EXERCISE - 4 : PREVIOUS YEARS JEE ADVANCED QUESTIONS

Heat, Temperature and Calorimetry

1. A cubical block of co-efficient of linear expansion α_s, is submerged partially inside a liquid of co-efficient of volume expansion γ_ℓ. On increasing the temperature of the system ΔT, the height of the cube inside the liquid remains unchanged. Find the relation between α_s, and γ_l. **(2004)**

Ans. $\gamma_\ell = 2\alpha_s$

Sol. When the temperature is increased, volume of the cube will increase while density of liquid will decrease. The depth upto which the cube is submerged in the liquid remains the same. Upthrust = Weight. Therefore, upthrust should not change

$F = F'$

$\therefore V_i\rho_L g = V_i'\rho_L' g$ (V_i = volume immersed)

$$\therefore (Ah_i)(\rho_L)(g) = A(1+2\alpha_s\Delta T)(h_i)\left(\frac{\rho L}{1+\gamma_\ell \Delta T}\right)g$$

Solving this equation, we get $\gamma_I = 2\alpha_s$

2. A piece of ice (heat capacity = 2100 J kg^{-1} $°C^{-1}$ and latent heat = 3.36×10^5 J kg^{-1}) of mass m gram is at –5°C at atmospheric pressure. It is given 420 J of heat so that the ice starts melting. Finally when the ice–water mixture is in equilibrium, it is found that 1 g of ice has melted. Assuming there is no other heat exchange in the process, the value of m is **(2010)**

Ans. (8.00)

Sol. Language of question is slightly wrong. As heat capacity and specific heat are two different physical quantities. Unit of heat capacity is J-kg^{-1} not J-kg^{-1} $°C^{-1}$. The heat capacity given in the question is really the specific heat. Now applying the heat exchange equation.

$420 = (m \times 10^{-3})(2100)(5) + (1 \times 10^{-3})(3.36 \times 10^5)$

Solving this equation we get,

$m = 8$ g

$\therefore$ The correct answer is 8.

3. Steel wire of length L at 40°C is suspended from the ceiling and then a mass m is hung from its free end. The wire is cooled down from 40°C to 30°C to regain its original length L. The coefficient of linear thermal expansion of the steel is 10^{-5}/°C, Young's modulus of steel is 10^{11}N/m^2 and radius of the wire is 1 mm. Assume that L>> diameter of the wire. Then the value of m in kg is nearly. **(2011)**

Ans. (3.00)

Sol. $\Delta L_1 = \frac{FL}{AY} = \frac{mgL}{\pi r^2 Y}$ = Increase in length

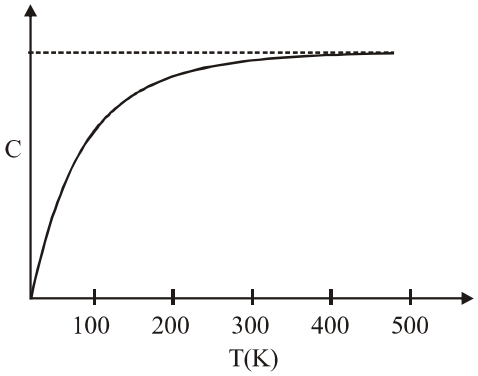
$\Delta L_2 = L\alpha\Delta\theta$ = Decrease in length

To regain its original length, $\Delta L_1 = \Delta L_2$

$$\therefore \frac{mgL}{\pi r^2 Y} = L\alpha\Delta\theta \Rightarrow \therefore m = \left(\frac{r^2 Y\alpha\Delta\theta}{g}\right)$$

Substituting the values we get, $m \simeq 3$ kg

4. The figure below shows the variation of specific heat capacity(C) of a solid as a function of temperature (T). The temperature is increased continuously from 0 to 500 K at a constant rate. Ignoring any volume change, the following statement(s) is (are) correct to reasonable approximation. **(2013)**



(a) the rate at which heat is absorbed in the range 0–100 K varies linearly with temperature T

(b) heat absorbed in increasing the temperature from 0–100 K is less than the heat required for increasing the temperature from 400–500 K

(c) there is no change in the rate of heat absorbtion in the range 400–500 K

(d) the rate of heat absorption increases in the range 200–300 K

Ans. (b, c, d)

Sol. $Q = mCT \Rightarrow \frac{dQ}{dt} = mc\frac{dT}{dt}$

R = rate of absorption of heat $= \frac{dQ}{dt} \propto C$

(i) in 0 – 100 K, C increase, so R increases but not linearly.

(ii) $\Delta Q = mC\Delta T$ as C is more in (400 – 500K) than (0 – 100K) so heat is increasing.

