

Solving Problems (Based on Quadratic Equations) – Summary Notes

Points to Remember

1. Method of solving problems on quadratic equations:

(i) Denote the unknown quantity in the given problem by x .

(ii) Form the quadratic equation with the help of the given condition or statement.

(iii) The solve for x .

Then the x will be the required answer.

Note: Lengths, areas, volume, money etc. is always taken as positive, neglecting the negative values.

Example 1:

The difference between two integers is 4. Their product is 221. Find the numbers.

Solution:

Let the required natural numbers be x and $(4-x)$.

Then, $x(4-x) = 221$

$$4x - x^2 = 221$$

$$\Leftrightarrow x^2 - 4x + 221 = 0$$

$$\Leftrightarrow x^2 - 17x + 13x + 221 = 0$$

$$\Leftrightarrow x(x-17) - 13(x-17) = 0$$

$$\Leftrightarrow (x-17)(x-13) = 0$$

$$\Leftrightarrow x = 17 \text{ or } x = 13$$

Hence, the required numbers are 17 and 13.

Example 2

Two years ago, a man's age was three times the square of his son's age. In three years' time, his age will be four times his son's age. Find their present ages.

Solution:

Let the present age of the man be x years, and that of his son be y years.

Two years ago,

The age of the man $= (x-2)$ years

and the age of his son $= (y-2)$ years

According to the given statement,

$$x-2 = 3(y-2)^2 \quad \dots(i)$$

After 3 years from now,

The age of the man $= (x+3)$ years

and the age of his son = $(y+3)$ years

According to the given statement,

$$x+3=4(y+3)$$

$$\Rightarrow x+3=4y+12$$

$$\Rightarrow x=4y+9 \quad \dots (ii)$$

Substituting, the value of x from (ii) in (i), we get:

$$3(y-2)^2 = 4y+9-2$$

$$\Rightarrow 3(y^2 - 4y + 4) = 4y + 7$$

$$\Rightarrow 3y^2 - 12y + 12 - 4y - 7 = 0$$

$$\Rightarrow 3y^2 - 16y + 5 = 0$$

$$\Rightarrow 3y^2 - 15y - y + 5 = 0$$

$$\Rightarrow 3y(y-5) - 1(y-5) = 0$$

$$\Rightarrow (y-5)(3y-1) = 0$$

$$\Rightarrow y-5=0 \text{ or } 3y-1=0$$

$$\Rightarrow y=5 \text{ or } y=\frac{1}{3}$$

But, $y=\frac{1}{3}$ is not possible, $\therefore y=5$

From (ii), $x=4 \times 5 + 9 = 29$.

$\therefore$ The present age of the man = 29 years and the present age of his son = 5 years.

Example 3

(i) A train covers a distance of 780 km at x km/hr. Had the speed been $(x-5)$ km/hr, the time taken to cover the same distance would have been increased by 1 hour. Write down an equation in x and solve it to evaluate x .

Solution:

(i) Total distance = 780 km

Speed of train = x km/hr

$$\therefore \text{Time taken} = \frac{780}{x} \text{ hrs.}$$

In second case,

Speed of the train = $(x-5)$ km/hr.

$$\text{Time taken} = \frac{780}{x-5} \text{ hr.}$$

According to the condition,

$$= \frac{780}{x-5} - \frac{780}{x} = 1$$

$$\Rightarrow 780 \left(\frac{1}{x-5} - \frac{1}{x} \right) = 1$$

$$\Rightarrow 780 \left(\frac{x-x+5}{x(x-5)} \right) = 1$$

$$\Rightarrow \frac{780 \times (5)}{x^2 - 5x} = 1$$

$$\Rightarrow 3900 = x^2 - 5x$$

$$\Rightarrow x^2 - 5x - 3900 = 0$$

$$\Rightarrow x^2 - 65x + 60x - 3900 = 0$$

$$\Rightarrow x(x-65) + 60(x-65) = 0$$

$$\Rightarrow (x-65)(x+60) = 0$$

[By cross multiplication]

$$\left\{ \begin{array}{l} \because -5 = -65 + 60 \\ -3900 = -65 \times 60 \end{array} \right\}$$

[Zero Product Rule]

Either $x - 65 = 0$, then $x = 65$

or $x + 60 = 0$, then $x = -60$, But it does not satisfy it.

$\therefore x = 65$.

- (ii) A train covers a distance of 600 km at x km/hr. Had the speed been $(x+20)$ km/hr, the time taken to cover the same distance would have been reduced by 5 hours, write down an equation in x and solve it to evaluate x .

