarrow : [(p \vee : q) \rightarrow (p \wedge : q)]$

$\equiv (p \vee : q) \wedge : (p \wedge : q)$ (Negation of implication)

$\equiv (p \vee : q) \wedge [: p \vee : (: q)]$ (Negation of conjunction)

$\equiv (p \vee : q) \wedge (: p \vee q)$ (Negation of negation)

Hence, $LHS = RHS$.