(iii) C remains constant so there no change in R from (400 – 500K)

(iv) C increases so R increases in range (200 – 300K)

5. A current carrying wire heats a metal rod. The wire provides a constant power (P) to the rod. The metal rod is enclosed in an insulated container. It is observed that the temperature (T) in the metal rod changes with time (t) as $T(t) = T_0(1+\beta t^{1/4})$ where β is a constant with appropriate dimension while T_0 is a constant with dimension of temperature. The heat capacity of metal is :

(2019)

(a) $\frac{4P(T(t)-T_0)^4}{\beta^4 T_0^5}$ (b) $\frac{4P(T(t)-T_0)^3}{\beta^4 T_0^4}$

(c) $\frac{4P(T(t)-T_0)}{\beta^4 T_0^2}$ (d) $\frac{4P(T(t)-T_0)^2}{\beta^4 T_0^3}$

Ans. (b)

Sol. dQ=HdT

$$\frac{dQ}{dt} = H.\frac{dT}{dt}$$

$$P = H.T_0.\beta.\frac{1}{4}.t^{-3/4}$$

$$\frac{4P}{T_0.\beta} = t^{-3/4}.H$$

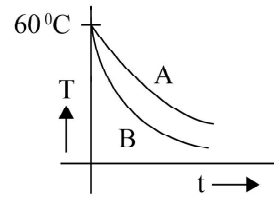
Now $T - T_0 = T_0\beta t^{1/4}$

So $t^{3/4} = \left[\frac{T-T_0}{T_0\beta}\right] \therefore H = \frac{4P(T-T_0)^3}{T_0^4\beta^4}$

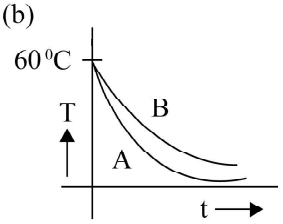
6. Two identical beakers A and B contain equal volumes of two different liquids at 60°C each and left to cool down. Liquid in A has density of 8×10^2 kg/m^3 and specific heat of 2000 Jkg^{-1}K^{-1} while liquid in B has density of 10^3 kg m^{-3} and specific heat of 4000 Jkg^{-1}K^{-1}. Which of the following best describes their temperature versus time graph schematically? (assume the emissivity of both the beakers to be the same)

(2019)

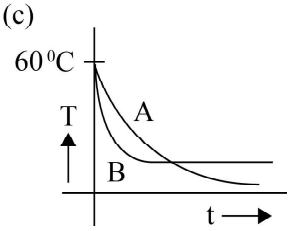
(a)



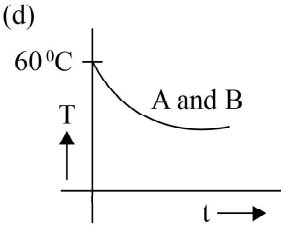
(b)



(c)



(d)



Ans. (b)

Sol. Rate of Heat loss = $mS\left(\frac{dT}{dt}\right) = e\sigma AT^4$

$$-\frac{dT}{dt} = \frac{e\sigma \times A \times T^4}{\rho \times Vol. \times S} \Rightarrow -\frac{dT}{dt} \propto \frac{1}{\rho S}$$

$$\frac{\left(-\frac{dT}{dt}\right)_A}{\left(-\frac{dT}{dt}\right)_B} = \frac{\rho_A}{\rho_B} \times \frac{S_A}{S_B} = \frac{10^3}{8\times10^2} \times \frac{4000}{2000} = 2.5 > 1$$

$$\Rightarrow \left(-\frac{dT}{dt}\right)_A > \left(-\frac{dT}{dt}\right)_B$$

So, A cools down at faster rate.

7. A liquid at 30° C is poured very slowly into a open Calorimeter that is at temperature of 110°C. The boiling temperature of the liquid is 80°C. It is found that the first 5 gm of the liquid completely evaporates. After pouring another 80 gm of the liquid the equilibrium temperature is found to be 50°C. The ratio of the Latent heat of the liquid to its specific heat will be _____ °C. (Neglect the heat exchange with surrounding]

(2019)

Ans. 270.00

Sol. CASE-I –If calorimeter is open and after evaporation liquid escapes

$5\times S\times 50\times 5L = W\times 30$

$80\times S\times 20 = W\times 30$

$80\times S\times 20 = 5\times S\times 50\times 5L$

$5L = 1350S$

$\frac{L}{S} = 270$

CASE-II –If calorimeter is closed (vapour not allowed to escape)

Heat gain = Heat loss

$5S(80-30)+5L = W(110-80)$

S=Specific heat of liquid

L=Latent heat of liquid

W=Water equivalent of calorimeter

$250S+5L = W\times 30$ .....(1)

Now 80gm liquid is poured

Heat gain= Heat loss

Here final temperature = 50°C

$80\times S\times 20 = 5L+5S\times 30+W\times 30$ .....(2)

From (1) & (2)

$\frac{L}{S} = 120$

8. When the temperature of a metal wire is increased from 0ºC to 10ºC, its length increases by 0.02%. The percentage change in its mass density will be closest to : **(2020)**

(a) 0.06 (b) 0.008

(c) 2.3 (d) 0.8

Ans. (a)

Sol. $\Delta\ell = \ell\propto \Delta t$

$\alpha = \frac{\Delta\ell}{\ell\Delta t} = \frac{0.02}{100\times 10} = 2\times 10^{-5}$