Solution:

Total distance = 600 km

Speed of the train = x km/hr.

$$\therefore \text{Time taken} = \frac{600}{x} \text{ hrs.}$$

In second case,

Speed of the train = $(x+20)$ km/hr.

$$\text{Time taken} = \frac{600}{x+20} \text{ hrs.}$$

According to the condition,

$$\frac{600}{x} - \frac{600}{x+20} = 5$$

$$\Rightarrow 600 \left[\frac{1}{x} - \frac{1}{x+20} \right] = 5$$

$$\Rightarrow 600 \left[\frac{x+20-x}{x(x+20)} \right] = 5$$

$$\Rightarrow \frac{600 \times 20}{x(x+20)} = 5$$

$$\Rightarrow 5x(x+20) = 600 \times 20 \quad (\text{By cross multiplication})$$

$$\Rightarrow x(x+20) = 600 \times 4 \quad (\text{Dividing by 5})$$

$$\Rightarrow x^2 + 20x = 2400$$

$$x^2 + 20x - 2400 = 0.$$

$$\Rightarrow x^2 + 60x - 40x - 2400 = 0$$

$$\begin{cases} \because 60 - 40 = 20 \\ 60 \times (-40) = -2400 \end{cases}$$

$$\Rightarrow x(x+60) - 40(x+60) = 0$$

$$\Rightarrow (x+60)(x-40) = 0 \quad (\text{Zero Product Rule})$$

Either $x+60=0$, then $x=-60$

Which is not possible being in negative or $x-40=0$, then $x=40$

$\therefore$ Speed of the train = 40 km/hr.

Example 4

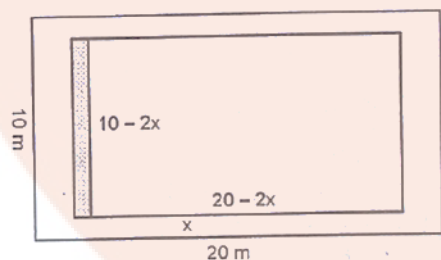
Within a rectangular garden 10 m wide and 20 m long, we wish to pave a walk around the borders of uniform width so as to leave an area of $96 m^2$ for flowers. How wide should the walk be?

Solution:

Length of garden = 20 m

Width of garden = 10 m

Let width of the walk = x m



$\therefore$ Inner length = $(20 - 2x)m$ and width = $(10 - 2x)m$

According to the condition,

$$(20 - 2x)(10 - 2x) = 96$$

$$\Rightarrow 200 - 40x - 20x + 4x^2 - 96 = 0$$

$$\Rightarrow 4x^2 - 60x + 200 - 96 = 0$$

$$\Rightarrow 4x^2 - 60x + 104 = 0$$

$$\Rightarrow x^2 - 15x + 26 = 0 \quad (\text{Dividing by 4})$$

$$\Rightarrow x^2 - 13x - 2x + 26 = 0$$

$$\Rightarrow x(x-13) - 2(x-13) = 0$$

$$\Rightarrow (x-13)(x-2) = 0 \quad (\text{Zero Product Rule})$$

Either $x - 13 = 0$, then $x = 13$

But it is not possible.

or $x - 2 = 0$, then $x = 2$

Hence, width of walk = 2 m.

Example 5

Rs. 6400 were divided equally among x persons. Had this money been divided equally among $(x+14)$ persons, each would have got Rs. 28 less. Find the value of x .

Solution:

Total money = Rs. 6400

In first case, no. of persons = x

$$\therefore \text{Amount to each person} = \text{Rs. } \frac{6400}{x}$$

In second case, no. of persons = $x+14$

$$\text{Amount to each person} = \text{Rs. } \left(\frac{6400}{x+14} \right)$$

According to the condition,

$$\frac{6400}{x} - \frac{6400}{x+14} = 28$$

$$\Rightarrow 6400 \left[\frac{1}{x} - \frac{1}{x+14} \right] = 28$$

$$\Rightarrow 6400 \left[\frac{x+14-x}{x(x+14)} \right] = 28$$

$$\Rightarrow \frac{6400 \times 14}{x(x+14)} = \frac{28}{1}$$

$$\Rightarrow 28x(x+14) = 89600$$

$$\Rightarrow x(x+14) = \frac{89600}{28} = 3200$$

$$\Rightarrow x^2 + 14x - 3200 = 0$$

$$\Rightarrow x^2 + 64x - 50x - 3200 = 0 \quad \left\{ \begin{array}{l} \because 14 = 64 - 50 \\ -3200 = 64 \times (-50) \end{array} \right\}$$

$$\Rightarrow x(x+64) - 50(x+64) = 0$$

$$\Rightarrow (x+64)(x-50) = 0 \quad \text{[Zero Product Rule]}$$

Either $x + 64 = 0$, then $x = -64$. But it is not possible.

or $x - 50 = 0$, then $x = 50$

$\therefore x = 50$.