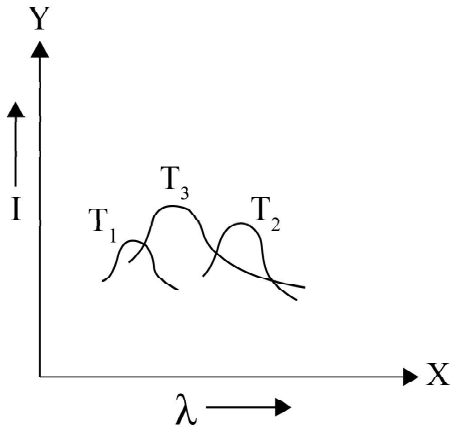
$v = 3\alpha = 6\times 10^{-5}$

Now, $\frac{\Delta v}{v}\times 100 = \gamma.\Delta t.100 = 6\times 10^{-5}\times 10\times 100$

$= 6\times 10^{-2} = 0.06$

Heat Transfer

9. The plots of intensity versus wavelength for three black bodies at temperatures T_1, T_2 and T_3 respectively are as shown. Their temperature are such that **(2000)**



(a) $T_1 > T_2 > T_3$ (b) $T_1 > T_3 > T_2$

(C) $T_2 > T_3 > T_1$ (d) $T_3 > T_2 > T_1$

Ans. (b)

Sol. Wien's displacement law for a perfectly black body is

$\lambda_m T$ = constant = Wien's constant b

Here, λ_m is the minimum wavelength corresponding to maximum intensity I.

or $\lambda_m \propto \frac{1}{T}$

From the figure $(\lambda_m)_1 < (\lambda_m)_3 < (\lambda_m)_2$

Therefore, $T_1 > T_3 > T_2$

10. When a block of iron floats in mercury at 0ºC, fraction k_1 of its volume is submerged, while at the temperature 60ºC, a fraction k_2 is seen to be submerged. If the coefficient of volume expansion of iron is γ_{Fe} and that of mercury is γ_{Hg}, then the ratio k_1/k_2 can be expressed as **(2001)**

(a) $\frac{1+60\gamma_{Fe}}{1+60\gamma_{Hg}}$ (b) $\frac{1-60\gamma_{Fe}}{1+60\gamma_{Hg}}$

(c) $\frac{1+60\gamma_{Fe}}{1-60\gamma_{Hg}}$ (d) $\frac{1+60\gamma_{Hg}}{1+60\gamma_{Fe}}$

Ans. (a)

Sol. Let volume of block = V

density of block = d_1

density of mercury = d_2

At 0 °C

upthrust = weight

$\Rightarrow K_1 Vd_2 g = Vd_1 g$

$\Rightarrow K_1 = \frac{d_1}{d_2}$

Of temperature increase then volume and density changes.

Let volume of block at 60°C = V^1

density of block at 60°C = d_1^1

density of mercury at 60°C = d_2^1

At 60°C : upthrust = weight

$\Rightarrow K_2 V^1 d_2^1 g = V^1 d_1^1 g$

$\Rightarrow K_2 = \frac{d_1^1}{d_2^1}$

$d_1^1 = \frac{d_1}{1+\gamma_{Fe}(60)}$

$d_2^1 = \frac{d_2}{1+\gamma_{Hg}(60)}$

Hence $\frac{K_1}{K_2} = \frac{d_1}{d_2}\left(\frac{d_2^1}{d_1^1}\right)$

$= \frac{1+\gamma_{Fe}(60)}{1+\gamma_{Hg}(60)}$

11. An ideal black-body at room temperature is thrown into a furnace. It is observed that **(2002)**

(a) initially it is the darkest body and at later times the brightest

(b) it is the darkest body at all times

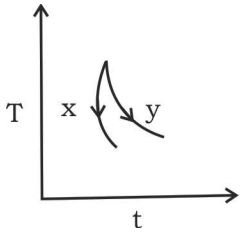
(c) it cannot be distinguished at all times

(d) initially it is the darkest body and at later times it cannot be distinguished.

Ans. (a)

Sol. Black body radiates maximum number of wavelength and maximum energy if all other conditions (eg. Temperature surface area etc.) are same. So, when the temperature of black body becomes equal to the temperature of the furnace, the black body will radiate maximum energy and it will be brightest of all. Initially, it will absorb all the radiant energy incident on it, so it is the darkest one.

12. The graph, shown in the adjacent diagram, represents the variation of temperature (T) of two bodies, x and y having same surface area, with time (t) due to the emission of radiation. Find the correct relation between the emissivity and absorptivity powers of the two bodies **(2003)**



(a) $E_x > E_y$ and $a_x < a_y$ (b) $E_x < E_y$ and $a_x > a_y$

(c) $E_x > E_y$ and $a_x > a_y$ (d) $E_x < E_y$ and $a_x < a_y$

Ans. (c)

Sol. Rate of cooling $\left(-\frac{dT}{dt}\right) \propto$ emissivity (e)

From the graph,

$\left(-\frac{dT}{dt}\right)_x > \left(-\frac{dT}{dt}\right)_y$

$\therefore E_x > E_y$

Further emissivity (E) = absorptive power (a) (good absorbers are good emitters also)

$\therefore a_x > a_y$

Hence, the correct answer is (c).

13. Two identical rods are connected between two conatiners. One of them is at 100°C containing water and another is at 0°C containing ice. If rods are connected in parallel then the rate of melting of ice is q_1 g/s. If they are connected in series then the rate is q_2. The ratio $\frac{q_2}{q_1}$ is **(2004)**

(a) 2 (b) 4

(c) $\frac{1}{2}$ (d) $\frac{1}{4}$

Ans. (d)

Sol. $\frac{dQ}{dt} = L\left(\frac{dm}{dt}\right)$

or $\frac{\text{Temperature difference}}{\text{Thermal resistance}} = L\left(\frac{dm}{dt}\right)$

or $\frac{dm}{dt} \propto \frac{1}{\text{Thermal resistance}} = L\left(\frac{dm}{dt}\right)$

In the first case rods are in parallel and thermal resistance is $\frac{R}{2}$ while in second case rods are in series and thermal resistance is 2R.

$$\frac{q_1}{q_2}=\frac{2R}{\frac{R}{2}}=\frac{4}{1}$$

Hence, the correct option is (c).

14. Three discs, A, B and C having radii 2 m, 4 m and 6 m respectively are coated with carbon black on their outer surfaces. The wavelengths corresponding to maximum intensity are 300 nm, 400 nm and 500 nm, respectively. The power radiated by them are Q_A, Q_B and Q_C respectively **(2004)**

(a) Q_A is maximum (b) Q_B is maximum
(c) Q_C is maximum (d) $Q_A = Q_B = Q_C$

Ans. (b)

Sol. $Q \propto AT^4$ and $\lambda_m T$ = constant

Hence,

$$Q \propto \frac{A}{(\lambda_m)^4} \text{ or } Q \propto \frac{r^2}{(\lambda_m)^4}$$

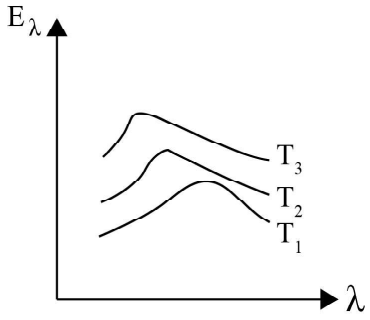
$$Q_A : Q_B : Q_C = \frac{(2)^2}{(3)^4} : \frac{(4)^2}{(4)^4} : \frac{(6)^2}{(5)^4}$$

$$= \frac{4}{81} : \frac{1}{16} : \frac{36}{625} = 0.05 : 0.0625 : 0.0576$$

i.e. Q_B is maximum.

Hence, the correct option is (b).

15. Variation of radiant energy emitted by sun, filament of tungsten lamp and welding arc as a function of its wavelength is shown in figure. **(2005)**



Which of the following option is the correct match?

(a) Sun-T_3, tungsten filament-T_1, welding arc-T_2
(b) Sun-T_2, tungsten filament-T_2, welding arc-T_3
(c) Sun-T_3, tungsten filament-T_2, welding arc-T_1
(d) Sun-T_1, tungsten filament-T_2, welding arc-T_3

Ans. (a)

Sol. $\lambda_m T$ = constant

From the graph $T_3 > T_2 > T_1$

Temperature of sun will be maximum

Also mention that temperature of welding arc is more than that of tungsten filament

Therefore, (a) is the correct option.

16. In which of the following processes, convection does not take place primarily? **(2005)**

(a) sea and land breeze
(b) boiling of water
(c) heating air around a furnace
(d) warming of glass of bulb due to filament.

Ans. (d)

Sol. Glass of bulb heats due to filament by radiation.

Hence, correct answer is (d).

17. A spherical body of area A and emissivity e = 0.6 is kept inside a perfectly black body. Energy radiated per second by the body at temperature T is **(2005)**

(a) 0.4 σAT^4 (b) 0.8 σAT^4
(c) 0.6 σAT^4 (d) 1.0 σAT^4

Ans. (c)

Sol. Energy radiated per second $E = e\sigma AT^4$

$E = 0.6\,\sigma AT^4$

18. Water of volume 2 litre in a container is heated with a coil of 1 kW at 27°C. The lid of the container is open and energy dissipates at rate of 160 J/s. In how much time temperature wil rise from 27°C to 77°C.

[Given specific heat of water is 4.2 kJ/kg] **(2005)**

(a) 7 min (b) 6 min 2 s
(c) 8 min 20s (d) 14 min

Ans. (c)

Sol. Energy gained by water (in 1s)

= energy supplied – energy lost = 1000 J – 160 J = 840 J

Total heat required to raise the temperature of water from 27°C to 77°C is $ms\Delta\theta$

Hence, the required time

$$t = \frac{ms\Delta\theta}{\text{rate by which energy is gained by water}}$$

$$= \frac{(2)(4.2\times10^3)(50)}{840} = 500\text{ s} = 8\text{ min }20\text{s}$$

19. Initially a black body at absolute temperature T is kept inside a closed chamber at absolute temperature T_0. Now the chamber is slightly opened to allow sun rays to enter. It is observed that temperatures T and T_0 remains constant. Which of the following statements is/are true? **(2006)**

(a) The rate of emission of energy from the black body remains the same

(b) The rate of emission of energy from the black body increases

(c) The rate of absorption of energy by the black body increases.

(d) The energy radiated by the black body equals the energy absorbed by it.

Ans. (a, d)

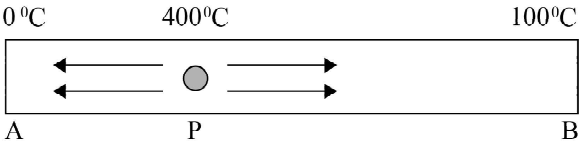
Sol. Since the temperature of black body is constant,

Total heat absorbed = total heat radiated.

20. A metal rod AB of length 10x has its one end A in ice at 0°C and the other end B in water at 100°C. If a point P on the rod is maintained at 400°C, then it is found that equal amounts of water and ice evaporate and melt per unit time. The latent heat of evaporation of water is 540 $calg^{-1}$ and latent heat of melting of ice is 80 $calg^{-1}$. If the point P is at a diatance of λx from the ice end A, find the value of λ. (Neglect any heat loss to the surrounding). **(2009)**

Ans. (0009)

Sol.



Heat will flow both sides from point P.

$$L_1\frac{dm_1}{dt}=\left(\frac{\text{Temperature difference}}{\text{Thermal resistance}}\right)=\frac{400}{\frac{(\lambda x)}{kA}} \quad ...(i)$$

$$L_1\frac{dm_1}{dt}=\frac{400-100}{(100-\lambda)\frac{x}{kA}} \quad ...(ii)$$

In about two equations, $\frac{dm_1}{dt}=\frac{dm_2}{dt}$ (given)

$L_1 = 80\ cal\ g^{-1}$ and $L_2 = 540\ cal\ g^{-1}$

Solving these two equations, we get λ = 9

21. Two spherical bodies A (radius 6 cm) and B (radius 18 cm) are at temperatures T_1 and T_2, respectively. The maximum intensity in the emission spectrum of A is at 500 nm and in that of B is at 1500 nm. Considering them to be black bodies, what will be the ratio of the rate of total energy radiated by A to that of B ? **(2010)**

Ans. (0009)

Sol. $\lambda_m \propto \frac{1}{T}$

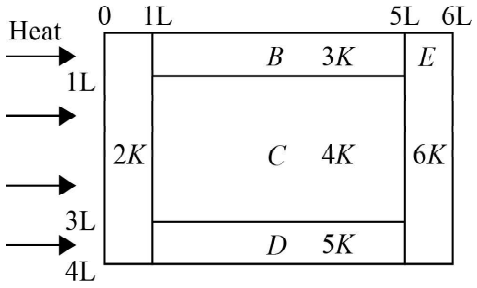
$$\therefore \frac{\lambda_A}{\lambda_B}=\frac{T_B}{T_A}=\frac{500}{1500}=\frac{1}{3}$$

$E \propto T^4 A$ (where, A = surface area $= 4\pi R^2$)

$\therefore E \propto T^4R^2$

$$\frac{E_A}{E_B}=\left(\frac{T_A}{T_B}\right)4\left(\frac{R_A}{R_B}\right)^2=(3)^4\left(\frac{6}{18}\right)^2=9$$

22. A composite block is made of slabs A, B, C, D and E of different thermal conductivities (given in terms of a constant K) and sizes (given in terms of length L) as shown in the figure. All slabs are of same width. Heat Q flows only from left to right through the blocks. Then in steady state **(2011)**



(a) heat flown through A and E slabs are same

(b) heat flown through slab E is maximum

(c) temperature difference across slab E is smallest

(d) heat flown through C = heat flow through B + heat flown through D.

Ans. (a, c, d)

Sol. Thermal resistance $R=\frac{\ell}{KA}$

$$\therefore RA=\frac{L}{(2K)(4Lw)} \text{ (Here w = width)}$$

$$=\frac{1}{8Kw},$$

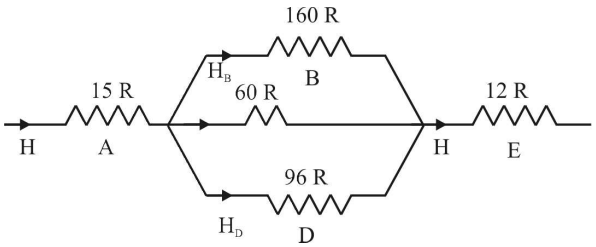
$$R_B = \frac{4L}{3K(Lw)} = \frac{4}{3Kw}$$

$$R_C = \frac{4L}{(4K)(2Lw)} = \frac{1}{2Kw}$$

$$R_D = \frac{4L}{(5K)(Lw)} = \frac{4}{5Kw}$$

$$R_E = \frac{L}{(6K)(Lw)} = \frac{1}{6Kw}$$

$R_A : R_B : R_C : R_D : R_E$
$= 15 : 160 : 60 : 96 : 12$
So, let us write, $R_A = 15\ R$, $R_B = 160\ R$ etc. and draw a simple electrical circuit as shown in figure



H = Heat current = Rate of heat flow.
$H_A = H_E = H$ (let)
In parallel current distributes in inverse ratio of resistance.

$$\therefore\ H_B : H_C : H_D = \frac{1}{R_B} : \frac{1}{R_C} : \frac{1}{R_D}$$

$$= \frac{1}{160} : \frac{1}{60} : \frac{1}{96} = 9 : 24 : 15$$

$$\therefore\ H_B = \left(\frac{9}{9+24+15}\right)H = \frac{3}{16}H$$

$$H_C = \left(\frac{24}{9+24+15}\right)H = \frac{1}{2}H$$

and $$H_D = \left(\frac{15}{9+24+15}\right)H = \frac{5}{16}H$$

$H_C = H_B + H_D$
Temperature difference (let us call it T)
= (Heat current) × (Thermal Resistance)
$T_A = H_A R_A = (H)(15R) = 15\ HR$

$$T_B = H_B R_B = \left(\frac{3}{16}H\right)(160\ R) = 30\ HR$$

$$T_C = H_C R_C = \left(\frac{1}{2}H\right)(60\ R) = 30\ HR$$

$$T_D = H_D R_D = \left(\frac{5}{16}H\right)(96\ R) = 30\ HR$$

$T_E = H_E R_E = (H)(12\ R) = 12\ HR$

23. Three very large plates of same area are kept parallel and close to each other. They are considered as ideal black surfaces and have very high thermal conductivity. The first and third plates are maintained at temperatures 2T and 3T respectively. The temperature of the middle (i.e., second) plate under steady state condition is : **(2012)**

(a) $\left(\frac{65}{2}\right)^{\frac{1}{4}} T$ (b) $\left(\frac{97}{4}\right)^{\frac{1}{4}} T$

(c) $\left(\frac{97}{2}\right)^{\frac{1}{4}} T$ (d) $(97)^{\frac{1}{4}} T$

Ans. (c)

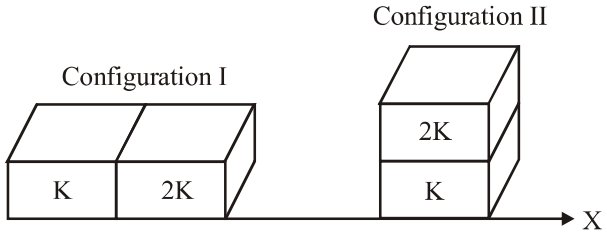
Sol. Let temperature of middle plate in steady state is T_0
$Q_1 = Q_2$
Q = net rate of heat flow

$$\therefore\ \sigma A(3T)^4 - \sigma A T_0^4 = \sigma A T_0^4 - \sigma A(2T)^4$$

Solving this equation, we get

$$T_0 = \left(\frac{97}{2}\right)^{\frac{1}{4}} T$$

24. Two rectangular blocks, having indentical dimensions, can be arranged either in configuration I or in configuration II as shown in the figure. One of the blocks has thermal conductivity K and the other 2K. The temperature difference between the ends along the x-axis is the same in both the configurations. It takes 9s to transport a certain amount of heat from the hot end to the cold end in the configuration I. The time to transport the same amount of heat in the configuration II is : **(2013)**



(a) 2.0 s (b) 3.0 s
(c) 4.5 s (d) 6.0 s

Ans. (a)

Sol. $R_I = R_1 + R_2 = \left(\frac{\ell}{KA}\right) + \left(\frac{\ell}{2KA}\right) = \frac{3}{2}\left(\frac{\ell}{KA}\right)$

$$\frac{1}{R_{II}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{KA}{\ell} + \frac{2KA}{\ell}$$

or $R_{II} = \frac{\ell}{3KA} = \frac{R_i}{4.5}$

Since thermal resistance R_{II} is 4.5 time less than thermal resistance R_I.

$$\therefore t_{II} = \frac{t_I}{4.5} = \frac{9}{4.5}s = 2s$$

25. Heater of an electric kettle is made of a wire of length L and diameter d. It takes 4 minutes to raise the temperature of 0.5 kg water by 40 K. This heater is replaced by a new heater having two wires of the same material, each of length L and diameter 2d. The way these wires are connected is given in the options. How much time in minutes will it take to raise the temperature of the same amount of water by 40 K?

(2014)

(a) 4 if wires are in parallel (b) 2 if wires are in series

(c) 1 if wires are in series (d) 0.5 if wires are in parallel

Ans. (b, d)

Sol. Joules law,

$$H = I^2Rt = \frac{V^2t}{R}$$

Also,

$$R = \rho\frac{l}{A} \Rightarrow R \propto \frac{1}{A} \Rightarrow R \propto \frac{1}{r^2}$$

$$\therefore R' = \frac{R}{4}$$

$$parallel \Rightarrow R_p = \left(\frac{1}{R/4} + \frac{1}{R/4}\right)^{-1} = \frac{R}{8}$$

$$series \Rightarrow R_s = \frac{R}{4} + \frac{R}{4} = \frac{R}{2}$$

$$\therefore \frac{V^2t}{R} = \frac{V^2t_p}{R_p} = \frac{V^2t_s}{R_s}$$

$$\Rightarrow t_p = 0.5 \text{ min}, t_s = 2 \text{ min}$$

26. Two spherical stars A and B emit black body radiation. The radius of A is 400 times that of B and A emits 10^4 times the power emitted from B. The ratio $\left(\frac{\lambda_A}{\lambda_B}\right)$ of their wavelengths λ_A and λ_B at which the peaks occur in their respective radiation curves is **(2015)**

Ans. (2)

Sol. Let T_A and T_B be the temperatures and R_A and R_B be the radii of the stars A and B. Stefan's law gives the power radiated by the two stars as

$$P_A = (dQ/dt)_A = \sigma 4\pi R_A^2 T_A^4, \ldots(1)$$

$$P_B = (dQ/dt)_B = \sigma 4\pi R_B^2 T_B^4, \ldots(2)$$

Given $R_A = 400\,R_B$ and $P_A = 10^4\,P_B$. Divide equation (2) by (1) to get

$$\frac{T_B}{T_A} = \left(\frac{P_B}{P_A}\right)^{1/4}\left(\frac{R_A}{R_B}\right)^{1/2} = 2 \ldots(3)$$

Apply Wien's displacement law to get

$$\lambda_A/\lambda_B = T_B/T_A = 2$$

27. The ends Q and R of two thin wires, PQ and RS, are soldered (joined) together. Initially each of the wires has a length of 1 m at 10°C. Now the end P is maintained at 10°C, while the end S is heated and maintained at 400°C. The system is thermally insulated form its surroundings. If the thermal conductivity of wire PQ is twice that of the wire RS and the coefficient of linear thermal expansion of PQ is $1.2 \times 10^{-5}\,K^{-1}$, the change in length of the wire PQ is

(2016)

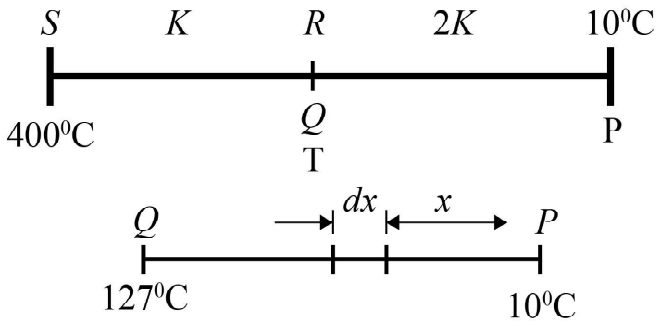
(a) 0.70 mm (b) 0.90 mm

(c) 1.56 mm (d) 2.34 mm

Ans. (a)

Sol. Let the temperature of the junction, in steady state, be T.

$KA(400 - T) = 2KA(T - 10)$



or, $T = \frac{380}{3} \simeq 127°C$.

Temperature gradient along PQ is $= \frac{127-10}{1} = 117°C/m$.

Temperature at a distance x from the end P is

$\theta = 10 + 117x$

Consider an elemental length dx as shown. Due to rise in temperature, the elongation in length of the element will be

$dl = \alpha \,.\, dx\,(10 + 117x - 10) = 117\,\alpha\, xdx$

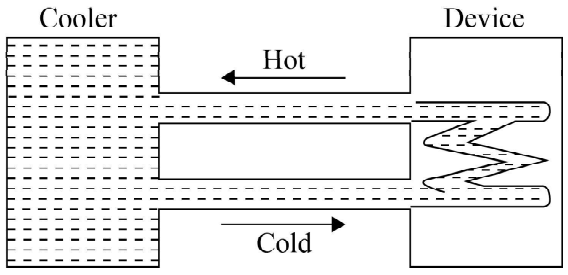
$\therefore$ Sum of elongation in each such element will be

$$\Delta l = 117\alpha \int_0^1 xdx = \frac{117}{2} \times 1.2 \times 10^{-5} \times 1 \simeq 70 \times 10^{-5}\, m$$

$= 0.7$ mm.

28. A water cooler of storage capacity 120 litres can cool water at a constant rate of P watts. In a closed circulation system (as shown schematically in the figure), the water from the cooler is used to cool an external device that generates constantly 3 kW of heat (thermal load). The temperature of water fed into the device cannot exceed 30°C and the entire stored 120 litres of water is initially cooled to 10°C. The entire system is thermally insulated. The minimum value of P (in watts) for which the device can be operated for 3 hours is

(2016)



(Specific heat of water is 4.2 kJ kg^{-1} K^{-1} and the density of water is 1000 kg m^{-3})

(a) 1600 (b) 2067

(c) 2533 (d) 3933

Ans. (b)

Sol. Let $P_d = 3$ kW be the power generated by the device. Let the water temperature increases from T to T + ΔT in a time interval Δt. In this time interval, the energy generated by the device is $P_d\Delta t$, the energy pumped out by the cooler is PΔt and the energy used to increase the temperature of water is mSΔT. The energy of the system is conserved because it is thermally insulated i.e.,

$P_d\Delta t = P\Delta t + mS\Delta T$,

which gives

$$P = P_d - \frac{mS\Delta T}{\Delta t} = P_d - \frac{\rho VS\Delta T}{\Delta t} . \qquad (1)$$

The minimum value of P occurs when ΔT attains its maximum allowed value i.e., $\Delta T = 30°C - 10°C = 20°C$. Substitute the values of various parameters in equation (1) to get

$$P = 3\times10^3 - \frac{(1000)\left(120\times10^{-3}\right)\left(4.2\times10^3\right)(20)}{3\times3600}$$

$= 2067$ W.

29. A metal is heated in a furnace where a sensor is kept above the metal surface to read the power radiated (P) by the metal. The sensor has a scale that displays $\log_2(P/P_0)$, where P_0 is a constant. When the metal surface is at a temperature of 487°C, the sensor shows a value 1. Assume that the emissivity of the metallic surface remains constant. What is the value displayed by the sensor when the temperature of the metal surface is raised to 2767°C?

(2016)

Ans. (9)

Sol. The power radiated by a metal surface is given by $P = \sigma eAT^4$, where σ is Stefan's constant, e is emissivity of the metal, A is surface area and T is the absolute temperature of the metal. Thus, the powers radiated at temperatures $T_1 = 487°C = 487 + 273 = 760$ K and $T_2 = 2767°C = 3040$ K are

$P_1 = \sigma eA(760)^4$,

$P_2 = \sigma eA(3040)^4$,

The sensor's measurement for these radiated powers are

$$x_1 = \log_2(P_1/P_0) = \log_2\left(\sigma eA(760)^4/P_0\right), \quad ...(1)$$

$$x_2 = \log_2(P_2/P_0) = \log_2\left(\sigma eA(3040)^4/P_0\right). \quad ...(2)$$

It is given that $x_1 = 1$. Subtract equation (1) from (2) and simplify to get

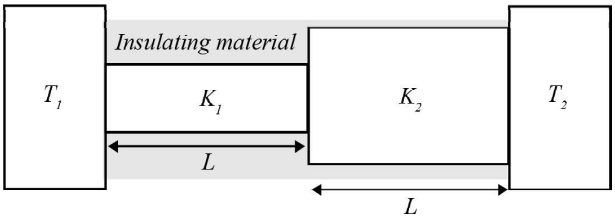
$$x_2 = x_1 + \log_2\left(\sigma eA(3040)^4/P_0\right)$$

$$-\log_2\left(\sigma eA(760)^4/P_0\right)$$

$$= 1 + \log_2\left(\frac{(3040)^4}{(760)^4}\right) = 1 + 4\log_2 4 = 9$$

30. Two conducting cylinders of equal length but different radii are connected in series between two heat baths kept at temperatures $T_1 = 300$ K and $T_2 = 100$ K, as shown in the figure. The radius of the bigger cylinder is twice that of the smaller one and the thermal conductivities of the materials of the smaller and the larger cylinders are K_1 and K_2 respectively. If the temperature at the junction of the two cylinders is the steady state is 200 K, then K_1/K_2 = _____.

(2018)



Ans. 4

Sol. The temperature of the source is $T_1 = 300$ K and that of the sink is $T_2 = 100$ K. The temperature at the junction of two cylinders is T = 200 K. The radius of the bigger cylinder, r_2, is twice the radius of the smaller cylinder, r_1 i.e., $r_2 = 2r_1$

The rate of heat conduction through a material, with conductivity K, cross-section area A, length Δx, and temperature difference between the two ends ΔT, is given by

$$\frac{\Delta Q}{\Delta t} = \frac{KA\Delta T}{\Delta x}$$

Thus, the rates of heat conduction through the two cylinders are

$$\frac{\Delta Q_1}{\Delta t} = \frac{K_1\left(\pi r_1^2\right)\left(T_1 - T\right)}{L_1},$$

$$\frac{\Delta Q_2}{\Delta t} = \frac{K_2\left(\pi r_2^2\right)\left(T - T_2\right)}{L_2}.$$

There is no heat loss from the cylinders because they are covered with an insulating material. Also, there is no heat accumulation in steady state. Thus, $\Delta Q_1/\Delta t = \Delta Q_2/\Delta t$, which gives

$$\frac{K_1}{K_2} = \frac{r_2^2\left(T - T_2\right)/L_2}{r_1^2\left(T_1 - T\right)/L_1} = \frac{\left(2r_1\right)^2\left(200 - 100\right)/L}{r_1^2\left(300 - 200\right)/L} = 4$$